Development and Application of Acoustic Metamaterials with Locally Resonant Microstructures

AFOSR grant #FA9550-10-1-0061
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AFOSR Annual Grantees’ Meeting
Arlington, VA
August 2, 2012
**Report Documentation Page**

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<th>13. SUPPLEMENTARY NOTES</th>
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<tr>
<th>14. ABSTRACT</th>
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<tr>
<td>32</td>
<td></td>
</tr>
</tbody>
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**Standard Form 298 (Rev. 8-98)**

Prescribed by ANSI Std Z39-18
Wave Propagation in Elastic Solids With Negative Mass Density or Modulus

What would happen if mass or modulus becomes negative?

• Dispersion equation:
  \[ q = \omega \sqrt{-\frac{\rho}{E}} = i \beta \omega \]

• Wave attenuates:
  \[ u = Ae^{i(qx - \omega t)} = Ae^{-\beta \omega x} e^{i\omega t} \]

  \[ \beta \] is attenuation factor

Wave cannot propagate without attenuation in elastic solids with negative mass density or modulus
Metamaterials with Local Resonators

Composite with Resonators

1D Lattice Model
Metamaterials with Negative Effective Mass

\[ u = Be^{i(\omega_0 x - \omega t)} \]

Effective mass for mass-in-mass lattice

\[ \rho_{\text{eff}} = \sqrt{\frac{k_2}{m_2}} \]

Displacement Envelope, (Local resonance \( \omega_0 \))

\[
\begin{align*}
\text{Number of unit cells} & \quad \omega/\omega_0 = 1.003 \\
& \quad \omega/\omega_0 = 1.073 \\
& \quad \omega/\omega_0 = 1.342 \\
& \quad \omega/\omega_0 = 2.013
\end{align*}
\]
Acoustic Metamaterial with Negative Effective Young’s Modulus

A Mechanical Unit Model and Its Representative Elastic Solid

\[ \sigma = E_{\text{eff}} \epsilon \]
Frequency-dependent Modulus (stress-strain curves)

\[ \frac{\varepsilon}{\varepsilon_0} \left( \frac{\sigma}{\sigma_0} \right) = E \]

(1) Static Modulus

(2) \( 0 < \omega < \omega^* \)

(3) Extreme Modulus: Very stiff

(4) Negative Modulus

(5) \( \omega^* < \omega < \omega_0 \)

(6) \( \omega > \omega_0 \)

\( \omega_0 = \sqrt{\frac{k_2}{m_2}} \) Is local resonance frequency

Strain

Stress

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Wave Attenuation in Metamaterial with Negative Effective Modulus

- Wave amplitude decays when its frequency falls inside the band gap, especially if frequency is near the frequency $\omega^*$. 

![Diagram showing wave attenuation in metamaterial with negative effective modulus](image)
Metamaterial with Double Negativity (DN)

Metamaterial with negative mass density (NMD)

Metamaterial with negative modulus (NEM)

Metamaterial with Double Negativity (DN)

Double Negativity
(green area)

Negative effective mass (Band Gap, red area)

Negative effective modulus (Band Gap, red area)
Wave Propagation in Metamaterial with Double Negativity

$$\omega / \omega_0^{NMD} = 1.56$$

Propogation direction of wavefront

Propagation direction of wave phase

Distance in number of unit cells
Double Positive Metamaterial

$T^* = 0.0000$
Double Negative Metamaterial
Derivation for Reflection and Transmission Coefficients

Material 1

\[ u_i = \hat{u}_i e^{i(\omega t - q_1 x)} \]
\[ u_r = \hat{u}_r e^{i(\omega t + q_1 x)} \]
\[ u_t = \hat{u}_t e^{i(\omega t - q_2 x)} \]

Material 2

\[ u_t \]

Assume \( x = 0 \)

If \( E_1 = E_2, \rho_1 = \rho_2 \), then \( R = 0, T = 1 \)

If \( E_2 = -E_1, \rho_2 = -\rho_1 \), then \( R = 0, T = 1 \)
Material 2

Material 1
(Regular Material)

Material 2
(Metamaterial)

Material 1
(Regular Material)

\[
E_{\text{eff}} = \frac{L}{A} \left[ k_1 + \frac{1}{2} \left( \frac{k_2 \omega^2}{\omega^2 - \omega_{0,\text{MOD}}^2} \right) \right] \left( \frac{L}{D} \right)^2
\]

\[
\rho_{\text{eff}} = \frac{1}{AL} \left[ m_1 + m_3 \left( \frac{\omega_{0,\text{MASS}}^2}{\omega_{0,\text{MASS}}^2 - \omega^2} \right) \right]
\]

\[
\omega_{0,\text{MOD}} = \sqrt{\frac{k_2}{m_2}}
\]

\[
\omega_{0,\text{MASS}} = \sqrt{\frac{k_3}{m_3}}
\]

\[A = 1\]
Material 1

Material 1
(Ordinary Material)

Material 2
(Metamaterial)

Material 1
(Ordinary Material)

\[ E_{\text{eff}} = \frac{k_4 L}{A} \]

\[ \rho_{\text{eff}} = \frac{m_4}{AL} \]

where \( A = 1 \)
Dispersion Curve for Metamaterial

Non-dimensionalized wave number $qL$

Frequency

DN Region

Negative Mass Region

Dispersion Curve

OP2

OP1

AC
Material Design

Case 1: \( \omega = 1200 \text{ (rad / s)} \) → Frequency for double negativity
Case 2: \( \omega = 650 \text{ (rad / s)} \) → Frequency for negative mass

\[
\begin{align*}
    m_1 &= 2.4 \times 10^{-4} \text{ (kg)} \\
    m_2 &= 1.2 \times 10^{-4} \text{ (kg)} \\
    m_3 &= 2.4 \times 10^{-4} \text{ (kg)} \\
    m_4 &= 9.0 \times 10^{-5} \text{ (kg)} \\
    k_1 &= 100.0 \text{ (N / mm)} \\
    k_2 &= 200.0 \text{ (N / mm)} \\
    k_3 &= 200.0 \text{ (N / mm)} \\
    k_4 &= 535.3 \text{ (N / mm)}
\end{align*}
\]
Case 1: Simulation Result in DN Region

\[ \omega = 1200 \text{ rad} / \text{s} \]

Material 1 (Regular Material)

Material 2 (Metamaterial)

Material 1 (Regular Material)

Distance in number of unit cells

t = 0.07 \text{ s}

t = 0.390 \text{ s}

t = 0.775 \text{ s}
Case 2: Simulation Result in Negative Mass Region

\[ \omega = 650 \text{ rad} / \text{s} \]

<table>
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<tr>
<th>Material 1 (Regular Material)</th>
<th>Material 2 (Metamaterial)</th>
<th>Material 1 (Regular Material)</th>
</tr>
</thead>
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Distance in number of unit cells

\[ t = 0.182 \text{ s} \quad t = 0.301 \text{ s} \quad t = 1.857 \text{ s} \]
Refraction of Metamaterials

- 2D Double-Negativity Metamaterial
Boundary Condition: Plane wave
Simulation Window (15x20 units)

Interface

Negative refraction

Normal to interface

Positive refraction

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Simulation: Plane wave (DN region: 1)

Interface

Normal to interface
Simulation: Plane wave (DN region: 2)
Simulation: Plane wave (DN region: 3)

Interface

Normal to interface
Simulation: Plane wave (DN region: 4)

Normal to interface
Simulation: Plane wave (DP region: 1)

Interface

Normal to interface
Simulation: Plane wave (DP region: 2)

Interface

Normal to interface
Simulation: Plane wave (DP region: 4)

Interface

Normal to interface
Plane Wave Comparison: DN vs. DP

Double Negativity

Interface

Double Positivity

Interface
List of Publications