# Model-based Structural Health Monitoring of Fatigue Damage Test-bed Specimens

**Authors:**
- Kincho H. Law, Ph.D.
- Jerome P. Lynch, Ph.D.
- Masahiro Kurata, Ph.D.

**Performing Organization:**
University of Michigan
2380 G G Brown Building
Ann Arbor, MI 48109-2125

**Sponsoring/Monitoring Agency:**
Office of Naval Research
Attn: Paul Hess
875 North Randolph Street
Arlington, VA 22203-1995

**Distribution/Availability Statement:**

**Abstract:**
This research project represents a preliminary investigation into a comprehensive analytical, numerical and experimental framework for the monitoring and life-cycle assessment of the structural integrity and performance of aluminum hull structure. Aluminum plate specimens are designed and fabricated to facilitate the investigation of system identification and damage detection methodologies, both of which are key components of future life-cycle analyses. The design of the plate specimens is intended to include the geometric complexity commonly found in aluminum ship hull structures. A powerful Bayesian probabilistic model updating framework is applied to validate its potential applicability for damage identification around critical weld zones where fatigue failure is likely to initiate. Extensive numerical simulation and experimental testing has been conducted to validate the model-based approach to crack damage detection in aluminum plate structures.
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Executive Summary
The U.S. Navy is currently exploring the adoption of novel ship design concepts that support offshore basing and littoral combat operations: examples include the Joint High-Speed Vehicle (JHSV) and Littoral Combat Ship (LCS) programs. As part of the JHSV and LCS programs, light-weight materials (e.g., aluminum) and non-conventional hull forms (e.g., catamaran-style hulls) are being evaluated for adoption. Compared to steel mono-hull ships, the U.S. Navy has comparatively less design and operational experience with high-performance aluminum vessels. Hence, a dire need exists for the development of comprehensive life-cycle monitoring and assessment strategies that can track the performance, state of health and expected lifespan of operational high-speed, aluminum ships exposed to demanding seaway loads. In response to this life-cycle monitoring framework, the Naval Surface Warfare Center-Carderock Division (NSWCCD) has proposed a comprehensive research program in improving the characterization of the structural materials for ship reliability. The life-cycle framework includes the development of effective structural health monitoring (SHM) methodologies to support early detection of structural failures (e.g., fatigue failure).

This research project represents a preliminary investigation into a comprehensive analytical, numerical and experimental framework for the monitoring and life-cycle assessment of the structural integrity and performance of aluminum hull structure. Aluminum plate specimens are designed and fabricated to facilitate the investigation of system identification and damage detection methodologies, both of which are key components of future life-cycle analyses. The design of the plate specimens is intended to include the geometric complexity commonly found in aluminum ship hull structures. A powerful Bayesian probabilistic model updating framework is applied to validate its potential applicability for damage identification around critical weld zones where fatigue failure is likely to initiate. Extensive numerical simulation and experimental testing has been conducted to validate the model-based approach to crack damage detection in aluminum plate structures. Future efforts include continuing collaboration with the researchers at NSWCCD in the fatigue testing of aluminum ship hull components and continued development of the model-based damage detection methodology as an integral part of a more general life-cycle design, monitoring and maintenance framework. The long-term benefit of the proposed SHM strategy is the reduced total-cost-of-ownership for aluminum littoral combat ships in addition to more resilient ship structures for use in demanding seaway environments.
Acknowledgement
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1. Objectives

The U.S. Navy is currently exploring the adoption of novel ship design concepts that support off-shore basing and littoral combat operations; examples include the Joint High-Speed Vehicle (JHSV) and Littoral Combat Ship (LCS) programs (Hess 2007). As part of the JHSV and LCS programs, light-weight materials (e.g., aluminum) and non-conventional hull forms (e.g., catamaran-style hulls) are being evaluated for adoption. In addition to the material properties being high corrosion resistant, the use of aluminum alloys in the design of naval structures offers the benefit of light-weight ship constructions that favor high speeds at sea. However, compared to steel mono-hull ships, the U.S. Navy has comparatively less design and operational experience with high-performance aluminum vessels. Hence, a dire need exists for the development of comprehensive life-cycle monitoring and assessment strategies that can measure the performance, state of health and expected lifespan of operational high-speed, aluminum ships exposed to demanding seaway loads. Current practice in engineering design and maintenance are primarily oriented towards serviceability and safety but typically exclude consideration of possible damage in the structure. For life-cycle design, it is critically important to go beyond the basic load scenarios, standard material behavior, and service and safety criteria. Methods for lifetime-oriented design must be able to describe damage initiation, damage evolution, and remaining life resistance, as well as to estimate the reliability and performance of a structural system including those with damage (Stangenberg et al. 2009).

The use of aluminum poses a number of technical challenges for the naval engineering community including a higher incidence of fatigue-related cracks. Fatigue-related damage in aluminum alloys often appear as widespread micro-cracks. High-speed aluminum hulls can remain in operation even after the initiation of micro-cracks because of the ductile mechanical characteristic of the material. However, fatigue cracks in aluminum can grow quickly resulting in rapid deterioration of the hull. Therefore, one key component to the life-cycle monitoring framework is the early detection of fatigue-induced cracks. Early detection of potential damage to the aluminum ship structure not only enhances the effectiveness of maintenance strategies, but also prevents the catastrophic failure of the hull. Furthermore, monitoring the integrity of the aluminum hull can provide valuable information that can be used to accurately estimate the residual life of hull components.

The Naval Surface Warfare Center - Carderock Division (NSWCCD) has proposed a comprehensive research program in improving the characterization of structural aluminum alloy materials for ship reliability. The NSWCCD effort places a particular emphasis on fatigue cracking in aluminum welded connections and assemblies common to LCS ship designs. In collaboration with researchers at NSWCCD, this study represents a preliminary investigation of a framework for the monitoring and performance assessment of aluminum hull components. Specifically, aluminum plate specimens have been designed and built with welded assemblies to facilitate the investigation of a system identification and damage detection methodology. This report describes a model-based damage detection methodology for autonomous structural health monitoring of aluminum hull structures monitored with sensors. The area of damage in the structure is estimated by comparing the structural characteristics of the “true” structure (damaged or undamaged) and hypothesized (or “trial”) finite element method (FEM) models. The probability associated with a hypothesized damage state (e.g., location and size) is evaluated through the calculation of an error between the “true” and “trial” models. The probable damage...
area (i.e., fatigue crack path) is identified by repeatedly applying a Bayesian inference algorithm
(Sohn and Law 2002). This inference framework has the advantage of providing a probabilistic
basis for guiding the selection of trial models with the aim of substantially reducing the
computational demands associated with exhaustive searches. The Bayesian probabilistic model-
updating framework is explored in this study for its potential applicability to damage
identification around critical weld zones on standard aluminum plate-stiffener specimens.
Specifically, extensive numerical simulations and experimental tests have been conducted to
examine the model-based Bayesian approach for the structural health monitoring of aluminum
hull components that are vulnerable to fatigue failure.

2. Theoretical Background and Algorithm

2.1. Model Updating for Damage Detection
A general class of damage detection algorithms is based on the concept of model updating
(Doebling et al. 1998). In model updating, parameters in an analytical model (typically based on
physical principles) are varied until the model approximates the behavior of the true observed
system. To identify an optimal model, the outputs from the observed system (namely, sensor
measurements) are used to evaluate a pre-defined objective (or error) function. An appropriate
objective function is one that takes into account the fundamental behavior of the system in both
its damaged and undamaged states, yet is compatible with the experimental data available.
Changes in the model parameters are then correlated to the (damaged versus undamaged)
condition of the structure.

Consider a simple plate structure. The governing equation describing the dynamic behavior of a
vibrating plate can be written in the following form (Chakraverty 2008):

\[ D \nabla^4 w + \rho h \frac{\partial^2 w}{\partial t^2} = q(x,y,t) \]

Eq. 1

where \( D \) is the plate flexural rigidity and is defined as:

\[ D = \frac{E h^3}{12(1-v^2)} \]

Eq. 2

Here, \( w \) denotes the vertical displacement of the plate, \( q(x,y,t) \) is the normal load distribution
function on the top of the plate, \( \rho \) is the plate density (mass per unit area), \( h \) is the plate
thickness, \( E \) is the Young’s (elastic) modulus and \( v \) is the Poisson ratio of the plate material.
The in-plane coordinates of the plate are denoted as \( x \) and \( y \) while \( t \) denotes time. To model the
plate under complex loading conditions or when the plate possesses non-uniform material
properties, the finite element method (FEM) can be used as an effective analytical method
(Hughes 2000).

For a finite element model with \( N \) elements, the elastic moduli for the elements can be denoted as:
For modeling purposes, this project models fatigue crack damage as a change in the flexural rigidity or stiffness of the plate, which, in turn, is reflected by a reduction in the elastic modulus of the damaged elements. If the model contains \( n \) damaged elements, their elastic moduli \( \{E_{i+1}, \ldots, E_{i+n}\} \) are replaced by

\[
E_d = \{k_{i+1}E_{i+1}, \ldots, k_{i+n}E_{i+n}\}
\]

where \( k_{i+j} \) denotes the reduction factor for the \((i+j)\)th damaged element.

Model updating as the primary means of performing damage detection is essentially a combinatorial optimization (CO) problem that seeks to find the optimal set of elastic moduli that minimizes a defined objective function by comparing the FEM model output and the actual system output derived from sensor measurements. Assuming an linear variation in \( k_{i+j} \), the CO problem is defined by an infinitely large state space that is impossible to exhaustively search to identify the best model parameters. In addition, the objective functions typically used in CO are rarely convex function; rather, they are defined by a family of local minima one of which is the true global minimum (i.e., the optimal model). As a result, gradient decent methods are prone to converge to local minima which can be far from the true global minimum. As a result, CO problems are extremely complex problems to solve. In this study, a probabilistic Bayesian methodology is adopted to balance computation effort with the accuracy of the model updating solution.

### 2.2. A Flexibility-Based Objective Function

Conceptually, damage detection is to identify changes (i.e., damage) in a structure by using specific structural characteristics as the basis for evaluation. Vibration data are among the most common sensor information easily obtained from dynamic excitations. Vibration-based damage identification methods using mode-shape information have been proposed (Doebling et al. 1998). One approach is to detect damage directly based on the changes in structural characteristics between damaged and undamaged structures. Examples of structural characteristics include curvature mode shape (Pandey et al. 1991), flexibility parameters (Pandey and Biswas 1995, Bernal 2006), strain and modal strain energy (Stubbs et al. 1995; Zonta and Bernal 2006).

Although current research in damage detection has made substantial progress, the methods developed so far are primarily restricted to simple beam or frame structures with a limited number of degrees-of-freedom. In contrast, continuum systems such as plate-like structures pose many challenges for current methods because of the large number of degrees-of-freedom that are required to properly model the structure. Additionally, it is not realistic to place a sufficient number of sensors to measure the system response at all of its degrees-of-freedom. An alternative approach is to construct a system model (such as a finite element model) of a targeted structure using measurement data. This "model-based" approach updates the system model by modifying the structural properties of the elements until the model resembles the dynamic characteristics estimated from sensor information. The objective is thus to define an objective (error) function and to compute the difference (error) between the trial finite element model and
the target structure. Damage, if any, is identified and revealed through subtle changes in the model parameters extracted during the model updating process.

For the plate problem, our study indicates that the use of flexibility matrices constructed based on modal properties (i.e., modal frequencies and mode shapes), as opposed to the direct use of modal properties, is one appropriate choice for the objective function (Kurata et al. 2010). The inverse relationship between the flexibility matrix and the square of the modal frequency renders the flexibility matrix less sensitive to high frequency modes which are difficult to identify in monitoring data (due to the electrical noise inherent to the data collection process). This unique characteristic allows for the inclusion of lower-order modes in a truncated flexibility matrix. This feature has attracted many researchers to explore flexibility as a core element in developing structural damage detection algorithms (Bernal 2006; Gao and Spencer 2002; Pandey and Biswas 1994; Pandey and Biswas 1995; Zonta and Bernal 2006). In the so-called “flexibility-based approach” (FBA) described herein, the objective (error) function is expressed in terms of the difference between the flexibility matrices that correspond to the true system(measured) and trial models (finite element).

When the mode shapes are mass-normalized (i.e., \( \phi^T M \phi \equiv I \)), the flexibility matrix \( F \) can be expressed in terms of the system modal properties as follows:

\[
F = \Omega^{-1} \phi^T = \sum_{i=1}^{N} \frac{1}{\omega_i} \phi_i \phi_i^T 
\]

Eq. 5

where \( M \) is the mass matrix, \( \phi = [\phi_1^T \phi_2^T ... \phi_N^T] \) is the mode shape matrix, \( \Omega = \text{diag}(\omega_i^2) \) is the spectral matrix consisting of the square of the modal frequencies, \( \omega_i \), and \( N \) is the number of degrees-of-freedom in the system. The mass-normalized modal vector, \( \phi_i \), is related to the arbitrarily scaled mode shape, \( \phi_i \), as:

\[
\phi_i = d_i \phi_i^m
\]

Eq. 6

where \( d_i = \frac{1}{\sqrt{\phi_i^T M \phi_i}} \) is a mass normalization constant for the \( i^{th} \) mode.

Suppose only a few (typically lower) modes are available, then a truncated flexibility matrix is obtained as:

\[
F_{\text{run}} = \sum_{i=1}^{n} \left( \frac{d_i}{\omega_i} \right)^2 \phi_i \phi_i^T
\]

Eq. 7

where \( n \) denotes the number of modes available.
Let us define the difference ($\Delta F_{\text{run}}$) between the flexibility matrices of the true (damaged) structure and the trial FEM model as:

$$\Delta F_{\text{run}} = F_{\text{run}}^{\text{true}} - F_{\text{run}}^{\text{trial}}$$  \hspace{1cm} \text{Eq. 8}

When a trial FEM model reasonably resembles the damaged structure, the difference in flexibility matrices is close to zero (exactly zero if there is no measurement noise nor modeling error). The scalar magnitude on the difference in the flexibility matrices can be measured by calculating the Frobenius norm of $\Delta F_{\text{run}}$:

$$\|\Delta F_{\text{run}}\|_F = \sqrt{\sum_i \sum_j x_{ij}^2}$$  \hspace{1cm} \text{Eq. 9}

which vanishes when all matrix elements $x_{ij}$ in $\Delta F_{\text{run}}$ are zero (meaning the FEM model perfectly matches the observed structure). The difference in the flexibility matrices can also be further decomposed into singular values by singular value decomposition (SVD):

$$\Delta F_{\text{run}} = USV^T$$  \hspace{1cm} \text{Eq. 10}

where $V^T$ and $U$ are matrices of singular vectors, and $S$ is the diagonal matrix whose elements are the singular values, $s_j$. Since the Frobenius norm is invariant under unitary transformation, the SVD of the $\Delta F_{\text{run}}$ yields

$$\|\Delta F_{\text{run}}\|_F = \|USV^T\|_F = \|[S]\|_F = \sqrt{s_1^2 + s_2^2 + \cdots + s_R^2}$$  \hspace{1cm} \text{Eq. 11}

where $R$ is the rank of $\Delta F_{\text{run}}$. Thus, the objective is to search for a trial FEM model that minimizes the Frobenius norm shown in Eq. 11.

Since the mode shapes experimentally obtained are arbitrarily scaled, the mass normalization constants ($d_i$) are required \textit{a priori} to properly compute the flexibility matrix of the real, true structure. One approach to extracting the mass normalization constants is based on testing the structure with a perturbed mass matrix (by adding a known mass at a certain location) and examining the sensitivity of the eigenvalues (Brinker and Anderson 2002; Parloo et al. 2002). In this study, mass normalization constants estimated using the FEM model are updated at each trial and applied to the experimentally obtained (and not mass normalized) mode shapes. That is, the constants are extracted by comparing the mass-normalized mode shapes and displacement-normalized mode shapes, where both mode shapes are available as options in many commercial FEM programs (e.g., ABAQUS).
2.3. Bayesian Probabilistic Approach
The model updating procedure adopted in this study is based on a Bayesian probabilistic approach which utilizes the parameters measured or estimated from a series of collected vibration signals or data. Unlike a deterministic optimization formulation, the state space search must reflect the relative degree of belief on the estimates of the optimal subset (i.e., $E_d$ in this case). Let $M$ denote the hypothesized damage states of the model. The calculation of the error that exists between the “true” structure and the FEM “trial” model is based on the measured or estimated structural parameters. The updated estimates on the damage of the structure are expressed as the posterior distribution based on Baye’s rule as follows:

$$p(M \mid s) = \frac{p(s \mid M)p(M)}{p(s)} \propto p(s \mid M)p(M)$$

where $p(M \mid s)$ is the posterior distribution function for a hypothesis $M$ given the measured or estimated parameters, $s$. $p(s \mid M)$ is termed the likelihood function, $p(M)$ is the prior probability of the hypothesis and $p(s)$ is treated here as some normalization constant. By collecting the likelihood function, the posterior distribution $p(M \mid s)$ would progressively become a better estimate than the prior probability $p(M)$ as the process goes on. For instance, if the initial estimate of the probable area of damage is assumed to be uniformly distributed over the structure before the model updating procedure, the most likely damaged areas are revealed with relatively higher posterior estimates by the repeated application of the Bayesian inference algorithm.

The selection of the most probable events from all conceivable possibilities using the Bayesian probabilistic approach can be systematically implemented using various “optimal” search methods; otherwise a random search in the state space becomes a computationally intractable task. To sample the posterior parameter distribution, the Bayesian approaches implemented with the Markov Chain Monte Carlo (MCMC) method and genetic algorithms (GA) have been reported with successful performance in detecting structural damage detection (Vanik et al. 2000; Cheung and Beck 2009; Stull et al. 2009; Nichols et al. 2009). However, these approaches are computationally expensive and they do not guarantee convergence to an optimal solution. To reduce the computational effort, the Bayesian damage detection algorithm proposed herein is enhanced with a branch-and-bound (BB) search technique where the state space searched is systematically narrowed through enumeration and pruning of candidate model solutions (Sohn and Law 2002).

The BB technique is a general search method originally developed for discrete optimization problems and is a powerful technique for controlling the size of a search space used in model updating (Norkin et al. 1998). As illustrated in Figure 1, the BB algorithm initially starts its search from some random subspace (i.e., leaf nodes associated with Branch 1 in Figure 1(a)). At each step, the algorithm takes an additional sample (i.e., an additional element in the hypothesized subset) at each leaf node and aims to improve its estimates (this process is called branching). As the search proceeds, the branches associated with large “errors” are eliminated from further consideration and the search is bounded by evaluating the remaining branches (this process is called pruning). Figure 1(b) schematically shows the application of the BB technique.
for FEM updating of a plate-type structure. Here, the crack is assumed to damage the entire section of the plate in the thickness direction and represented in the model by a set of finite elements with reduced elastic modulus for simplicity. The collection of survived leaf nodes (or “trial” model with small error) is mapped into a damage map at every branching process. Therefore, the probable damage area in a system can be systematically updated by implementing the BB technique in the Bayesian formulation. The computational effort and accuracy of the estimates for the optimal subset (i.e. $E_d$) can be controlled by selecting an optimal pruning rate at each branch.

2.4. Summary of the Model Based Bayesian Damage Detection Approach
The model-based structural health monitoring framework consists of three phases as shown in Figure 2. The first phase is the construction of the FEM model with structural properties and boundary conditions close to that of the target structure. In the second phase, the monitoring system acquires information about the status of the target structure through a network of sensors installed in the structure. Once the variation in the modal properties exceeds a pre-defined threshold value, the third phase for damage detection begins. The model-based damage detection
algorithm computes the error associated with an initial set of hypotheses using an objective function. Engineering judgment based on knowledge about common damage patterns can be very useful to account for the selection of the initial set of hypotheses; for instance, heat affected zones (HAZ) around the hull welds or notches along component edges are good initial candidates for the hypothetical damage initiation areas. The branching process adds one more damaged element to each hypothesis following the branching rule shown in Figure 1(b). The hypotheses with relatively large error are pruned before the next branching process. The branching, error computation and pruning processes make a finite loop until the damage identification algorithm terminates. At the end of each loop, the elements surviving in the current and previous pruning processes are classified and stored in a bin based on their error. The histogram-like plot of survived elements serves as a damage map showing the probable area(s) of damage. The search process is terminated when the deviation between the current and last damage areas converges within a predefined tolerance.
Figure 2. Flowchart of the model-based Bayesian damage detection system
3. Numerical Simulation and Experimental Validation

The Bayesian model-updating procedure is applied to the problem of damage detection on a stiffened aluminum plate, as shown in Figure 3(a). The design of the structural plate is intended to include the geometric complexity that is commonly found in aluminum ship hull structures. The aluminum plate includes an area with high stress concentrations due to the presence of a welded stiffener. Such areas are the likely locations for fatigue-related damage initiation (i.e., fatigue crack). Knowledge of this fact allows one to customize the model updating algorithm to prioritize the search of this area for damage. The aluminum plate is 24-in by 48-in and is 1/4-in thick. In addition, the plate has a 2-in by 18-in by 1/4-in stiffener plate welded to it off-center. The base plate and the stiffener plate are rigidly welded by a tungsten inert gas (TIG) weld. Three different crack paths initiating from the HAZ around the welded stiffener plate are considered as example damage scenarios as shown in Figure 3(b) and denoted as crack paths 1, 2 and 3.

![Figure 3. A Test-bed Plate Structure](image-url)
The plate structure is modeled using ABAQUS, a general-purpose FEM analysis program. The base plate and stiffener plate are modeled with 4-node reduced integration, doubly curved shell elements with hourglass control (S4RSW) (ABAQUS 2010). These plates are assumed to be rigidly connected. The mesh size of the plate elements are 1 in × 1 in. The elastic modulus $E$ of the undamaged elements is 10300 ksi while for the cracked elements, the elastic modulus is reduced $10^6$ times from the original modulus ($E_d = E \times 10^6$). The mass density of the aluminum is assumed to be $2.489 \times 10^{-7}$ slug/in$^3$. The Lanczos frequency analysis method in ABAQUS is employed to compute the modal properties (i.e., modal frequency and mode shape) of the base structure as well as the structural model with different damage scenarios introduced. Only the first five modes are considered in the numerical, as well as experimental simulations. Additionally, sensors are placed (or assumed to be placed) at strategic locations as shown in Figure 3(c) by the “x” marks.

3.1 Numerical Simulations

For the numerical simulation study, the test-bed plate structure as shown in Figure 3(a) is employed for evaluating the model-based damage detection algorithm. Three separate cases are considered. First, damage detection of the plate with individual cracks and initial probable damage zones as shown in Figure 3(b) and 3(c), respectively, is investigated. Second, similar to the first damage detection case with the three individual crack paths, the plate is assumed to have another welded stiffener and the initial probable damage zone is expanded to include other possible damage initiation areas. For the third case, damage detection with multiple simultaneous cracks is studied.

**Damage Detection of Individual Crack Paths with a Single Stiffener:** The modal properties for the undamaged plate structure and the damaged plate with cracks are obtained first. Table 1 summarizes the modal frequencies obtained for the undamaged and damaged plates. As an example, Figure 4 shows the mode shapes of the model with damage introduced as crack path 1. It can be seen that the first and third modes are bending modes, the second and fourth are torsional modes, and the fifth is a bending mode in both directions of the plate. Using the computed modal frequencies and mode shapes, the flexibility matrices of the damaged models are constructed using Eq. 7.

### Table 1 Modal frequencies of the damaged and undamaged plates

<table>
<thead>
<tr>
<th>Mode</th>
<th>No crack (Hz)</th>
<th>Crack 1 (Hz)</th>
<th>Crack 2 (Hz)</th>
<th>Crack 3 (Hz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Diff (%)</td>
<td>Diff (%)</td>
<td>Diff (%)</td>
</tr>
<tr>
<td>1</td>
<td>26.88</td>
<td>26.52</td>
<td>-1.34</td>
<td>26.82</td>
</tr>
<tr>
<td>2</td>
<td>41.76</td>
<td>39.89</td>
<td>-4.48</td>
<td>41.60</td>
</tr>
<tr>
<td>3</td>
<td>72.87</td>
<td>70.89</td>
<td>-2.72</td>
<td>72.02</td>
</tr>
<tr>
<td>4</td>
<td>94.93</td>
<td>89.49</td>
<td>-5.73</td>
<td>94.88</td>
</tr>
<tr>
<td>5</td>
<td>128.95</td>
<td>126.93</td>
<td>-1.57</td>
<td>128.64</td>
</tr>
</tbody>
</table>
The model-updating procedure (as described in Figure 2) is employed to identify the individual cracks introduced in the aluminum hull specimen. Figure 5 illustrates the basic process in locating crack path 1 shown in Figure 3(b). Starting from probable damaged elements in the vicinity of the weld (i.e., HAZ area shown in Figure 3(c)), at each branching step the probabilistic branch-and-bound scheme proceeds to search for the most likely set of damaged elements based on the objective function shown in Eq. 11, which evaluates the difference between the trial FEM model and the true damaged FEM model. The pruning rate is set to 50% in this simulation (i.e., only the lower 50% of the hypotheses tested defined by the smallest errors are retained in the next branching process). In the damage maps, the elements filled with darker color have the higher probabilities of being damaged. These damage maps are created by counting the number of instances an element is included in a retained trail FEM model; in effect, it is a histogram of frequency of inclusion of an element in the hypothesized damage set. Probabilistically, the more frequently an element appears in the Bayesian hypothesized states, the more likely it is a truly damaged element (i.e., an element through which the fatigue crack has propagated). As shown in Figure 5, the procedure progressively converges to the crack region and is terminated when the variation between successive branching steps is negligible; for this study, this was found between the 8th and 9th branching steps.

The probabilistic model-based updating procedure is also applied for the detections of crack paths 2 and 3 as shown in Figure 3(b). These two cracks are slightly shorter than crack 1 and they are located internally inside the plate. The detection of these two cracks is more challenging because of the relatively small changes in the modal properties between the undamaged and damaged plates as shown in Table 1. As shown in Figure 6(a), the model-updating algorithm is able to identify accurately the location of crack path 2. It can be seen from the damage map that the search for probable damaged elements converges quickly with a modest number of branching steps.

Crack path 3 is a particularly challenging case because the crack is oriented in the longitudinal direction (i.e., along the longer side of the plate). It may also be interesting to point out that the finite elements are uniformly and regularly distributed in the mesh. As shown in Table 1, there are very little changes in the modal properties between the damaged and the undamaged plates. 

Figure 4. Mode shapes of the baseline structure with crack 1
Figure 5. Histogram-like damage map showing the basic process for locating probable damage region for crack path 1.

Figure 6. Damage detection results for individual crack paths 2 and 3

Nonetheless, as shown in Figure 6(b), the model-updating procedure is able to locate the probable damage area that includes the crack but the area is distributed in the transverse direction (i.e., along the shorter side of the plate). Furthermore, the probable damage area converges very quickly with very few branching processes.

**Damage Detection of Individual Cracks with Multiple Stiffeners:** The model-updating procedure is applied to detect damage in a plate with two welded stiffeners as shown in Figure 7(a). Similar to the previous cases, the three crack paths as shown in Figure 3(b) are considered individually. However, the initial probable damage zone is expanded to include the weld toes (i.e., HAZ effect) around the two stiffeners and along the longitudinal edges (i.e., notch defects) of the plate. Figure 7(b) shows the probable damage areas identified. For the damage detection for crack path 1, areas around the longer stiffener and along the top edge are suspected for possible damage very early in the process. The hypothetical branches grow both from the weld
(a) A baseline plate structure with two stiffeners

(b) Damage detection results with individual cracks and multiple stiffeners

Figure 7. Damage detection of cracks on a plate with multiple welded stiffeners

toe and edges but quickly coalesce (by branch step 6) around the most probable damage area in the vicinity of crack 1. By branch step 8, the model updating process converges and stopped.

For the detection of crack 2, the model-updating algorithm is not able to pin-point the damage area with high probability. Instead, the detection result suggests two damage prone areas as shown in Figure 7(b). Nevertheless, the area around crack 2 is included in one of the probable damage areas. For detecting the damage in crack 3, the model-updating procedure identifies the damage area successfully with high probability, as noted with darker color elements shown in Figure 7(b). All in all, the damage detection results show the ability of the probabilistic model-updating algorithm in detecting cracks on a plate with multiple stiffeners and the rapid convergence in identifying the damage areas even when the initial probable damage areas include relatively wide spread regions, as in this case, around the stiffeners and along the edges of the plate.

Damage Detection of Multiple Cracks: In this numerical simulation study, the model-updating procedure is applied to detect multiple cracks on the plate structure. Specifically, two spatially isolated cracks situated in the same plate are simulated for the one- (Figure 3) and two-stiffener (Figure 7(a)) hull assemblies. Figures 8(a) and 8(b) show two different multi-crack cases introduced within the single-stiffener plate system. As shown in Figure 8(a), the model-updating
Figure 8. Damage detection of multiple cracks

procedure is able to successfully identify both cracks next to the stiffener which are located internally within the plate. However, as shown in Figure 8(b), only the longer and more “dominant” crack situated diagonally to the top edge of the plate is identified by the damage detection algorithm. Similar results for the damage detection of two cracks with the two-stiffener plate system are shown in Figures 8(c) and 8(d). As shown in Figure 8(c), the model-updating procedure is able to detect both internal cracks with similar size located next to each of the stiffeners. However, as shown in Figure 8(d), only the damage from the longer crack originating from the bottom edge of the plate is identified. It is suspected that the positioning of a crack at the edge of the plate may “dominate” the plate dynamics such that any crack away from the edge is hard to identify due to its low participation in the global plate dynamics. As a result, further investigation of the Bayesian finite element model updating framework for the undamaged and damaged plates may be necessary to resolve this issue.

3.2 Experimental Simulation

A number of single-stiffener plate specimens have been fabricated in full-scale according to the schematic shown in Figure 3(a). The intent of the experimental specimens is to test the applicability of the probabilistic model-updating procedure for crack damage detection. A more detailed overview of the experimental setup is shown in Figure 9. The specimens are made of marine grade aluminum alloy (5086). As shown in Figure 9(a), an 18 in long stiffener plate has been welded to each of the tested plates with 0.625 in long discrete TIG welds at 5 locations (with a spacing of 4.5 in) to avoid excessive distortions that might result from weld heat. The base plates have been pre-heated using a gas torch to expedite the welding process and to minimize local warping. As shown in Figure 9(b), the base plates are rigidly fixed along the shorter edges using 8-3/8 in stainless screws with round aluminum washers; furthermore, the assemblage is rigidly connected to a large steel beam post-tensioned to the concrete strong floor in the laboratory. Figure 9(c) shows a diagonally cut (resembling crack path 2 as shown in Figure 3(b)) next to the stiffener. As shown in the figure, MEMS-based accelerometers (Crossbow CXL02LF1Z) are used to measure the out-of-plane vibrations of the specimen during forced vibration testing. Furthermore, the Narada wireless system developed at the University of Michigan, which has been successfully implemented on a naval vessel for hull monitoring (e.g.,
all-aluminum FSF-1 SeaFighter), is deployed as the primary data acquisition system for the collection of plate acceleration data (Swartz and Lynch 2009).

Modal Property Extraction and Calibration: Both impact hammer tests (i.e., pure impulsive loading) and hand tapping (i.e., colored broadband noise) tests were applied on each plate specimen. For each plate specimen, albeit the undamaged plate or the plates with cracks introduced, the hammer test is applied 6 times at the locations away from the weld and crack damage area as shown in Figure 9(a). Additionally, each plate specimen is hand tapped 5 times at random locations to simulate colored ambient excitation. The frequency domain decomposition (FDD) technique is employed to extract the modal properties of the plate specimens using the measured acceleration data. The FDD technique is widely used for modal parameter estimation and is based on the classical complex mode identification function (Brinker et al. 2001; Shih et al. 1998). As illustrated in Figure 10, the estimated modal properties are very close for both the impact hammer and hand tapping tests.
For the calibration of the FEM model, the modal properties of the undamaged plate specimen (without cracks) are extracted from the vibration tests. Slight differences are observed on the modal frequencies between the experimental results and the original finite element model (used in the numerical simulation). Such differences are expected because of the variation in the idealized finite element model, plate material properties, and boundary conditions encountered in the experimental setup. To better calibrate between the numerical model and the experimental specimens, the elastic (Young’s) modulus used in the finite element model is reduced to 90% of the nominal values (10,300 ksi) so that the first mode frequencies of the undamaged plate from both the FEM model and the experimental tests are matched. Table 2 summarizes the modal frequencies of the plate structure with crack path 1 obtained from the experimental hammer and hand tapping tests as well as from the updated finite element model. It can be seen that the results match with the modal assurance criteria (MAC) exceeding 90%.

**Damage Identification Results:** The model updating procedure is applied to identify the damage area (cracks) based on the modal properties extracted from the plate specimens. Figure 11 shows the damage identification results for the three crack paths from both impact hammer and hand tapping tests. It can be seen that the model updating procedure successfully detects crack paths 1 and 3. However, for crack path 2, although the most probable damage area is identified near the damage area, the procedure fails to pin-point the actual crack location. It can also be observed
Table 2. Summary of modal properties extracted for specimen with crack path 1

<table>
<thead>
<tr>
<th>Mode</th>
<th>Impact hammer test Modal frequency</th>
<th>Hand-tapping test Modal frequency</th>
<th>Mode shape MAC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Test (Hz)</td>
<td>Updated FEM (Hz)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>24.22</td>
<td>25.17</td>
<td>0.998</td>
</tr>
<tr>
<td>2</td>
<td>35.89</td>
<td>37.89</td>
<td>0.983</td>
</tr>
<tr>
<td>3</td>
<td>65.43</td>
<td>67.36</td>
<td>0.992</td>
</tr>
<tr>
<td>4</td>
<td>85.57</td>
<td>85.14</td>
<td>0.982</td>
</tr>
<tr>
<td>5</td>
<td>122.07</td>
<td>120.43</td>
<td>0.932</td>
</tr>
</tbody>
</table>

Figure 11. Damage identification results from experimental test specimens

that, in general, the damage identification results based on impact hammer tests are more precise, as illustrated from the darker colors shown in the damage maps. It should be cautioned that, as in any experimental test, the fabrication qualities of the plate specimens and the welds do vary. The modal frequencies could vary up to 5-7%. Further study on the impact of modeling uncertainties is needed. The effects of noise on the probabilistic model-updating procedure are also worth further investigation.
4. Summary and Future Work
A model-based damage detection algorithm has been explored for structural health monitoring of aluminum ship hulls. A particular emphasis was placed on the detection of fatigue-related crack damage in plate-type hull assemblies. The algorithm combines vibration-based damage detection with a finite element method model to provide the basis for high-fidelity damage detection using a limited number of sensors installed in the assembly. The FEM model of the interested structure is successively updated in a Bayesian inference scheme employing a Branch-and-Bound technique as the primary systematic searching strategy. Numerical simulations and experimental tests have been conducted to illustrate the model-based monitoring framework and to validate the potential application of the probabilistic-based damage detection approach. Preliminary numerical and experimental results indicate that the model-based damage detection procedure is able to successfully identify crack damage areas, especially when the size of the crack is relatively large and strongly influences the global plate dynamics.

Currently, the Bayesian model updating framework is being applied to a more complex, but realistic hull component. Specifically, a series of fatigue tests of the MAHI (Monitored Aluminum Hull Integrity) specimens (welded aluminum assemblages of a bulkhead element and wide flange column) are underway at the Naval Surface Warfare Center-Carderock Division (NSWCCD). Our research efforts have included active engagement with researchers at NSWCCD on the fatigue tests both in instrumentation, data collection and finite element modeling of the specimens. Our future plan is to further apply and validate the model-based damage detection framework using the MAHI project.

Frequent inspection of ship hulls can extend the operational life of a ship since the detection of the onset of damage can reduce overall ship life-cycle costs. However, current visual inspections of the entire hull are both costly and labor-intensive. Therefore, the instrumentation of a structural health monitoring (SHM) system coupled with an effective damage detection methodology such as the approach presented in this report can reduce the cost of inspection by providing inspectors with a prioritized list of probable areas of damage. Such a list would effectively narrow down the areas of the hull requiring detailed inspections (perhaps by conducting nondestructive testing with ultrasonic waves, etc.) and repairs. Our future investigation will extend the current damage detection methodology as an integral component of a life cycle design, monitoring and maintenance framework for ship structure.
References


