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CONTROL-ORIENTED AEROELASTIC REDUCED-ORDER MODELING OF FIGHTERS FA9550-10-1-0539

Charbel Farhat and David Amsallem
Aeronautics and Astronautics
Stanford University
FSI: STATE OF THE ART

🌟 Structural dynamics

- multibody dynamics
- geometrical nonlinearities (large displacements, rotations and strains)
- material nonlinearities (nonlinear constitutive models)
- crack propagation (failure)

🌟 Computational fluid dynamics

- shocks
- turbulence

🌟 Coupling

- static (steady) and dynamic (unsteady)
- eigen
**IMPACT ON ENGINEERING**

- **Strong**
  - analysis of carefully selected critical configurations

- **Weak (or not as strong)**
  - routine analysis
  - design (and test)
  - control

  significant CPU time issues

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"[If I am not getting the NASTRAN answer after 4 hours on a Cray, then God is sending me the message I have the wrong design]"

Burt Rutan, 1993
Model reduction (MOR)

- build the lowest dimensional model that can capture the dominant behavior of the system of interest by projecting the high-fidelity model onto a well-chosen subspace

\[ \text{drastic CPU time reduction} \]

Complex, time-dependent problems (with a CFD component)

- Perturbation problems (stability, trends, control, etc.)
  - linearized \[ \rightarrow \text{linear ROMs} \]

- Response problems (behavior, performance, etc.)
  - nonlinear \[ \rightarrow \text{linearized} \]
  - Newton \[ \rightarrow \text{linear ROMs} \]
What this is NOT about

- building the simplest model
- building a variable (or multi) fidelity model
- adopting the coarsest mesh
- substructuring
- constructing a “surrogate” or “meta” model
LINEARIZED DYNAMICAL SYSTEMS

\[ \mu = [\mu_1, \ldots, \mu_p] \]

\[ y = \int_{-\infty}^{t} h(t - \tau) \mu(\tau) \, d\tau \]

\[ u = A(\mu_1, \ldots, \mu_p)u + f \]

\[ y = g(u, f, \mu_1, \ldots, \mu_p) \]

\[ u(t_0) = u_0 \]
**SURROGATE MODEL**

* External description

\[ \mu = [\mu_1, \ldots, \mu_p] \]

\[ y = [y_1, \ldots, y_p] \]

- meta-model, surrogate model, response surface, kriging
- lower-dimensionality is not guaranteed
- it is not a model of the system but of the output
MODEL REDUCTION

★ Internal description

\[ \dot{u} = A(\mu_1, \ldots, \mu_p)u + f \text{ (dimension } = N) \]

★ Projection onto a subspace of dimension \( k << N \)

- right Reduced-Order Basis (ROB) \( V_k \), \( k << N \)

\[ u \sim V_k y \quad \rightarrow \quad V_k \dot{y} = A(\mu_1, \ldots, \mu_p)V_k y + f + r \]

- left Reduced-Order Basis (ROB) \( U_k \), \( k << N \)

\[ \text{constraints} \quad \rightarrow \quad \dot{y} = U_k^T A(\mu_1, \ldots, \mu_p) V_k y + U_k^T f \]

parameterized linear(ized) Reduced-Order Model (ROM)
\[ \dot{u} = A(\mu_1, \ldots, \mu_p) u + f \]

\[ \dot{y} = U_k^T A(\mu_1, \ldots, \mu_p) V_k y + U_k^T f \]

- The outcome is a dynamical system of lower dimension

- Key issues
  - choice of the lower dimension $k$ and the ROBs $U_k$ and $V_k$
  - dependence of the resulting ROM on the parameters $\{\mu_i\}$
General purpose linearized aeroelastic high-fidelity model (HFM)

\[
\begin{bmatrix}
\dot{w} \\
\ddot{u} \\
\dot{u}
\end{bmatrix}
= \begin{bmatrix}
-H & -B & -C \\
M^{-1}P & O & -M^{-1}K \\
0 & I & 0
\end{bmatrix}
\begin{bmatrix}
w \\
\dot{w} \\
\dot{u}
\end{bmatrix}
+ \begin{bmatrix}
0 \\
M^{-1}T_i^T \\
0
\end{bmatrix} F
\]

\[A(\mu); \quad \mu = (\mu_1, \ldots, \mu_q)\]
MULTIDISCIPLINARY ROM

Fluid ROM

- CFD-based HFM
- Balanced POD-based ROB
  (with stability guarantee)

Structural ROM

- FE-based HFM
- Eigen-based ROB
  (with truncation)
**FLUID-STRUCTURE ROM**

**Fluid**

ROBs: \( U_k, V_k / U_k^T V_k = I \)

\[ w = V_k w_r \]

ROM: \( H_r = U_k^T H V_k \)

**Structure**

ROB: \( X_m, (X_m^T M X_m = I) \)

\[ u = X_m u_m \]

ROM: \( \Omega_m^2 = X_m^T K X_m \)

**Coupling**

\[ B_r = U_k^T B X_m \]

\[ C_r = U_k^T C X_m \]

\[ P_r = X_m^T P V_k \]

*General purpose aeroelastic ROM (flutter, response, control, …)*

\[
\begin{pmatrix}
\dot{w}_r \\
\dot{u}_m \\
\ddot{u}_m
\end{pmatrix}
= \begin{pmatrix}
-H_r & -B_r & -C_r \\
0 & P_r & 0 \\
0 & 0 & \Omega_m^2
\end{pmatrix}
\begin{pmatrix}
\dot{w}_r \\
\dot{u}_m \\
u_m
\end{pmatrix}
+ \begin{pmatrix}
0 \\
X_m^T T_i T_i^T \\
0
\end{pmatrix} F
\]

\( A_r(\mu); \quad \mu = (\mu_1, \ldots, \mu_q) \)

- Aeroelastic response
- Control, flutter
- Aeroservoelastic analysis, …
Natural mode \textit{shapes} (ground vibrations) \((n_m = 9 \text{ modes})\)

Excitations in a sampled frequency range

\[ 0 < \kappa = \frac{\omega L_R}{v_R} \sim 1 \quad \rightarrow \quad n_f = 5 \text{ frequencies} \]

Responses of \textit{linearized} flow (frequency domain)

\[ (H + i\omega_q I) \mathbf{w}_{jq} = -(C + i\omega_q B) \mathbf{u}_j \quad j = 1, \ldots, n_m, \quad q = 1, \ldots, n_f \]

\[ (-H^T + i\omega_q I) \mathbf{w}_{*jq} = -P^T \mathbf{u}_j \quad j = 1, \ldots, n_m, \quad q = 1, \ldots, n_f \]

\[ \mathbf{W} \quad \text{data compression} \rightarrow \mathbf{V}_k \]

\[ \mathbf{W}^* \quad \text{data compression} \rightarrow \mathbf{U}_k \]
DATA COMPRESSION

🌟 Modal superposition (Fourier decomposition)
  - limited range of applications

🌟 Proper Orthogonal Decomposition (POD)
  - lacks stability

🌟 Balanced Proper Orthogonal Decomposition (POD)
  - more robust than POD but still lacks stability
**GUARANTEED STABILIZATION METHOD**

* ROM stabilization method (*Amsallem and Farhat, 2011*)
  - universal
  - non intrusive
  - computational complexity scales with the size of the ROM
  - preserves accuracy of original (unstable) ROM

**input: ROM** → **Stabilization Method** → **output: stable ROM**
Higher-fidelity (higher-dimensional) models (HFM)

- structure: FEM with 168,799 dofs
- fluid: Euler CFD model with 403,919 grid points

ROMs

- structure: projection on ROB consisting of first 9 natural mode shapes
- fluid: projection on ROB of dimension 60 generated by POD using 99 snapshots (9 shapes x 5 ωs x 2 + 9 shapes at 0 Hz)
POD-based fluid ROM (60) built at $M = 0.710$ (trimmed angle)
ANALOGY

Configuration 1

HFM$_1$

ROM$_1$

Reduced-order basis $V_{k_1}$

Configuration 2

HFM$_2$

ROM$_2$

Reduced-order basis $V_{k_2} = V_{k_1}$
TURNAROUND TIME

- F-16 Block 40 — 1 operating point — 2nd-order discretization
- 64-processor Linux cluster

Construction and processing of aeroelastic ROM in [0, 1.0] s

- Fluid steady-state computation: 6 minutes
- Generation of 99 fluid snapshots and POD-based fluid ROM: 50 minutes
- Construction of fluid ROM: 0.25 minute
- Processing aeroelastic ROM: 0.10 minute
- Total CPU time: 57.25 minutes
**APPROACH**

* Database of fixed-size stable **ROMs**

![Diagram showing Parameter 1 and Parameter 2 with precomputed ROMs and instantaneous ROMs, along with zonal interpolation.]
CONSISTENCY

Eigenvalues

\[ \lambda_{1,2}(\mu) = \frac{\mu + 1.2 \mp \sqrt{((\mu - 0.9)^2 + 0.01}}}{2} \]

Eigenvectors

precomputed ROMs are not necessarily computed in a consistent set of generalized coordinates
**Congruence transformations**

- Reference ROB: \( V_{\text{ref}} = V_k(\mu_{\text{ref}}) \)
- \( V_i \leftarrow V_i Q_i \) \( Q_i = \arg \min_{Q \in O(k)} \| V_i Q - V_{\text{ref}} \|_F \)

**Orthogonal Procrustes problem**

- SVD: \( V_i^T V_{\text{ref}} = U_i \Sigma_i Z_i^T \)

\( Q_i = U_i Z_i^T \)
QUOTIENT AND EMBEDDED MANIFOLDS

- standard interpolation of a ROM does not produce a ROM
- interpolation must be carried out on a manifold characterizing the ROM

* Manifolds of interest (quotient = blue, embedded = yellow)

- \( \text{span}(V_k) \) belongs to the Grassmann manifold \( G(k,N) \)
- \( A_r(\mu) = V_k^T A(\mu) V_k \) belongs to
  * manifold of invertible matrices \( GL(k) \) [fluid]
  * manifold of (reduced-order) symmetric positive definite matrices \( SPD(k) \) [structure]
Online Interpolation on a Manifold

- Given a p-parameter system, the appropriate Riemannian manifold, and its logarithmic and exponential maps.

\[ \tau_{R_0}M \]

\[ \text{Log}_{R_0}(R_2) \quad \text{Log}_{R_0}(R_3) \]

\[ R_4 = \text{Exp}_{R_0}(X) \]

\[ X = \text{Log}_{R_0}(R_3) \]
SHOWCASE APPLICATION

✨ F-16 Block 40

- CFD model
- FEM structural model

✨ Assisting flight test
- single aircraft configuration ➔ single structural ROM
- hundreds of flight conditions ➔ database of fluid ROBs
83 pairs of flight conditions (operating points)
- 70.6 hours CPU on a 64-processor Linux cluster
Fast responses to 5 queries

- possible scenarios: flight test, flutter clearance, optimization
- interpolation of fluid ROMs
Deep transonic point \((M_\infty, \alpha) = (0.930, 1.3^\circ)\)

Graph showing lift (lbf) over time (sec) for different methods:
- L-HFM
- D-ROM
- I-ROM

Delivered accuracy
### TURNAROUND TIME

* F-16 Block 40 — 1 operation point — 1-processor (desktop)

**CPU time for interpolating and processing**

<table>
<thead>
<tr>
<th>Description</th>
<th>Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluid ROM interpolation (5 points)</td>
<td>0.2 second</td>
</tr>
<tr>
<td>Aeroelastic ROM processing (FD)</td>
<td>0.3 second</td>
</tr>
<tr>
<td>Total CPU time</td>
<td>0.5 second</td>
</tr>
</tbody>
</table>

**real-time, parameterized, CFD-based flutter analysis**
IMPLEMENTATION ON MOBILE DEVICES
IMPLEMENTATION ON MOBILE DEVICES

![Graph showing Flutter Speed Index vs Mach Number]

**Flutter Speed Index**
- Fill Level = 50.00%
US Air Force Office of Scientific Research, T&E Program