“Effect of grain scale properties on bulk deformation of granular deposit due to high speed projectile impact”

Sept. 16, 2012

Name of Principal Investigators: Takashi Matsushima
- e-mail address: tmatsu@kz.tsukuba.ac.jp
- Institution: University of Tsukuba
- Mailing Address: 1-1-1, Tennodai, Tsukuba, Ibaraki, 305-8573, Japan
- Phone: +81-29-853-5269
- Fax: +81-29-853-5207


Abstract:
In order to understand the behavior of granular deposit subjected to high-speed projectile impact, we perform some experiments and grain-scale numerical simulations. In the high-speed impact experiment, we adopted a new technology to measure the deceleration of projectile after penetrating into sand layer. The observed results together with simple one-dimensional model with Rankine-Hugoniot equations suggests the importance of grain-scale mechanics such as grain crushing to evaluate the evolution of material parameters. Then a series of numerical simulation using Discrete Element Method were performed and the compression behavior of granular materials was analyzed in terms of grain crushing. The results suggest the transition of three material phases; (1) elastic regime as a granular matter, (2) plastic regime due to grain crushing, and (3) Hugoniot solid regime, and they are the important ingredients for modeling the equation of state of granular materials subjected to high speed compression.

Introduction:
High speed impact problem has been of great concern in various science communities. In particular, the high speed projectile impact to granular deposit (e.g., sand deposit) has been extensively studied in many research fields. Planetary science, for example, requires the mechanical information on meteoroid impact to planets and asteroids in order to understand the formation and evolution process of their surface morphology and material [1, 2]. Material science has dealt with the high impact behavior for new material productions and processes [3]. Civil engineering researchers have been studied the stability of soil ground and earth structures against the projectile impact or explosion [4]. In spite of such tremendous history, we still do not fully understand the whole phenomena, primarily because the granular material itself is a material full of complexity [5,6], and also because some additional physics such as shock wave transmission [7] and material melting [8] are included in the impact phenomena.

According to Reference [2], the behavior of granular deposit due to high speed projectile impact is divided into three stages for better comprehension: (a) contact and compression stage, (b) excavation stage, and (c) modification stage. In stage (a), material compressibility plays an important role, while incompressible fluid model [9] is often assumed in stage (b). Stage (c) is related to the more solid-like nature of granular materials. These stages are characterized by the deformation velocity range of the material. In order to simulate the whole behavior it is inevitable not only to observe the phenomenon of each stage in details, but also to develop the unified description being valid throughout the wide range of deformation speed; from quasi-static deformation to high-speed deformation over 1km/sec.

Quasi-static deformation has been studied mainly in geotechnical engineering field, while high-rate shear has been studied in physics and powder engineering. In addition,
# Effect of Grain Scale Properties on Bulk Deformation of Granular Deposits Due to High Speed Projectile Impact

In order to understand the behavior of granular deposit subjected to high-speed projectile impact, we perform some experiments and grain-scale numerical simulations. In the high-speed impact experiment, we adopted a new technology to measure the deceleration of projectile after penetrating into sand layer. The observed results together with simple one-dimensional model with Rankine-Hugoniot equations suggest the importance of grain-scale mechanics such as grain crushing to evaluate the evolution of material parameters. Then a series of numerical simulation using Discrete Element Method were performed and the compression behavior of granular materials was analyzed in terms of grain crushing. The results suggest the transition of three material phases; (1) elastic regime as a granular matter, (2) plastic regime due to grain crushing, and (3) Hugoniot solid regime, and they are the important ingredients for modeling the equation of state of granular materials subjected to high speed compression.
high-impact problem associated with grain crushing and melting due to shock wave has been studied in astrophysics and military engineering. These fields have their own disciplines and there is no unified perspective so far. Therefore, it is quite important to make an interdisciplinary discussion over the societies [10].

This research project aims at better understanding and modeling the whole process of projectile impact to the granular deposit using experimental and numerical tools. First, we present some results of impact experiments in which a new technology is adopted to measure the deceleration of projectile after penetrating into sand layer. The observed velocity-displacement relation is compared with a conceptual one-dimensional model at first, and then is quantitatively evaluated with Rankine-Hugoniot equations. Through this analysis, it is pointed that the material parameters for the equation of state of granular materials should be examined in more details. In particular, considerable grain crushing is observed in the impact experiment, which has to be taken into account. For this purpose, a series of numerical simulation using Discrete Element Method have been performed, and the relation between the compressive stress and strain is examined in terms of grain crushing behavior. The results suggest the transition of three material phases; (1) elastic regime as a granular matter, (2) plastic regime due to grain crushing, and (3) Hugoniot solid regime.

Finally we summarize this annual report with some future research plan.

High-speed impact experiment:

Large scale vertical powder gun is used to perform impact experiments in which a cylindrical projectile (15mm in diameter and 26mm in length) vertically hit to a sand deposit in a PMMA container (100mm in inner diameter and 300mm in inner depth) with a speed of about 500 m/sec (Figs.1 and 2). Sand material is the Florida coastal sand largely consisting of quartz whose grain size ranges from 0.1mm to 1mm (Figs 3 and 4). A high resolution high speed camera is used to observe the surface deformation of the granular deposit (Figs.5 and 6). The earth pressure change due to projectile penetration is measured by high response pressure gauges. Small thermocouples are also used to heat transfer in the granular deposit. Moreover, an originally developed devices using electromagnetic induction (Fig.7) are applied to clarify the location of projectile after penetrating into the granular deposit. It allows us the multi-point measurement of the location of a moving magnet in opaque media like sands. We just embed a magnet in a projectile which results in a minimal disturbance of sand and projectile. The setup is relatively inexpensive and easy to handle. We have already confirmed that it has sufficient measurement accuracy.
Fig. 8 shows an example of the deceleration of the projectile during the penetration into the sand deposit.

In order to understand the feature of this time history of the projectile velocity $v$, we consider the following simple equation of motion [11,12]:

$$\frac{dv}{dt} = \alpha v^2 + \beta v + \gamma$$

(1)

where $\alpha$, $\beta$ and $\gamma$ are the positive constants. $\alpha v^2$ represents the drag force, $\beta v$ represents the viscous damping and $\gamma$ represents the constant friction, respectively. The analytical solution of this
differential equation is available. For \( \alpha \neq 0 \),

\[
  t = \begin{cases} 
    C + \frac{2}{2 \alpha v + \beta} \tan^{-1} \frac{2 \alpha v + \beta}{\sqrt{|D|}} & (D < 0) \\
    C + \frac{2}{2 \alpha v + \beta} \tan^{-1} \frac{2 \alpha v + \beta}{\sqrt{|D|}} & (D = 0) \\
    C - \frac{1}{\sqrt{|D|}} \ln \left( \frac{2 \alpha v + \beta - \sqrt{|D|}}{2 \alpha v + \beta + \sqrt{|D|}} \right) & (D > 0)
  \end{cases}
\]

where \( D = \beta^2 - 4 \alpha \gamma \). For \( \alpha = 0 \) and \( \beta \neq 0 \),

\[
  t = C - \frac{1}{\beta} \ln |\beta v + \gamma|
\]

Fig. 7 Penetration velocity measurement system using magnet-coil gage (left) and the examples of the output signals of coil (right)

Fig. 8 Time history of projectile velocity during penetration (left) and the relation between penetration velocity and penetration depth (right)
and for $\alpha = \beta = 0$ and $\gamma \neq 0$, 
$$v = -\gamma t + C \quad (4)$$
Moreover, the displacement of the projectile $u$ can be obtained by integrating $v$ for $t$. Fig. 9 shows some examples of the solution. Here we set $\nu(0) = 1$ and $du/dt \big|_{t=0} = (1/\nu)du/dt \big|_{t=0} = -1$. Compared with Fig. 8, it is realized that the initial stage of the penetration is mainly governed by the drag force term, and the latter stage is controlled by the other two terms.

Here we consider the one-dimensional Rankine-Hugoniot equations. the pressure applied to the projectile is 
$$P = P_0 = \rho_0 U u \quad (5)$$
where $\rho_0$ and $P_0$ is the pressure before impact ($P_0 \approx 0$), $U$ is the shock wave speed and $u$ is equal to the projectile speed if we assume that the stiffness of the projectile is sufficiently larger than that of sand deposit. According to the previous experimental data [13-16], the shock wave velocity of most of the materials can be described as follows:
$$U = C + Su \quad (6)$$
where $C$ is the sound speed of the material, and $S$ is the dimensionless material parameter. Accordingly we obtain
$$P = \rho_0 (C + Su)u \quad (7)$$
and together with the simple equation of motion (eq(4)),
$$\alpha = \frac{\rho_0}{\rho_p} \frac{S}{L_p} \quad , \quad \beta = \frac{\rho_0}{\rho_p} \frac{C}{L_p} \quad (8)$$
where $\rho_p$ and $L_p$ are the density and the length of the projectile, respectively.

In our impact experiment, $\rho_0 = 1.35(g/cm^3)$, $\rho_p = 3.48(g/cm^3)$, $L_p = 26(mm)$. The bulk sound speed of the sand deposit under a low confining pressure (1.0 to 3.0 (kPa)) can be roughly estimated as 50 to 100 (m/s) [17, 18]. Fig. 10 shows the analytical results using the above physical parameters. Good agreement with experimental observation (Fig. 8) is obtained when $S = 0.75$, which is about half of the typical value in rock materials ($S \approx 1.5$). This reduction may come from the nature of granular materials that contains a lot of voids. The bulk material parameters of granular assembly, $C$ and $S$ are different from those of solids consisting of the grains. For the first-order approximation neglecting the structural details, the difference is a function of void ratio, and in turn, it is largely related to the grain size distribution.

Fig. 11 shows the evidence of grain crushing after the impact experiment. Crushed grains are found not only in front of the projectile but also in its circumferential area. It is also found that the densely compacted crushed grains cone is formed in front of the
projectile. This experimental result implies the importance to consider three-dimensional deformation of granular materials. Also, the energy dissipation due to grain crushing and the evolution of grain size distribution are related to the origin of $C$ and $S$ (there is no reason why they are constant throughout the impact process). In the following sections, we explore the grain scale mechanics involving grain crushing.

**Numerical simulation using DEM (Discrete Element Method):**

In order to investigate the grain crushing behavior due to compressive force and the resulting evolution of grain size distribution, a series of one-dimensional compression test are performed using 2-D Discrete Element Method. Crushable grains are modeled with random packing of slightly polydispersed circular elements bonded together (Fig.12(a)). According to the Brazilian test (a single grain crushing test by compressing it with two horizontal plates), the crushing stress $\sigma_f = \frac{p_f}{D^2}$, where $p_f$ is the compression force and $D$ is the grain diameter, decreases with increasing grain size (Fig.12(b)). This tendency is basically in agreement with the real geological grains.

Using such grains, four specimens of different grain size distribution are prepared (B1, B2, B3 and B4 in Figs. 13 and 14). Then gradually increased vertical load, $p$, is applied to the specimens to obtain the relation between $p$ and void ratio $e$ (Fig.15). It is found that the grain crushing occurs at around $\sigma_v = 10$ to 20 (N/m), which is 1/2 or 1/3 of the single grain crushing stress, primarily because of the force chain structure formed in the granular assembly. Moreover, all the specimens converge into a unique curve after sufficient
vertical load application. Fig. 16 shows the initial and the final (\( p = 200(kN) \)) grain size distributions of all the specimens. The final distributions are almost unique, which is referred to as "critical grading" [19,20]. This critical distribution implies that a certain percentage of the larger grains survive under the very high compressive stress even though their crushing stress is rather smaller than the smaller grains. This is because the larger grains surrounded by smaller grains are subjected to the isotropic internal stress condition from a large number of contact points. Since the grain crushing is caused by the deviatoric stress component, the larger grains survive and eventually critical grading is attained. If such a mechanism works in various scale at the same time, the critical grading must have a fractal nature. Fig. 17 is the cumulative grain size distribution which indicates the distribution has the fractal dimension, \( D_f \) of about 1.8. The both edge of the distribution (related to the smallest and the largest grains) is affected by the computational limitation.

![Graph showing critical grading](image)

![Fractal nature of critical grading](image)
Fig. 15  Void ratio decrease with increasing vertical load

Fig. 16  Evolution of grain size distribution for four specimens

Fig. 17  Fractal grain size distribution
Grading change directly leads to the change of the stable void ratio range, which may cause the plastic compression observed in Fig.15. After getting the critical grading, grain crushing hardly occur, and the material behaves not as a granular material any more, but as a continuum solid with little pore. In this phase, usual Hugoniot equation of state can be applied.

Consequently, it is found to be important to consider the three phases; (1) elastic regime as a granular matter, (2) plastic regime due to grain crushing, and (3) Hugoniot solid regime. Based on these phases, more realistic equation of states of granular materials can be modeled.

**Future plan:**

We prolonged the research period by six months. During this extended period, we are planning to work on the following topics:

(1) A series of impact experiments widely changing the impact speed (from quasi-static to >1 km/s) are planned to be performed. Various advanced technologies are used to measure velocity, pressure, stress, temperature, etc. In particular, we try to clarify the grain crushing region and the resulting grain size distribution in terms of impact velocity.

(2) Numerical simulation of impact experiments using Smooth Particle Hydrodynamics (SPH) are planned to be performed. SPH is one of the meshless particle methods [21], which is suitable for simulating large deformation of granular materials. We can study three dimensional effect including the grain crushing in the circumferential area of the projectile and the formation of crushed grains cone observed in the impact experiment. This effect may result in the emergence of the term in Eq.(1).

**REFERENCES:**

1980.


**List of Publications:**


