Strain- and Temperature-Dependence of Electromagnetic Metamaterials

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**ABSTRACT**
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**LIMITATION OF ABSTRACT**
Same as Report (SAR)

**NUMBER OF PAGES**
14
Agenda

• Motivation
• Analytic Expression for Constitutive Parameters
• Equivalent Circuit Expressions
• Strain-Dependence
• Temperature-Dependence
• Low Modulus Substrate
• Testing
• Process
• Conclusions
Motivation

• Tailored EM Response
  - Engineered Constitutive Props: Permittivity, Permeability, Magneto-electric coupling
  - Frequency-dependent
  - Anisotropic
  - Inhomogeneous

• Impressive Results: Lab Env.

Motivation

• Defense Systems Operate in Extreme Environments

• Require ability to understand and predict performance before transitioning into Operational Platforms
  - Temperature Changes
  - Mechanical Loading

• Large Structures, Dynamic Environment, Many Unique Unit Cell Designs
Analytic Expressions for Constitutive Parameters

• Analytic Expressions for $\varepsilon$ and $\mu$
  - ELC Unit Cell
  - Source is external
  - Prediction of full Structure’s Performance

\[
\varepsilon = \bar{\varepsilon} \frac{\theta d}{2} \frac{\theta d}{\sin \frac{\theta d}{2} \cos \frac{\theta d}{2}}
\]
\[
\mu = \frac{\theta d}{2} \frac{\theta d}{\sin \frac{\theta d}{2} \cos \frac{\theta d}{2}}
\]
\[
\bar{\varepsilon}(f) = \varepsilon_b - \frac{f_p^2}{f^2 - f_0^2 + i\Gamma_e f}
\]
\[
\theta = n_{eff} \frac{\omega}{c}
\]

• Alternate Form of the Lorentzian Term

\[
\bar{\varepsilon} = 1 + \frac{C_{ext}}{d \varepsilon_0} \frac{\omega_0^2 - \omega^2}{\omega_0^2 - \omega^2 \left(1 + \frac{C_{ext}}{C_{int}}\right)}
\]
\[
\omega_0^2 = \frac{1}{LC_{int}}
\]
\[
L = L + \frac{R}{j\omega}
\]
\[
\sin \frac{\theta d}{2} = \sqrt{\varepsilon} \frac{kd}{2}
\]

Metamaterial’s Strain- and Temperature-Dependence can be FULLY described via $R$, $L$, $C_{int}$, $C_{ext}$
Equivalent Circuit Expressions

• Equivalent circuits expressions are functions of geometry and materials properties
  - Mechanical Strain: Change in Geometry
  - Temperature Change: Mechanical Strain and Changes in Material Properties

\[
C = C_a + C_z \\
C_a = \varepsilon_0 \frac{2}{\pi} \ln \left( 2 \beta \frac{H}{s} \right) W \\
C_z = \varepsilon_0 \frac{\varepsilon_z - 1}{s + \frac{4}{3} \ln \beta} \frac{W}{h_z} \\
L = \frac{\mu_0 l}{2\pi} \left[ \ln \left( \frac{2l}{b} \right) + \frac{1}{2} + \frac{b}{3l} - \frac{b^2}{24l^2} \right] \\
R = \frac{1}{\sigma A} \\
A = b \delta \quad \left( \delta = \sqrt{\frac{2}{\omega_0 \mu \sigma}} \right)
\]

• Utilized full wave simulation to assess parameter values at baseline condition
  - Expressions utilized to determine changes in value as a function of strain and temperature
  - Minimizes errors from inaccurate expressions
Strain-Dependence

Exx=Eyy=-5%

Exx=Eyy=+5%
Temperature-Dependence

![Graphs showing temperature-dependence of real relative permittivity and real relative permeability across different frequencies and temperatures.](image-url)
Low Modulus Substrates

• Previous analysis utilized thick, high-modulus substrates
  - Homogeneous Strain Profile
  - Simplified integration into analytic expressions

• A soft substrate complicates the strain profile
  - Utilize shear-lag models to describe the different strain levels in the copper and dielectric
  - Modifies geometry from previous equivalent circuit expressions

\[
\varepsilon_{\text{xxc}} = \frac{\sigma_{\text{xxc}}}{Y_c} = \frac{E_{\text{xx}}}{Y_c} \left[ \frac{S_{\text{t}} Y_{\text{S}}}{t_{\text{c}} (1 + S)} \right] = E_{\text{xx}} \left[ \frac{Y^* t^*}{Y^* t^* + 1} \right] \\
\varepsilon_{\text{iiS}} = \chi E_{\text{ii}} = \frac{d - l_c \beta}{d - l_c} E_{\text{ii}} = \frac{d - l_c \left( \frac{Y^* t^*}{Y^* t^* + 1} \right)}{d - l_c} E_{\text{ii}} = \frac{1 - \frac{l_c}{d}}{1 - \frac{l_c}{d}} E_{\text{ii}}
\]
Low Modulus Substrate

Change in Resonant Frequency as a Function of Strain ($E_{xx}$) and Modulus Ratio (Analytic)

Change in Resonant Frequency as a Function of Strain ($E_{yy}$) and Modulus Ratio (Analytic)

Change in Resonant Frequency as a Function of Strain ($E_{xx}$) and Modulus Ratio (Numeric)

Change in Resonant Frequency as a Function of Strain ($E_{yy}$) and Modulus Ratio (Numeric)
Testing

• Facility at Duke University
  - Loadframe
  - RF Characterization

• Photogrammetry
  - Large area strain mapping

• Mechanical Characterization
  - At AFRL
  - Material Props did not meet vendor specifications
Test Results

- Predicted different shifts for the different samples
- Understanding EM performance requires knowledge of the full strain vector

<table>
<thead>
<tr>
<th></th>
<th>Pyralux, ½oz Cu, Sample 1, 2400 lbs</th>
<th>Pyralux, ½oz Cu, Sample 2, 2400 lbs</th>
<th>Pyralux, 1oz Cu, Sample 1, 2400 lbs</th>
<th>Pyralux, 1oz Cu, Sample 2, 2400 lbs</th>
<th>5880, Sample 1, 1200 lbs</th>
<th>5880, Sample 1, 1600 lbs</th>
<th>5880, Sample 2, 1250 lbs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{XX}$ (%)</td>
<td>-1.12 to -1.06</td>
<td>-1.12 to -0.99</td>
<td>-1.25 to -1.19</td>
<td>-1.12 to -1.06</td>
<td>-0.73 to -0.59</td>
<td>-1.26 to -0.99</td>
<td>-0.79 to -0.66</td>
</tr>
<tr>
<td>$E_{YY}$ (%)</td>
<td>4.16 to 4.3</td>
<td>4.03 to 4.1</td>
<td>3.76 to 3.96</td>
<td>4.03 to 4.1</td>
<td>1.10 to 1.14</td>
<td>1.83 to 1.87</td>
<td>1.10 to 1.14</td>
</tr>
<tr>
<td>Predicted $\Delta f_0$ (GHz)</td>
<td><strong>-0.032</strong></td>
<td><strong>-0.030</strong></td>
<td><strong>-0.020</strong></td>
<td><strong>-0.029</strong></td>
<td><strong>+0.009</strong></td>
<td><strong>+0.018</strong></td>
<td><strong>+0.012</strong></td>
</tr>
<tr>
<td>StDev $\Delta f_0$ (GHz)</td>
<td>0.002</td>
<td>0.003</td>
<td>0.002</td>
<td>0.001</td>
<td>0.006</td>
<td>0.003</td>
<td>0.003</td>
</tr>
<tr>
<td>Test Results (GHz)</td>
<td><strong>-0.039</strong></td>
<td><strong>-0.032</strong></td>
<td><strong>-0.026</strong></td>
<td><strong>-0.029</strong></td>
<td><strong>+0.011</strong></td>
<td><strong>+0.018</strong></td>
<td><strong>+0.013</strong></td>
</tr>
</tbody>
</table>
The Process

• Baseline EM Parameters Extracted from Full-Wave Simulations
• Strain/Temperature Profiles pulled from Finite Element Software
• Simple Scripts executed to determine EM Parameters at given strain/temperature condition
Conclusions

• Analytic Expressions are powerful tools for describing metamaterial strain/temp-dependence
  - Provide insight into physics behind linkage
  - Enable accurate prediction over the continuum of strains/temps
  - Rapid description of properties; $>10^5$ redux in model complexity
  - Rapidly predict strain/temp-dependence for unit cells in same design “family”

• Enable efficient determination of EM performance of large structures, with multiple unit cell designs, under complicated strain/temp profiles

• Care must be exercised in choosing appropriate analytic expressions
  - Circuit elements
  - Constitutive properties

• Process extendable to other unit cell designs
  - Magnetic metamaterials/SRRs
  - Owing to similar analytic expressions and circuit elements