Optimization-based wireless network architectures: complexity, decentralization and performance guarantees

The goal of the project was to use optimization-based techniques to develop new architectures, algorithms and performance analysis tools for wireless networks. Our main results are as follows: (i) we developed a novel architecture using virtual backlog queues to accommodate real-time and elastic flows in the same network, (ii) we developed a distributed CSMA algorithm, using a statistical physics idea called Glauber dynamics, which achieved 100% throughput in wireless networks with low complexity, and (iii) we used techniques used to bound Markov chain mixing times to understand the performance of our algorithms. The sum total of our effort led to new algorithms for wireless networks which significantly improved the state-of-the-art.

wireless networks; optimization; stochastic networks; Markov chains
Final Report

Project Title: Optimization-based wireless network architectures: complexity, decentralization and performance guarantees
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Objectives: The focus of this project was to derive wireless network architectures that can support both elastic and inelastic flows, design low-complexity/distributed resource allocation algorithms and to rigorously understand the performance of these algorithms by establishing fundamental limits on their throughput and delay performance. The proposed research drew upon techniques and tools from several areas of applied mathematics such as convex optimization, Markov chains, queueing theory and stochastic control.

Our goal in this project was not to make incremental changes to existing protocols and algorithms that result in small performance gains. Rather we are interested in clean-slate designs that would result in fundamentally new architectures and algorithms that can result in large performance gains in the networks of the future. The project was organized around three major themes:

1. Architectural Issues: We developed a new theory for deriving optimal architectures to handle inelastic flows by using a combination of convex optimization and stochastic network theory.
2. Complexity and Decentralization Issues: We designed distributed, randomized algorithms that achieved near-optimal performance with high probability.
3. Performance Issues: We used techniques for Markov chain mixing times to understand the performance of our algorithms.

Significant Results: In the rest of this report, we will summarize our key findings.

Architectural Results:

1. Resource allocation problems in wireless networks typically impose an upper bound on the average delay experienced by a packet. This is consistent with the multi-commodity flow optimization viewpoint where constraints essentially ensure flow balance across each resource in this network. In a significant departure from this framework, we have recently been able to incorporate strict deadlines into the optimization framework. The solution identifies fictitious queues in the network (called deficit counters) with Lagrange multipliers corresponding to the delay constraints. The resulting scheduling algorithm uses these deficit counters, in addition to real queues, to make link activation decisions. Next, we considered single-hop wireless networks with real-time traffic, where the wireless channel is subject to errors. Our analysis results in an optimal scheduling algorithm which fairly allocates data rates to all flows while meeting long-term delay demands. We also proved that, in this
scenario, our solution translates into a greedy strategy that makes optimal decisions with low complexity.

2. We considered multiuser scheduling in wireless networks with flow-level dynamics. It was shown by others that the widely-studied MaxWeight algorithm, which is throughput-optimal in networks with a fixed number of users, fails to achieve the maximum throughput in the presence of flow-level dynamics. They, however, do not provide practical alternatives of MaxWeight scheduling for networks having flow-level dynamics. We proposed a class of algorithms, called workload-based scheduling with learning, which are provably throughput-optimal under flow-level dynamics, require no prior knowledge of channels and user demands, and perform significantly better than MaxWeight algorithm and the algorithm suggested by others previously.

**Distributed algorithms:**

1. It had been shown by others that CSMA-type random access algorithms can achieve the maximum throughput in wireless ad hoc networks. Central to these results is a distributed randomized algorithm which selects schedules according to a product-form distribution. The product-form distribution is achieved by considering a continuous-time Markov model of an idealized CSMA protocol under which collisions cannot occur. In this work, we present an algorithm which achieves the same product-form distribution in a discrete-time setting where collision of data packets is avoided through the exchange of control messages (however, the control messages are allowed to collide as in the 802.11 suite of protocols). In our discrete-time model, each time slot consists of a few control mini-slots followed by a data slot. We show that two control mini-slots are sufficient for our distributed scheduling algorithm to realize the same steady-state distribution as in the continuous-time case. Under some assumptions, the scheduling algorithm can be made throughput optimal by appropriately choosing the link activation probabilities as functions of the queue lengths.

2. The throughput-optimality is established under the perfect or ideal carrier sensing assumption, i.e., each link can precisely sense the presence of other active links in its neighborhood. We also investigated achievable throughput of the CSMA algorithm under imperfect carrier sensing. Through the analysis on both false positive and negative carrier sensing failures, we showed that CSMA can achieve an arbitrary fraction of the capacity region if certain access probabilities are set appropriately. To establish this result, we use the perturbation theory of Markov chains.

3. In the above-mentioned algorithms, each link of the wireless network has to choose two parameters: a transmission probability and an access probability. The transmission probability of each link is chosen as an appropriate function of its queue-length, however, the access probabilities are simply regarded as some arbitrary numbers since they do not play any role in establishing the network stability. We showed that the access probabilities control the mixing time of the CSMA Markov chain and, as a result, affect the delay performance of
the CSMA. In particular, when each link $i$ only knows the number of its neighboring interferers $d$, we showed that $1/(d + 1)$ is a good choice for the access probability of link $i$.

**Complexity and Performance analysis:**

1. Due to its low complexity, Greedy Maximal Scheduling (GMS), also known as Longest Queue First (LQF), has been studied extensively for wireless networks. However, GMS can result in degraded throughput performance in general wireless networks. We proved that GMS achieves $100\%$ throughput in all networks with eight nodes or less, under the two-hop interference model. Further, we obtained performance bounds that improve upon previous results for larger networks up to a certain size. We also provided a simple proof to show that GMS can be implemented using only local neighborhood information in networks of any size.

2. As mentioned earlier, Glauber dynamics is a powerful tool to generate randomized, approximate solutions to combinatorially difficult problems. It has been used to analyze and design distributed CSMA (Carrier Sense Multiple Access) scheduling algorithms for multi-hop wireless networks. We derived bounds on the mixing time of a generalization of Glauber dynamics where multiple links are allowed to update their states in parallel and the fugacity of each link can be different. The results can be used to prove that the average queue length (and hence, the delay) under the parallel Glauber dynamics based CSMA grows polynomially in the number of links for wireless networks with bounded-degree interference graphs when the arrival rate lies in a fraction of the capacity region. We also showed that, in specific network topologies, the low-delay capacity region can be further improved.

3. The Foster-Lyapunov theorem and its variants serve as the primary tools for studying the stability of queueing systems. In addition, it is well known that setting the drift of the Lyapunov function equal to zero in steady-state provides bounds on the expected queue lengths. However, such bounds are often very loose due to the fact that they fail to capture resource pooling effects. We recently showed that the approach of "setting the drift of a Lyapunov function equal to zero" can be used to obtain bounds on the steady-state queue lengths which are tight in the heavy-traffic limit. The key is to establish an appropriate notion of state-space collapse in terms of steady-state moments of weighted queue length differences, and use this state-space collapse result when setting the Lyapunov drift equal to zero. As an application of the methodology, we prove the steady-state equivalent of the heavy-traffic optimality result of Stolyar for wireless networks operating under the MaxWeight scheduling policy.

**Honors and Awards Received During Reporting Period**

- Distinguished Lecturer, IEEE Communications Society, 2011-12
- Invited Speaker, Stochastic Networks Conference, Isaac Newton Institute for Mathematical Sciences, University of Cambridge, March 2010
• Keynote Speaker, the 4th International Wireless Internet Conference (WICON), Maui, Hawaii, November 2008
• Plenary Speaker, Fourth International Conference on Queueing Theory and Network Applications (QTNA), July 2009
• Keynote Speaker, INFOCOM Student Workshop, March 2010

**List of Journal Papers**


