RISK-BASED COMPUTATIONAL PROTOTYPING
(BRIEFING CHARTS)
Philip Beran, José Camberos, Ned Lindsley, and Bret Stanford
Multi-Disciplinary Technologies Branch
Structures Division

OCTOBER 2010
Interim Report

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*//Signature//
PHILIP S. BERAN
Principle Research Aerospace Engineer
Multi-Disciplinary Technologies Branch
Structures Division

//Signature//
DENIS P. MROZINSKI, Chief
Multi-Disciplinary Technologies Branch
Structures Division

//Signature//
DAVID M. PRATT
Technical Advisor
Structures Division
Air Vehicles Directorate

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We are developing computational methods that will enable the computational design of air vehicles accounting for inherently nonlinear dynamic behaviors. These behaviors fall into two categories: behaviors that are beneficial for vehicle operation, such as could be observed for micro air vehicles propelled by wing flapping (e.g., a productive energy transfer between the unsteady vortical flow produced by a flapping wing and the associated nonlinear deformation of the wing), and behaviors that constrain vehicle operation, such as in the dangerous limit-cycle oscillation of large aircraft. In either case, the design space is large and the analysis multi-disciplinary. We have investigated different ways of computing sensitivities of vehicle dynamics to a large number of design variables, compressing the computation using model reduction, and assessing the impact of variability on the reliability of the system.
Risk-Based Computational Prototyping

Dr. Philip Beran, Principal Research Aerospace Engineer, PI
Dr. José Camberos, Aerospace Engineer, Co-PI
Dr. Ned Lindsley, Aerospace Engineer, Co-PI
Dr. Bret Stanford, Post-Doctoral Research Associate

Multidisciplinary Science & Technology Center (MSTC)
Air Vehicles Directorate

September, 2010

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MSTC Organization & Activity

Mission: *Integrate multiple disciplines to discover and exploit new phenomena for system optimization and assessment of revolutionary aerospace vehicles*

Prototype Representation & Design Exploration Methods
- Parametric Geometry & Mesh
- Subsystem Representation
- Design Space Exploration & Optimization
- Risk-based Design

Analysis Methods for Prototypes
- Multidisciplinary Analysis
- Appropriate-fidelity Solutions and Sensitivities
- Nondeterministic Models

Prototype Validation & Assessment
- HiFi QTA
- Prototype Experimental Validation
- TRL Assessment

Shared Activity - Utilize a Unified Framework (SORCER, MODEL Center)

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Some Significant Collaborations

MSTC Collaborative Center with VPI & SU, WSU, and University of Maryland (Formed March 2009)

Prof Kapania, Director
Dr. Kolonay, PM

AFRL/RB and WSU Center for Micro Air Vehicle Studies (Formed June 2010)

Prof Huang, Director
Dr. Beran, PM

• Prof Missoum, Mr. Basudhar (UA, Tucson) and Dr. Lambe (MSSRC) – RBDO with LCO
• Prof Dong and Mr. Gaston (WSU) – ROM and Simulation of falling bodies
• Prof McFarland and Mr. Hubbard (UIUC) – Transmission design with nonlinearity

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Internal Collaborations in MAVs

**Math (6.1)**
- Risk-Based Computational Prototyping
  - Beran (PI), Camberos, Lindsley

**Physics (6.1)**
- Physics-Based Design Analysis of MAVs
  - Snyder (PI), Beran, Kolonay

**Basic Research in Computational Design (2009-2011)**

- NRC: Chabalko, Kurdi, McClung, Stanford

**Basic and Applied Research in MAVs**
- *Structural and Flight Testing* (Parker) – validation of structural and system models
- *CFD* (Visbal) – verification of aero models
- *Controls Science* (Doman) – integration of controls models
- *Unsteady Aerodynamics* (OL) – validation of aero models
- *Perching Technologies* (Reich) – application of aero models

**Flapping Sciences Integration (2009-2011): 6.2**
- Service-oriented framework
- In-house computer scientists
- Design tools (*Transition*)
- Funded follow-on design program (MPP, FY12+)

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Role of Computational Mathematics

PDF

Distribution of critical speeds arises via variability in air vehicle or environment

Speed at which LCO achieves maximum value

Target Flight Speed

$U_\infty$

$PF = \text{Probability of Failure (large LCO)}$

Must be sufficiently small

$P_F = \text{Probability of Failure (large LCO)}$

Must be sufficiently small

Computational mathematics needed for physics-based design of reliable vehicles

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Role of Computational Mathematics (cont.)

• Exploit nonlinear aeroelastic interactions for small aircraft


Unsteady Deformations

• Numerous challenges for design of Micro air vehicles (MAVs)
  – Physics Rich (must be a physics-based approach)
  – Complex and time-dependent actuations (unsteady)
  – Non-conventional geometries and structural topologies
  – Power-based integration of propulsion, structure, control components

Computational mathematics needed for physics-based design of MAVs


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Monolithic-time collocation

Requires increased resolution

• Uses a local basis instead of global basis

\[ X_e(\zeta) = \sum_{q=0}^{m} X_e(\zeta_q)\Psi_q(\zeta) \]

- Order of the spectral element
- Zeroes of the Lobatto-Legendre polynomials
- Lagrange polynomial of order \( m \)


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Monolithic-Time Collocation

Arrays corresponding to a discrete 2D field variable

\[
X_{\text{mon}} = [X^0, X^1, X^2, X^3, \ldots]
\]

Context for time-periodic and transient solutions
Adjoint-Variable Approach

1. Solve $F_{mon}(X_{mon}, \lambda) = 0$

   $H(X_{mon}) = \text{objective}$
   $F_{mon} = \text{equation residual}$

2. $\begin{bmatrix} \frac{\partial F_{mon}}{\partial X_{mon}} \end{bmatrix}^T C_{mon} = \begin{bmatrix} \frac{\partial H}{\partial X_{mon}} \end{bmatrix}^T$

3. \[ \frac{dH}{d\lambda} = - C_{mon}^T \frac{\partial F_{mon}}{\partial \lambda} \]

Sensitivity:
\[ \frac{dH}{d\lambda} = - \frac{\partial H}{\partial X_{mon}} \left( \frac{\partial F_{mon}}{\partial X_{mon}} \right)^{-1} \frac{\partial F_{mon}}{\partial \lambda} \]

High cost: computed once

Inexpensive:
analytic or finite-difference (repeat for each variable) about monolithic solution

Goal: Examine challenge of storing $X_{mon}$ between step 1 and 2
Adjoint Computation for Transient Sensitivity Analysis

Goal: Develop a sensitivity analysis process that scales well with total # DOFs

- Interested in the adjoint-variable approach in anticipation of:
  - many design variables (not true of direct and sampling based approaches)
  - use of gradient-based optimization (trade global effectiveness for efficiency)

- Some relevant literature

- Create a sample problem to explore a POD-based approach to eliminate challenge of storing the forward solution
Problem Description

 transient analysis of incompressible flow in a square cavity with unsteady lid

Flow at rest initially

\begin{align*}
\frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} &= \frac{1}{Re} \nabla^2 \omega \\
\nabla^2 \Psi &= -\omega \\
u &= \frac{\partial \Psi}{\partial y}, \quad v = -\frac{\partial \Psi}{\partial x}
\end{align*}

• Steady: \( U = 1 \) (impulsive)
  – verify; assess accuracy

• Transient: \( U = \frac{1}{2}(1-\cos(f t)) \)
  – define \( H \), a function of the transient solution
  – compute sensitivity of \( H \) to frequency, \( f \)

• Streamfunction-vorticity form

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Discretization and Time Integration

1. \[
\frac{\omega^{n+1} - \omega^n}{dt} + \left( u \delta_x \omega + v \delta_y \omega \right)^n = \frac{1}{\text{Re}} \left( \delta_{xx} + \delta_{yy} \right) \omega^{n+1}
\]

2nd-order-accurate, central-difference operators

2. \[
\left( \delta_{xx} + \delta_{yy} \right) \Psi^{n+1} = -\omega^{n+1}
\]

Explicit/implicit formulation

3. Repeat for next time step

\[
\omega(x,1) = -\frac{2}{\Lambda_y^2} \left( \Psi(x,1 - \Delta_y) + U(t)\Delta_y \right) + O(\Delta_y)
\]

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Adjoint-Variable Approach

\[ \Delta X_{\text{mon}} = -F_{\text{mon}} \]

\[ C_{\text{mon}} = \left( \frac{\partial H}{\partial X_{\text{mon}}} \right)^T \]

Reverse-time

Linear, time invariant

Vorticity BC coupling terms

Jacobians arising from convective terms [apply data compression]
Verification (Steady State)

\[ \Psi_{\text{min}} \in [-0.1026, -0.1035] \text{ Collected*} \]

\[-0.1035 \text{ Current} \]

\[ \Psi_{\text{min}} \in [-0.1163, -0.1188] \text{ Collected*} \]

\[-0.1180 \text{ Current} \]

\[ t_{\text{final}} = 20, \text{ time step} = 0.001 \]

\[ \text{Re} = 100 \]

\[ t_{\text{final}} = 50, \text{ time step} = 0.001 \]

\[ \text{Re} = 1000 \]


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Verification (Transient)

- Re=10000
- U(t) = 1
- Contour plots of \( \Psi \)
- \( t=2 \): agree within 2.8%
- \( t=8 \): agree within 4.4%
- Need to explore mesh and time step refinements
Verification (Sensitivity)

- Re=1000 with baseline mesh (101 × 101)
- U varies in time
- Determine sensitivity of H₂ about f = 1
- H₂ evaluated at t = 10
- Finite-difference sensitivity: δf = 0.0001
- *Sensitivities match to 6 significant digits*

\[
U(t) = \frac{1}{2} \left(1 - \cos(ft)\right)
\]

\[
H_2(x_{mon}) = \sum_k \left(\Psi_k^n\right)^2
\]

<table>
<thead>
<tr>
<th>(\frac{\partial H_2}{\partial f}) (Adjoint)</th>
<th>(\frac{\partial H_2}{\partial f}) (Finite Difference)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.70771958780</td>
<td>4.7077182309</td>
</tr>
</tbody>
</table>

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POD Data Compression for Sensitivity Analysis

- Same conditions as verification case
- Integration time of 10; 1000 time steps
- Collect snapshots once every 10 time steps
- Decimate snapshot set to coarsen
- Evaluate efficiency and accuracy of POD-based adjoint sensitivity analysis as function of number of snapshots and modes

On-the-fly, use the modes and the instantaneous modal amplitudes to reconstruct the flow solution

Convergence of $\Psi$-modes from 100 snapshots

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Solution and POD Modes (Streamfunction)

Mode 1

Mode 2

Mode 3

Mode 4

Mode 5

1M ode 18 2

Mode 3

Mode 4

Mode 5

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Efficiency and Accuracy

\[ \frac{\partial H_2}{\partial f} \text{ using 100 snapshots} \]

<table>
<thead>
<tr>
<th></th>
<th>Full order</th>
<th>50 modes</th>
<th>20 modes</th>
<th>10 modes</th>
<th>5 modes</th>
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<tr>
<td></td>
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<td>4.724862963</td>
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<table>
<thead>
<tr>
<th>% Error in [\frac{\partial H_2}{\partial f}]</th>
<th>50 modes</th>
<th>20 modes</th>
<th>10 modes</th>
<th>5 modes</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 snapshots</td>
<td>0.00012</td>
<td>0.078</td>
<td>0.36</td>
<td>-34</td>
</tr>
<tr>
<td>20 snapshots</td>
<td>–</td>
<td>1.4</td>
<td>3.2</td>
<td>-31</td>
</tr>
<tr>
<td>10 snapshots</td>
<td>–</td>
<td>–</td>
<td>1.6</td>
<td>2.8</td>
</tr>
</tbody>
</table>

20 snapshots = 2\% of time-history data
10 modes = 1\% of time-history data

Greatly decrease memory requirement at 2× cost: explore other POD uses

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Goal: study transient sensitivity analysis in context of DOF reduction

- Identify best thickness distribution for rapidly actuated plate
- Nonlinear modeling of a flapping plate
- Minimize $\delta_{\text{ave}} = \text{time-averaged}\ \delta$
- 256 variables (element thicknesses w/ constraints)
- GBO via MATLAB ($fmincon$)

Resultant thickness distribution

Extension: Kinematic/Structural design for wing design with airloads (Stanford et al., SDM10)

$\omega = 7\text{Hz}$


ROM Adjoints $\approx$ Free

5X design speed-up

Normalized design cost

$\delta_{\text{ave}}/L$
Beam Design (Inertial Loads Only)

- Identify best area distributions for minimum and maximum time-averaged tip displaced.
- Co-rotational FEA formulation; 50 beam elements, each with a different sectional area.
- Side constraints on area; GBO via MATLAB (`fmincon`).
- Compute sensitivities with the adjoint formulation.

Some benefits of combining SE and ROM in time-periodic formulation.

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Reliability-Based Design Optimization (RBDO)

Goal: Examine use of transient sensitivity analysis to design a plate wing that is both light and reliable

- Reliable: wing does not exhibit too severe a limit-cycle oscillation
- $U_\infty > U_{\text{flutter}} \rightarrow$ limit cycle oscillation
- Piston theory aerodynamics ($M_\infty > 1$)
- Nonlinear von Kármán plate FEA

Minimize mass of plate; constrain the probability that $LCO_{\text{amp}} > \delta$ ($P_F \leq \sigma$)

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Contrasting Approaches

**Deterministic Optimization**

- **Success:** \( LCO_{\text{amp}} < \delta \)
- **Failure:** \( LCO_{\text{amp}} > \delta \)

Generally, the designed plate “moves” to the constraint boundary \( (P_F \approx \frac{1}{2}) \)

\[
\begin{align*}
\text{min} \quad & \text{weight} = f(d) \\
\text{subject to:} \quad & g(x(d, E, M_\infty)) > 0; \text{ side constraints on } d \\
& g(x(d, E, M_\infty)) = \delta - LCO_{\text{amp}} = 0
\end{align*}
\]

\( x = \text{response variables} \)
\( d = \text{design variables} \)

**RBDO**

- **Failure:** \( LCO_{\text{amp}} > \delta \)
- **Success:** \( LCO_{\text{amp}} < \delta \)

Generally, the designed plate “moves” away from the constraint boundary a “safe” distance \( (P_F = \sigma) \)

\[
\begin{align*}
\text{min} \quad & \text{weight} = f(d) \\
\text{subject to:} \quad & 1 - \text{Prob}(g < 0)/\sigma \geq 0; \text{ side constraints on } d
\end{align*}
\]


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RBDO Formulation

- $M_\infty$ and $E$ are chosen to be uncertain (normal)
- Map to uncorrelated random variables $u_1$ and $u_2$ in standard normal space
- Compute Most Probable Point (MPP) and reliability index $\beta$
- Approximate failure surface as linear: First Order Reliability Method (FORM)
- Compute probability of failure, $P_F = P_F(\beta)$
- Meet $P_F$ constraint using analytical gradients

1. For a given structure, compute MPP using gradient based optimization: require sensitivities of $g$ to $u_1$ and $u_2$

2. Reduce weight while meeting $P_F$ constraint using gradient based optimization: require sensitivities of $P_F$ to $d_i$ (found from sensitivities of $g$ to $d_i$)

Adjoints of transient solutions used to compute sensitivities of $g$ to $d_i$
RBDO and SVM Results

1. Basudhar used Support Vector Machine and adaptive sampling to approximately construct failure surface
2. Computed $P_F$ with MCS on SVM boundary (55 samples)
3. Computed $P_F$ with QMCS (Lambe, MSSRC)

<table>
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<tr>
<th>Method</th>
<th>$P_F$</th>
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<tr>
<td>FORM</td>
<td>0.0197</td>
</tr>
<tr>
<td>MCS ($10^6$)</td>
<td>0.0248</td>
</tr>
<tr>
<td>QMCS ($10^4$)</td>
<td>0.0244</td>
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</table>


**RBDO Step**     **Cost (MATLAB, single CPU)**
Simulation         10 minutes
Adjoint            5 minutes
MPP                1 hour
Optimization       4 hours (deterministic), 12 hours (probabilistic)
Recent Activities: Rigid-Body MAV Motions

- Start to investigate impact of rigid-body motion on MAV performance
- Prof. Haibo Dong (WSU), Mr. Zachary Gaston (WSU)
- Mr. Tim Broering (UL)


Need to include rigid-body motions and body flexibility in bio-inspired MAV models

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Plan for Rigid-Body Coupling

• Emphasize passive motions first: falling bodies in quiescent flow
  – Modify high-fidelity tools to repeat 2D simulations and extend in 3D; validate at WSU with high-speed photography (want comparisons)
  – Calibrate quasi-steady models (like those used in flapping)

• Examine influences of gust and variability on falling motions
  – Introduce variability into quasi-steady models (e.g., how is seed dispersal impacted by winds?)

• Re-examine design procedures that have been developed so far: want MAVs that are robust to gust
Some Typical Motions

- $c =$ chord
- $h =$ thickness
- $\rho_f =$ water
- $\rho_s =$ aluminum

$\beta = h/c = 1/14$

flutter: $\beta = h/c = 1/5$

- Will explore impact of variability (physical and model) on distribution of landing locations
- Developed transient sensitivities (direct): role in selecting bodies with more desirable falling characteristics?
High-Fidelity Results

Re = 40 (Stationary)

Overture

CD = 1.88

Re = O[10^3] (Falling)

Preliminary VICAR3D result

CD = 1.84

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Recent Activities (cont.)

• Start to explore role of actuation mechanism in MAV design
  – Investigate physical interactions between a flapping wing and the mechanism that flaps the wing (e.g., transmission of inertial loads)

• Developing compliant mechanisms via topological optimization
  – Link mechanism with generated inertial/aero loads (MAO 2010)

Understand/modeling energy transfers between mechanism and wing critical
Concluding Remarks

• Sensitivity analysis of transient/time-periodic systems serves an important role for design of both large and small aircraft
  – Constraint boundaries often nonlinear (LCO and aeroelastic response in gust); strive for physics-based approaches not reliant on safety factors
  – Essential for design of flapping wing MAVs; strive for physics-based approaches that account for gust

• Lessons learned through unsteady sample problems
  – POD is a straightforward means for data compression in sensitivity analysis for large systems; extensions using POD ripe for study
  – Adjoint vectors in ROM formulation computed virtually for free (tailoring of structure for nonlinear response during rotary actuation)
  – Adjoint-based sensitivities work well in an RBDO context; want to extend (e.g., transonic, SVM, SORM) based on lessons learned

• Interesting departure points for further study: variability in motion subject to gust, mechanism design
Recent Publications


Questions?
AMP Team Composition (WPAFB)

Mission: *Integrate multiple disciplines to discover and exploit new phenomena for system optimization and assessment of revolutionary aerospace vehicles*

**Analysis Methods for Prototypes**
- Dr. José Camberos – On detail as RB Deputy Chief Scientist
- *Dr. Chris Chabalko – Postdoc (NRC, UTC)*
- Dr. Ned Lindsley – Supporting prototype validation/assessment
- *Dr. Aaron McClung – Civil Servant, formerly NRC*
- Mr. John Moore – Undergraduate Co-op (University Florida)
- Mr. Michael Robbeloth – Computer Scientist, DSA
- Dr. Rich Snyder
- *Dr. Bret Stanford – Postdoc (NRC)*
- Dr. Phil Beran - Lead

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Methods Development Strategy

**Goal:** Multifidelity framework built on new methods

**Development of New Methods**
- Time-Periodic Analysis
- Sensitivity Analysis
- Reduced Order Modeling
- Uncertainty Characterization

**Application and Extension of High-Fidelity Methods**
- Navier-Stokes (OVERFLOW)
- Beams, Plates, Shells models
- Aeroelasticity
- Vortex methods (medium fidelity)

**Validation through physical experiment**
- Water channel (OL, AVT-149)
- Free flight (TU Delft, AVT-184)
- Aeroelastic ground-test facility (Parker)

Develop methods: start with low-dimensional formulations and move towards high-dimensional.

Characterize physical limitations of lower fidelity approaches.

Assess validity of all methods.

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Application to Insect Wing


Feathering angle
Wing axis (in X-Y plane)

Prescribed (ψ) and realized (η) angles
- mass-spring-damper
- inertial & aero loads

Quasi-steady (QS) aerodynamics (Berman & Wang)

Power reduction from initial design:
- 55% for unconstrained acceleration
- 40% for constrained acceleration

Looking at inertial power contribution


Optimized fruitfly wing kinematics (235 Hz)

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High-Fidelity Analysis

Understanding Complex Physics

- Study Hawkmoth physics using Navier-Stokes (NS) simulation
- Collaboration with AFIT
- Hawkmoth kinematics (hover)
- What’s new?
  - OVERFLOW 2.1 Elastic (5th/2nd-order in space/time)
  - Prescribed wing deformations
  - Variations in kinematics
- Moderate flexibility increases hover efficiency

<table>
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<th>Fy (N)</th>
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