TA5 Project 5.3


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# ACCELERATED TESTING AND PREVENTIVE MAINTENANCE IN ACQUISITION, MAINTENANCE AND OPERATION OF VEHICLE SYSTEMS USING TIME-DEPENDENT RELIABILITY/DURABILITY PRINCIPLES

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Standard Form 298 (Rev. 8-98)
Prescribed by ANSI Std Z39-18
Army Needs in Reliability, Maintenance and Logistics

- Reduce operations and maintenance costs
- Increase effectiveness of fleet logistics
- Control lifecycle cost and also use it in design and procurement
- Improve availability; schedule maintenance
- Use both analytical and experimental / field data to estimate reliability
Random Process leads to Time-Dependent Reliability
Research Statement

Develop methodologies to obtain a **preventive maintenance schedule** and to assess and improve the **reliability / durability** of vehicle systems using

- Experimental (field) data
- “Expert” opinion
- Predictive tools (physics-of-failure data)

Previously and currently at TARDEC

Current research
Overview

Part 1:

Optimal preventive maintenance schedule using time-dependent reliability and lifecycle cost

Part 2:

Accelerated testing method based on importance sampling using few tests which run for only a short time
Part 1: Optimal Preventive Maintenance Schedule
What is Reliability?
Cumulative Probability of Failure

Reliability at time \( t \) is the probability that the system has not failed before time \( t \).

\[
F_T^c(t_L) = P(\exists t \in [0, t_L], \text{such that} \ g(X(t), t) \leq 0)
\]

Cumulative Prob. of Failure

\[
F_T^i(t_L) = P(g(X(t_L), t_L) \leq 0)
\]

Instantaneous Prob. of Failure

Calculation Methods for \( F_T^c(t) \)

- Maximum Response Method
- Niching GA & Lazy Learning Local Metamodelling
- MCS / Importance sampling

\[
F_T^c(t) = 1 - \exp \left[ - \int_0^t \lambda(t) \, dt \right]
\]
Definition of Lifecycle Cost

Lifecycle Cost = Production Cost + Inspection Cost + Expected Variable Cost

Quality

Time-Dependent System Reliability
Definition of Lifecycle Cost

\[ C_L(d, X, t_f, r) = C_P(d, X) + C_I(d, X, t_0) + C_V^E(d, X, t_f, r) \]

- **Lifecycle Cost**
- **Production Cost**
- **Inspection Cost**
- **Expected Variable Cost**

\[ C_V^E(d, X, t_f, r) = \int_{0}^{t_f} c_F(t) e^{-rt} f_T^c(t) dt \]

- Final time
- Interest rate
- Cost of failure at time \( t \)
- PDF of time to failure time

\[ F_T^c(t_L) = P(\exists t \in [0, t_L], such\,\, that\,\, g(X(t), t) \leq 0) \]
Preventive Maintenance Schedule

Estimation of Time for Preventive Maintenance

\[
\begin{align*}
\max_{d, \mu_X, \sigma_X, t_M} & \quad t_M \\
\text{s. t.} & \quad C_L(d, \mu_X, \sigma_X, t_M, r) \leq C_L^t \\
& \quad F_T^c(d, X, t_M) \leq 1 - R^t(t_M) \\
& \quad d_L \leq d \leq d_U \\
& \quad \mu_{X_L} \leq \mu_X \leq \mu_{X_U} \\
& \quad \sigma_{X_L} \leq \sigma_X \leq \sigma_{X_U}
\end{align*}
\]
A Roller Clutch Example

Constraints:

- Contact angle $\alpha = 0.11 \pm 0.06 \text{ rad}$
- Torque $\tau \geq 3000 \text{ Nm}$
- Hoop stress $\sigma_h \leq 400 \text{ MPa}$

Random Variables: $D$, $d$, $A$

Due to degradation:

$D \rightarrow D(1 - kt)$

$d \rightarrow d(1 - kt)$

$A \rightarrow A(1 + kt)$

with: $k = 2.5E-04 \text{ mm/ year}$

$$g_1(D,d,A) = 0.05 - \cos^{-1}\left(\frac{D-d}{A-d}\right) \leq 0$$

$$g_2(D,d,A) = \cos^{-1}\left(\frac{D-d}{A-d}\right) - 0.17 \leq 0$$

$$g_3(D,d,A) = 3000 - NL\left(\frac{\sigma_c}{c_1}\right)^2 \frac{D^2d}{4(D+d)} \sqrt{1 - S^2} \leq 0$$

$$g_4(D,d,A) = \frac{N}{2\pi}\left(\frac{\sigma_c}{c_1}\right)^2 \left(\frac{Dd}{(D+d)}\right) S \left(\frac{B^2 + A^2}{A\left(B^2 - A^2\right)}\right) - 400E06 \leq 0$$
Roller Clutch: Lifecycle Cost

\[ C_L = C_P + C_I + C^E_V \]

where:

\[ C_P = \left( 3.5 + \frac{0.75}{3\sigma_D} \right) + \left( 3.0 + \frac{0.65}{3\sigma_d} \right) + \left( 0.5 + \frac{0.88}{3\sigma_A} \right) \]

\[ C_I = 20F^i_T(X, t_0) \quad \text{Scrap cost/unit} \]

\[ C^E_V = \int_0^{t_f} 20e^{-rt} f^c_T(t)dt \quad \text{Failure cost/unit (warranty cost)} \]

\[ t_f = 10 \text{ years} \quad r = 3\% \]
Roller Clutch: Reliability vs Time-to-Maintenance

System Reliability vs Time

- Initial design
- Optimal design

Maximum $t_M$ limited by side constraints

$t_M = 7.01$ years
$t_M = 12.77$ years

$R_i = 0.9$

$C_L^t = \$18$
$C_L^t = \$19$
$C_L^t = \$20$
$C_L^t = \$21$
$C_L^t = \$22$
$C_L^t = \$23$
$C_L^t = \$24$
$C_L^t = \$25$
$C_L^t = \$26$
$C_L^t = \$27$
$C_L^t = \$28$
Roller Clutch: Pareto Optimality between Time-to-Maintenance and Cost

Target Lifecycle Cost vs Time for Maintenance

Maximum $t_M$ limited by side constraints

$t_M = 12.77$ years
Roller Clutch: Pareto Optimality between Time-to-Maintenance and Cost

Design Variables:

\[ \boldsymbol{\mu}_X = \{\mu_D, \mu_d, \mu_A\} \quad \sigma_X = \{\sigma_D, \sigma_d, \sigma_A\} \]

Side Constraints:

\[
\begin{align*}
55.0973 & \leq \mu_D \leq 55.4973 \\
22.66 & \leq \mu_D \leq 23.06 \\
101.49 & \leq \mu_A \leq 101.89 \\
0.04 & \leq \sigma_D \leq 0.08 \\
0.03 & \leq \sigma_d \leq 0.1 \\
0.07 & \leq \sigma_A \leq 0.113
\end{align*}
\]

<table>
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<th>(c_L^t)</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
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<tr>
<td>(\sigma_A)</td>
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Part 2: Accelerated Testing using Importance Sampling
Problem Description

Vehicle speed: 20 mph; Mission distance: 100 miles

Simulation can be practically performed for a short-duration time
A novel MC-based method to calculate the time-dependent reliability (cumulative probability of failure) based on:

- short-duration data and an exponential extrapolation using MCS or Importance Sampling (Infant Mortality)
- Poisson’s assumption (Useful Life)
Exponential Extrapolation

\[ \hat{\lambda}(t) \approx \lambda_0 e^{-bt} \]

The Bathtub Curve

- Hypothetical Failure Rate versus Time
- Increased Failure Rate
- Normal Life (Useful Life)
- Low "Constant" Failure Rate
- End of Life Wear-Out
- Increasing Failure Rate

Poisson’s Assumption

\[ F_T^c(t) = \begin{cases} 
-\int_0^t \hat{\lambda}(t)\,dt, & t \in [0, t_{int}] \\
1 - e^{-\int_0^t \hat{\lambda}(t)\,dt}, & t \in [t_{int}, t_f] \\
1 - (1 - F_T^c(t_{int})) e^{-\nu_m(t-t_{int})}, & t \in [t_{int}, t_f] 
\end{cases} \]
Poisson Assumption

\[ F_T^c(t_{\text{min}}, t) = 1 - (1 - F^i(t_{\text{min}}))e^{-m_1} \]

where:

\[ m_1 = E[N^+(t_{\text{min}}, t)] = \int_{t_{\text{min}}}^{t} \nu^+(t) \, dt = \nu_m(t - t_{\text{min}}) \]

Number of out-crossings

\[ \nu^+(t) = \lim_{\Delta \tau \to 0, \Delta \tau > 0} \frac{P[g(d, X, t) > 0 \cap g(d, X, t + \Delta \tau) \leq 0]}{\Delta \tau} \]

Out-crossing rate
Quarter-Car Model on Stochastic Terrain

Constant design parameters:
\[ m_s = 1000 \text{ kg} \]
\[ m_u = 100 \text{ kg} \]
Vehicle speed = 20 mph

Random Input variables
Damping, \( b_s \sim N(7000,1400^2) \)
Stiffness, \( k_s \sim N(40 \times 10^3,(4 \times 10^3)^2) \)

Random Input Process: Experimental Stochastic Terrain from Yuma Proving Grounds.

Random Output Process:
(Vertical Acceleration, G’)
Threshold = 2G
AR(3) model was identified based on:

Autocorrelation Function

Sample Autocorrelation Function (ACF)

Lag
Sample Autocorrelation

-0.2
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50

Autocorrelation of Residual process

Sample Autocorrelation Function (ACF)

Lag
Sample Autocorrelation

-0.2
0
0.2
0.4
0.6
0.8

\[ u_i = 1.2456 \, u_{i-1} - 0.2976 \, u_{i-2} - 0.1954 \, u_{i-3} + \varepsilon_i(0, \, 0.5132^2) \]

Statistical tests were performed to verify the model
Quarter-Car Model: Results
(Failure Rate Estimation for Threshold = 2G)

Estimated parameters:
\[ \lambda_0 = 0.1708 \]
\[ b = 0.0818 \]

Estimation requires **short duration** MCS

Exponential extrapolation

\[ \hat{\lambda}(t) \approx \lambda_0 e^{-bt} \]
Quarter-Car Model: Results
Cumulative Probability of Failure for Threshold = 2G

Efficient MCS (blue) approach is close to true MCS results (red)
Principle of Importance Sampling: Random Variable Case

Feasible Region

\[ g_1(x_1, x_2) = 0 \]

\[ g_2(x_1, x_2) = 0 \]

Increased Performance

Reliable Optimum

\[ f(x_1, x_2) \] contours
Importance Sampling for Random Process

Instantaneous **Conditional** Probability of Failure:

\[ p_f^\lambda(t_i) = \int_{\Omega} \theta(x; t_i) f_x(x; t_i) dx \]

\[ x = \{x_1, x_2, \ldots, x_i\} \text{ where } x_i \text{ is a realization of R.V. } X_i = X(t_i) \]

\[ p_f^\lambda(t_i) = \int_{\Omega} \theta(x; t_i) \frac{f_x(x; t_i)}{f_{x^s}(x; t_i)} f_{x^s}(x; t_i) dx \]

**Sampling Joint PDF**

\[ p_f^\lambda(t_i) = \frac{\sum_{n=1}^{N_{f}(t_i)} \theta(x; t_i) \omega(x, t_i)}{N_S(t_{i-1})} = \frac{\sum_{n=1}^{N_f(t_i)} \omega(x, t_i)}{N_S(t_{i-1})} \]
Importance Sampling for Random Process

\[
P_f^\lambda(t_i) = \frac{\sum_{n=1}^{N_s(t_{i-1})} \theta(x; t_i) \omega(x, t_i)}{N_s(t_{i-1})} = \frac{\sum_{n=1}^{N_f(t_i)} \omega(x, t_i)}{N_s(t_{i-1})}
\]

\[
\lambda(t_i) = \lim_{\Delta t \to 0} \frac{p_f^\lambda(t_i)}{\Delta t} = \lim_{\Delta t \to 0} \frac{\sum_{n=1}^{N_f(t_i)} \omega(x, t_i)}{\Delta t \cdot N_s(t_{i-1})}
\]

\[
\omega(x, t_i) = \frac{f_x(x; t_i)}{f_{xS}(x; t_i)} : \text{Likelihood ratio at } t_i
\]

\[
N_s(t_{i-1}) : \text{Safe sample points at } t_{i-1}
\]

\[
N_f(t_i) : \text{Number of failures in } \Delta t = t_i - t_{i-1}
\]
Likelihood ratio:

\[
\omega(x; t_i) = \frac{f_X(x; t_i)}{f_X^S(x; t_i)} = \frac{f_X(x_i, x_{i-1}, \ldots, x_{i-d})}{f_X^S(x_i, x_{i-1}, \ldots, x_{i-d})}
\]

**Decorrelation length**: Maximum number of lags over which realizations of \(x_i\) are significantly correlated

\[
x_i - \mu = \phi_1(x_{i-1} - \mu) + \phi_2(x_{i-2} - \mu) + \ldots + \phi_p(x_{i-p} - \mu) + \epsilon_i(N(0, \sigma^2_s))
\]

To generate sampling PDF

\[
f_X(x) = \frac{1}{(2\pi)^{k/2} |\Sigma|^{1/2}} \exp\left(-\frac{1}{2} (x - \mu)^T \Sigma^{-1} (x - \mu)\right)
\]

From Yule-Walker Eqs
Estimation of Safe Sample Functions

\[ \lambda(t_i) = \lim_{\Delta t \to 0} \frac{\sum_{n=1}^{N_f(t_i)} \omega(x, t_i)}{\Delta t \cdot N_S(t_{i-1})} \]

\[ \frac{\sigma_e}{\sigma_S} x_f > S_{\text{threshold}} \]

“Inflated” response
Quarter-Car Example

\[ u_i = 1.2456u_{i-1} - 0.2976u_{i-2} - 0.1954u_{i-3} + \varepsilon_i (0, 0.5132^2) \]

Original PDF \( \sigma_e = 0.51 \)

Sampling PDF \( \sigma_s = 0.7 \)

The sampling PDF results in more failures
Quarter-Car Example

Sample Autocorrelation Function (ACF)

Sample Autocorrelation

Lag

\( d = 7 \)

UNCLASSIFIED
Quarter-Car Example

Stationary Case

- ECS 1,500,000 samples
- IS 10,000 samples
Quarter-Car Example

Stationary Case

![Graph showing failure rate over time for different scenarios with varying parameters.]

- **MCS**
- **IS, d=7**
- **IS, d=12**
- **IS, d=6**
- **IS, d=5**

Stationary Case - Quarter-Car Example
Quarter-Car Example

Non-Stationary Case

Threshold = 2 g

Threshold = 2.65 g
Observations / Practical Issues

- Analytical methods can be used under the Poisson’s assumption.
- IS at initial time may need a few thousand output sample functions.
Ongoing Work Plan

- Improve the current accelerated testing method based on importance sampling so that only 5-10 tests are needed (Q3)
  - Characterize the “inflated” output random process in importance sampling using “generalized” Kriging and MLE and/or time series

- Demonstrate the accelerated testing methodology using the N-post (or 4-post) Reconfigurable Road Simulator of the Physical Simulation Laboratory at TARDEC (Q3 and Q4)
TARDEC N-post Reconfigurable Road Simulator
Thanks for your attention!

Q & A

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