Performance Analysis of Adaptive Probabilistic Multi-Hypothesis Tracking With the Metron Data Sets

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Abstract-
The Probabilistic Multi-hypothesis Tracking (PMHT) algorithm [1] is a batch type multi-target tracking algorithm based on the Expectation-Maximization (EM) method [2]. Unlike other popular batch methods (e.g., Multi-Hypothesis Tracking, MHT) the computational burden of PMHT grows linearly in the size of the batch, the number of clutter detections, and the number of targets tracked. This is achieved by employing the independent assignment model for assigning measurements to tracks which gives rise to a different likelihood function that that used by the other methods. In practice, however, the PMHT often exhibits slow convergence to a non-global local peak of the relevant likelihood function [3]. The authors have modified the E-M based optimization method and significantly improved the convergence behavior.

This study investigates the ability of Adaptive PMHT to hold track on contacts in a field of active receivers. Metron Inc. has constructed a collection of simulated multi-static active sonar data sets designed to approximate the performance of a buoy field. Each scenario contains multiple maneuvering targets that exhibit frequent dropouts and aspect dependent SNR and these situations are of particular interest.

Index Terms- Adaptive Probabilistic Multi-hypothesis Tracker, multi-static active sonar, batch target tracking, centralized and distributed processing systems.

1. INTRODUCTION

Batch tracking algorithms (e.g., Multi-Hypothesis Tracking, MHT) are currently being developed and investigated for multi-static active sonar systems to improve overall tracking performance in general and track hold during contact maneuvers and temporary loss of detection in particular. The cost of estimating sequence of target states instead of just the current state (i.e., recursive tracking) is increased computational burden and for many batch algorithms the computational load increases factorially with the length of the batch. The Probabilistic Multi-Hypothesis Tracking (PMHT) algorithm considered here enjoys the advantage of having a computational load that grows linearly in the length of the batch, the number of contacts and the density of clutter. This is true even if the targets tracked are in close proximity or crossing.

Most methods exhibit combinatorial growth in their computational burden because they are based on the conventional assumption that each target generates at most one detection per scan per sensor which requires the enumeration of every possible sequence of detection to track assignments to determine the Maximum Likelihood estimate of target trajectory. By contrast PMHT is based on a more flexible assignment model that allows for the possibility of more than one target detection. Many schemes have been developed to control the computational burden of MHT type methods by limiting the selection of assignment sequences to the most likely, [4]. Such pruning methods, however, inevitably sacrifice any optimality property of the full algorithm. The computational burden of any algorithm increases with batch size and therefore system designers will need to know the minimum batch size required to achieve the desired performance.

The analysis presented here utilizes a centralized processing architecture where the measurements (i.e., clustered echo detections) are registered to a common frame of reference and synchronized; no measurement fusion of any kind is employed. Figure 1 illustrates the fundamental cycle of a centralized tracking architecture that performs sequential updates to a set of tracks using registered synchronized measurements from two sensors. This approach performs more updates to the tracks per scan but allows the pulse repetition interval for each source receiver combination to be varied individually and can handle lost receivers more gracefully.

In this analysis an implementation of PMHT based on the centralized architecture depicted in figure 1 is used to evaluate...
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**Abstract**

The Probabilistic Multi-hypothesis Tracking (PMHT) algorithm [1] is a batch type multi-target tracking algorithm based on the Expectation-Maximization (EM) method [2]. Unlike other popular batch methods (e.g., Multi-Hypothesis Tracking, MHT) the computational burden of PMHT grows linearly in the size of the batch, the number of clutter detections, and the number of targets tracked. This is achieved by employing the independent assignment model for assigning measurements to tracks which gives rise to a different likelihood function that that used by the other methods. In practice, however, the PMHT often exhibits slow convergence to a non-global local peak of the relevant likelihood function [3]. The authors have modified the E-M based optimization method and significantly improved the convergence behavior. This study investigates the ability of Adaptive PMHT to hold track on contacts in a field of active receivers. Metron Inc. has constructed a collection of simulated multi-static active sonar data sets designed to approximate the performance of a buoy field. Each scenario contains multiple maneuvering targets that exhibit frequent dropouts and aspect dependent SNR and these situations are of particular interest.
track hold performance. The PMHT algorithm is an adaptation of the Expectation-Maximization (EM) method [2] that is formulated to estimate batch state sequences of multiple maneuvering targets in clutter from active sonar detections. EM is explicitly designed for estimation problems where the data is fundamentally incomplete. In this application the measurements contain no information on their origin (e.g., target or clutter). The derivation of the original PMHT algorithm is well described in [1] and is based on the so called independent assignment model; each measurement has some non-zero prior probability of being from any one of the targets present independent of the origin of all the other measurements. Although this assignment model may seem inappropriate, real world active sonar data often exhibits multiple detections for targets in multi-path propagation environments and when the processing over resolves the target. The independent assignment model gives rise to a different likelihood function than the conventional assignment model and PMHT is an iterative procedure based on EM for optimizing that likelihood function.

![Diagram](image)

Figure 1. Centralized Multi-Sensor Tracking and Detection

The advantage of the independent assignment model is that when it is used in conjunction with the Expectation Maximization method it avoids having to enumerate a large number of candidate measurement assignment hypotheses and instead only requires the calculation of the posterior probabilities that the \( r^{th} \) measurement at time \( t \) originated from target \( s \) as

\[
W_{str} = \frac{\pi_s \sum_{m=1}^{M} \pi_m N(z_{rt}; x_{ts}, R_{ts})}{\sum_{m=1}^{M} \pi_m N(z_{rt}; x_{tm}, R_{tm})} 
\]

(1)

where \( \pi_s \) is the prior probability that a measurement originated from the \( i^{th} \) target being tracked and \( M \) is number of targets. In [4] the above formula is modified to employ amplitude information and account for uniformly distributed clutter. \( V \) is the volume of the association gate, and \( f_0(a_{rt}) \) and \( f_i(a_{rt}) \) are the distributions for the echo amplitudes for clutter and target respectively,

\[
W_{str} = \frac{\pi_s f_i(a_{rt}) N(z_{rt}; x_{ts}, R_{ts})}{\sum_{m=1}^{M} \pi_m f_i(a_{rt}) N(z_{rt}; x_{tm}, R_{tm})} 
\]

(2)

Each iteration of PMHT uses these weights to form synthetic measurements (a.k.a., measurement centroids) which are then used in a simple Kalman smoother to obtain updated state estimates. This process is iterated until a suitable convergence criteria is satisfied, typically less than 10 iterations.

In practice, however, PMHT often fails to converge to the global maximum of the likelihood function; it often converges to a local maximum thereby returning inaccurate estimates of target state [3]. This phenomenon is caused by the lack of any error covariance information about the current state estimate in the weights calculation and by the use of a badly mismatched measurement error covariance in the Kalman smoother. The authors have made three modifications to PMHT to improve convergence: modified the Kalman smoother to use a more accurate measurement error covariance and compute an accurate estimate of state error covariance, incorporated the state error covariance in the calculation of the weights. These modifications appear to have an annealing effect on the optimization of the likelihood function and much more reliable convergence to the global maximum has so far been observed.

The basic procedure of the Adaptive PMHT algorithm used in this study amounts to iterating the following four steps:

1. Compute the association weights, \( w_{str} \), for each measurement and target at each time step in batch.
2. Using the weights compute a measurement centroid and effective error covariance matrix (a.k.a. the approximate true measurement error covariance) for each target at each time step in the batch.
3. Update the track (i.e., the batch sequence of state estimates) for each target with a Kalman smoother on the synthetic measurements and error covariance matrices.
4. Compute the error covariance of the state estimates using the true measurement error covariance and the Kalman gains from step 3.

2. PURPOSE

The purpose of the effort reported here is to assess the target tracking performance of Adaptive PMHT using simulated multi-static data sets from Metron Inc. This study investigates the ability of Adaptive PMHT to hold track on constant velocity and maneuvering contacts in a field of buoy receivers. Situations involving contact maneuvers or...
temporary loss of detection (a.k.a., drop outs) are of particular interest. The Metron data sets appear to be designed to be especially challenging in those respects.

In order to focus on the track hold performance the tracking conditions are assumed to be ideal in most other respects: independent and identically distributed zero mean measurement errors with known covariance and a benign environment with identical interference level, propagation loss and target strength at all sensors. Although the clutter is assumed to be uniformly distributed the spatial densities and amplitude distributions for clutter are not assumed to be the same for both FM and CW waveforms. The FM and CW data used in this study contains a significant amount of clutter that exhibited markedly different spatial densities and amplitude distributions. The reconstruction information was used to separate the clutter detections from the target detections for each waveform and estimate the spatial density and amplitude distribution for use in the PMHT tracker. This was necessary because the amplitudes of the clutter data were not well modeled by standard ideal normalizer output (i.e., unit mean Rayleigh).

3. THE METRON DATA SETS

The Metron data sets model the performance of a field of 25 receiver buoys in a hexagonal packing arrangement as shown in figure 2; the diagonal separation between neighboring buoys is approximately 14km. Four of the buoys also serve as source buoys that ping sequentially every 180 seconds. Each source buoy alternately transmits FM and CW type waveforms. The bearing errors, time of arrival errors, and Doppler errors (only for CW detections) for target detections are normally distributed with zero mean and standard deviations of 8.0 degrees, 0.4 seconds, and 0.5 meters per second respectively. For clutter detections the Doppler error standard deviation was observed to be approximately 1.25 meters per second.

The distribution of the amplitudes for both target and clutter detections in the Metron data was observed to be highly non-Rayleigh and was instead modeled by exponential distributions. In this study the target echo and clutter amplitudes are assumed to be Exponentially distributed with different parameters;

\[
f_\tau(a) = \lambda_\tau e^{-\lambda_\tau(a-\tau)} \quad \text{for } a > \tau
\]

is the target amplitude distribution. The clutter distribution is;

\[
f_0(a) = \lambda_0 e^{-\lambda_0(a-\tau)} \quad \text{for } a > \tau
\]

For the FM waveform \(\lambda_\tau=0.2083\) and \(\lambda_0=0.4963\) and for the CW waveform \(\lambda_\tau=0.2222\) and \(\lambda_0=0.5673\). The effective per scan probability of target detection by an individual buoy was taken to be 0.15.

The spatial distribution of the clutter is clearly non-uniform as shown in figure 3. It appears that the clutter data was uniformly distributed in range a bearing about each receiver. Converting such data to Cartesian space generates apparent clusters of clutter centered on each receiver. In this study this characteristic of the data was not explicitly modeled or exploited in the Adaptive PMHT algorithm because it was regarded as an artifact of the simulation.

The data in each test set were registered to a common frame of reference before tracking to investigate a centralized tracking architecture. All target tracks were initialized by a method based on the Maximum Likelihood PMHT algorithm [6]. Target tracks were rapidly initialized in both data sets; the initialization latency was less than 12 pings. The performance metric considered here is essentially track hold. Other popular metrics (e.g., false alarm rate) were also evaluated.

Figure 2. Plot of sensor layout for the Metron data sets. Receiver buoys are depicted with small circles and the source buoys are depicted by solid dots. Ground truth for the four targets in scenario 1 is also shown.

Figure 3. Plot of all detections from Metron Data Set 1 Pings 1 to 100. Clutter detections are shown in black and target detections are shown in red. Clusters of clutter echoes are clearly evident at each receiver buoy location.
4. RESULTS

The authors have applied the Adaptive PMHT algorithm with a batch size of 11 scans to two of the Metron data sets: data sets 1 and 4. Figure 4 shows the ground truth and tracks produced by the Adaptive PMHT algorithm in the first 50 pings of data set 1. All four targets are continuously tracked through multiple maneuvers and dropouts without any false tracks.

Figure 4. Plot of ground truth and all tracks for Metron Data Set 1 pings 1 to 50. All four targets are continuously tracked. Clutter detections for one ping are shown in black and target detections are shown in red.

Figure 5 shows the tracks produced in the first 100 pings of data set 1. The two northern targets are continuously tracked twice around their rectangular trajectories and through repeated maneuvers and dropouts without any false tracks.

Figure 5. Plot of ground truth and all tracks for Metron Data Set 1 pings 1 to 100. The two northern targets are continuously tracked.

Figure 6 shows the ground truth for data set 4. There are four targets, two of which move with constant velocity from east to west, a third which also starts out heading west and then turns north, and the fourth target which moves from west to east along a serpentine trajectory.

Figure 6. Plot of ground truth for Metron Data Set 4 which include four targets and one fixed persistent scatterer. The serpentine target moves from west to east, the two constant velocity targets head west, and the fourth target also starts heading west but eventually turns north.

Figure 7 shows the tracks produced by Adaptive PMHT on the Metron Data Set 4. All of the targets are tracked intermittently because this data set exhibited an extensive number of dropouts, especially for the target following the serpentine trajectory. Tracks for the two constant velocity targets we routinely reinitialized and held until the next extended dropout. The third target was tracked intermittently while it was heading west but was held continuously after it turned north. The serpentine target was tracked briefly at the beginning, followed by an extended dropout and then intermittently during the last 100 pings. This data set also produced some false alarms. The persistent scatterer in the southern part of the buoy field produced a group of false tracks all in close proximity to one another. A small number of false tracks were also produced in the vicinity of the start of the serpentine target.

Figure 7. Plot of all tracks from Metron Data Set 1 Pings 1 to 100. All of the targets are tracked intermittently and the false alarm rate is very low.
5. Conclusions

The results in the preceding section clearly show that PMHT can achieve adequate to exceptional track hold performance at a very low false alarm rate with a small batch lengths. Moreover, the track hold performance was robust to target maneuvers, temporary loss of target detection, aspect dependent target SNR.

This study focused on the low false alarm rate performance. the authors intend to investigate the target tracking performance improvement that can be achieved at somewhat higher false alarm rates. Another factor that affected target tracking performance was the non-uniform spatial distribution of the clutter. Modeling the true spatial distribution of the clutter in Adaptive PMHT should also improve track hold performance.

The results presented here clearly show that PMHT provides at least competitive, and possibly impressive, multi-static tracking performance on simulated active sonar data. Moreover, PMHT offers computational efficiency and system flexibility; it can be implemented in either distributed or centralized architectures and combined with almost any track management logic. Appropriate track initialization methods have been presented in [7] and [9]. PMHT is a viable multi-target tracking method appropriate for use in multi-static active sonar systems.

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References


