CHAIN DYNAMIC FORMULATIONS FOR MULTIBODY SYSTEM TRACKED VEHICLES

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**Abstract:**
MBS based computer simulations are necessary for the design and performance evaluation of complex tracked vehicles. The development of accurate and efficient tracked vehicle models can be achieved with a better understanding of MBS dynamic formulation. Three different chain formulations will be discussed using the Augmented Formulation Ideal Joint Formulation Compliant Discrete-based Formulation Compliant Continuum-based Formulation.
OUTLINE

- Background
- Objective
- Augmented Formulation
- Chain Formulations
- Tracked Vehicle Model
- Simulation
- Numerical Results
- Summary
- Future Work
Multibody Systems (MBS) consist of many components interconnected by joints and force elements.

Examples of multibody tracked vehicles include: bulldozers, military battle tanks, armored personnel carriers.

Investigations on the dynamic analysis of tracked vehicles has been limited due to the complexity of forces resulting from interactions between components.

MBS algorithms have been developed to solve systems in computer programs, such as SAMS/2000.
OBJECTIVE

• MBS based computer simulations are necessary for the design and performance evaluation of complex tracked vehicles

• The development of accurate and efficient tracked vehicle models can be achieved with a better understanding of MBS dynamic formulation

• Three different chain formulations will be discussed using the Augmented Formulation
  – Ideal Joint Formulation
  – Compliant Discrete-based Formulation
  – Compliant Continuum-based Formulation
Augmented Formulation

- Employs technique of Lagrange Multipliers
- Constraint relationships are used with the differential equations of motion to solve for unknown accelerations and constraint forces

Equations of motion for body $i$: $M_i \ddot{q}_i = Q_{e}^i + Q_{c}^i + Q_{v}^i$

- $\ddot{q}_i$ = vector of accelerations
- $C_q$ = constraint Jacobian matrix
- $Q_e$ = vector of external forces
- $Q_c$ = vector of constraint forces
- $Q_v$ = vector of inertia forces
- $Q_d$ = RHS of constraint acceleration equations
• Ideal Joint Formulation
  – Constrained Dynamic Approach
    • Uses algebraic constraints at position/velocity/acceleration levels and eliminates degrees of freedom between track links
  – Penalty Method
    • Does not eliminate degrees of freedom
    • Joint constraints enforced using high stiffness penalty coefficients cannot be satisfied at the acceleration level

\[
C(q^j, q^j) = \begin{bmatrix} v_1^j v^j & v_2^j v^j & v_{1P}^j r_{1P} & v_{2P}^j r_{2P} & r_{1P}^j r_{2P} - k_e \end{bmatrix}^T = 0
\]
**CHAIN FORMULATIONS**

- Compliant Discrete Element Joint Formulation
  - Bushing Element Formulation
    - No algebraic equations are used to describe joints
    - User-defined force elements describe connectivity between bodies

\[
\begin{bmatrix}
\mathbf{F}_R^b \\
\mathbf{M}_\theta^b
\end{bmatrix} =
\begin{bmatrix}
\mathbf{K}_r & 0 \\
0 & \mathbf{K}_\theta
\end{bmatrix}
\begin{bmatrix}
\dot{\delta}^{bij} \\
\dot{\theta}^{bij}
\end{bmatrix} +
\begin{bmatrix}
\mathbf{C}_r & 0 \\
0 & \mathbf{C}_\theta
\end{bmatrix}
\begin{bmatrix}
\ddot{\delta}^{bij} \\
\ddot{\theta}^{bij}
\end{bmatrix}
\]
• Compliant Continuum-Based Joint Formulation
  – ANCF Finite Elements
    • Finite Element (FE) meshes allow for constant inertia matrix and zero Coriolis and centrifugal forces
    • \( r_i = r_j, r_\alpha = r_\alpha \): defines linear chain connectivity and eliminates 6 degrees of freedom (three translations, two rotations, one deformation mode)
• M113 armored personnel carrier
  – Made up of 1 chassis and a left and right track system each consisting of:
    • 64 track links
    • 1 sprocket
    • 1 idler
    • 5 road wheels
    • 5 road arms, each placed between road wheels and chassis
    • 5 shock absorbers, each connected between the road arms and chassis
    • 1 track tensioner
Simulation
NUMERICAL RESULTS

- Sprocket angular velocity

- Chassis forward displacement
  - Constrained joint model
  - Penalty method model
  - Bushing element model

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NUMERICAL RESULTS

- Trajectory motion of a track link in the chassis coordinate system
  - Constraint joint model
  - Penalty method model
  - Bushing element model
NUMERICAL RESULTS

- Joint longitudinal forces
  - Constrained joint model
  - Penalty method model \((k = 10^7 \text{ N/m})\)

- Joint vertical forces
  - Constrained joint model
  - Penalty method model \((k = 10^7 \text{ N/m})\)
NUMERICAL RESULTS

• Joint longitudinal forces
  — Constrained joint model
  --- Penalty method model ($k = 10^9$ N/m)

• Joint vertical forces
  — Constrained joint model
  --- Penalty method model ($k = 10^9$ N/m)
SUMMARY/CONCLUSION

- The penalty method and bushing element models have good, although not exact, agreement with the constrained model.
- Penalty force based joint construction is shown to be sensitive to the selection of stiffness coefficients.
- Larger stiffness coefficients lead to more accurate results but also to longer CPU time (Penalty is 2:1 to constrained model and Bushing is 4:1 in CPU time).
- Simulations show that the constrained (revolute) model has the best overall results with the shortest CPU time and can be used to model the most accurate and efficient tracked vehicle models.
FUTURE WORK

• Developing and testing models made up of ANCF finite elements
• Numerical comparisons between rigid body chain formulations and ANCF finite elements
• Comparing these models driving over different types of ground formations - bumps, ramps, etc.
Questions?