Time-Evolution of Maritime Domain Awareness

Maritime Domain Awareness and Counter Piracy, 26-29 October 2009, Ottawa, Canada
Maritime Domain Awareness

- Maritime Domain Awareness (MDA) depends upon Intelligence, Surveillance, and Reconnaissance (ISR)
- MDA is generated by completing ISR tasks that characterize contacts
ISR Experiment

- ISR in Maritime context
  - Littoral waters near Tofino, BC
- Live experiment, including UAV
- July 7 to 11, 2003
Dynamical IISRA Model (DIISRAM)

- Four contact characterization states:
  - Detection: $x_1$
  - Length measurement: $x_2$
  - Classification: $x_3$
  - Identification: $x_4$

- Predict the time-evolution of $x = (x_1, x_2, x_3, x_4)$

- Nonlinear system:
  $$\frac{d}{dt}x = F(x)$$
Postulate 1

- Capability Limit $\Rightarrow$ MDA evolves toward a steady-state
Postulate 2

- Object availability $\Rightarrow$ MDA evolution responds to the number of objects in each contact characterization state
Postulate 3

• Task Activation \( \Rightarrow \) MDA evolution depends on the stability of precursory and/or competitive tasks
Postulate 4

• Capability overreach $\Rightarrow$ MDA evolution can temporarily exceed its steady-state limits
Postulates 1, 2, 3
Math Summary

- To first order, rates are proportional to
  - the difference from steady-state
  - the number of targets
  - the stability of precursory or competitive processes

\[
\dot{x}_k \propto \left(a_k N - x_k\right)
\]

\[
\dot{x}_k \propto N
\]

\[
\dot{x}_k \propto \frac{x_j}{a_j N}, \quad j \neq k
\]
Capability Overreach
Math Summary

- Steady-state can be temporarily exceeded
  \[ \dot{x}_k \propto (a_k N + g_k(a, x) - x_k) \]

- Excess contacts lost during subsequent processing
  \[ g_k(a, x) = \sum_{j \neq k} a_{m(j,k)} \frac{a_{i(j,k)} N - x_{i(j,k)}}{a_{i(j,k)} N - a_{n(j,k)}} \]
Best-Fit Solution

• Simple

• Text-book methods
  – Runge-Kutta solver
  – Downhill simplex search (Nelder & Mead)
  – Least squares

• Constraints: non-negative contact counts, not more than the number detected

• Penalty-function: unconstrained non-linear optimization
Best-Fit Solution
Math Summary

- Textbook Methods: Downhill Simplex Search & Least Squares

\[ \chi^2 = \sum_{k=1}^{N_D} \left[ \frac{F(a, x(t_k)) \cdot u_k - y_k}{\sigma_k} \right]^2 \]

- Numerical integration subject to constraints on characterization-state counts

\[ 0 \leq x_k \leq x_1 \]

- Penalty-function checks constraints on 19 model parameters

\[ P(a) = \begin{cases} 
0, & a \in \Omega \\
\text{otherwise}, & P_{\text{max}} 
\end{cases} \]
A Tale of Two Pictures

- Tofino Littoral Picture (TLP)
  - Tactical-level
  - Located at Tofino airport (UAV’s base)
  - Closest node to airborne sensors

- Experimental Littoral Picture (XLP)
  - Operational-level
  - Located at Canadian Forces Base Esquimalt
  - Furthest node from airborne sensors
Case 1: Capability Overreach

\[ \nu = 32 \]
\[ \chi^2 = 40.0 \]
\[ Q = 15.7\% \]

\[ \nu = 7 \]
\[ \chi^2 = 18.5 \]
\[ Q = 0.9\% \]
Case 2: Sensitive Solution

\[ \nu = 2 \]
\[ \chi^2 = 0.8 \]
\[ Q = 66.4\% \]

\[ \nu = 2 \]
\[ \chi^2 = 1.0 \]
\[ Q = 60.7\% \]
Case 4: Capability Under-Reach

\[ \nu = 42 \]
\[ \chi^2 = 58.0 \]
\[ Q = 66.4\% \]

\[ \nu = 15 \]
\[ \chi^2 = 28.7 \]
\[ Q = 1.8\% \]
Case 6: Double Overreach

\[ \nu = 32 \]
\[ \chi^2 = 11.5 \]
\[ Q = 99.9\% \]

\[ \nu = 9 \]
\[ \chi^2 = 9.6 \]
\[ Q = 38.1\% \]
Case 9: Large Under-Reach

\[ \nu = 32 \]
\[ \chi^2 = 32.6 \]
\[ Q = 42.8\% \]

\[ \nu = 2 \]
\[ \chi^2 = 0.6 \]
\[ Q = 72.4\% \]
Case 12: Inversion of Reach

\[ \nu = 29 \]
\[ \chi^2 = 8.12 \]
\[ Q = 99.9\% \]

\[ \nu = 12 \]
\[ \chi^2 = 10.7 \]
\[ Q = 56.1\% \]
## Results Summary

**Tofino Littoral Picture (TLP)**

<table>
<thead>
<tr>
<th>Period</th>
<th>Data Points $N_D$</th>
<th>DIISRAM Parameters $N_p$</th>
<th>Degrees of Freedom $v$</th>
<th>DIISRAM Solution</th>
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<tbody>
<tr>
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<td>Critical Numbers</td>
<td>Provisional</td>
<td>Best-Fit</td>
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<tr>
<td></td>
<td>$\chi^2/\nu$</td>
<td>$Q(\chi^2, v)$</td>
<td>$\chi^2/\nu$</td>
<td>$Q(\chi^2, v)$</td>
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## Results Summary

**Experimental Littoral Picture (XLP)**

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<th>Data Points $N_D$</th>
<th>DIISSRAM Parameters $N_p$</th>
<th>Degrees of Freedom $\nu$</th>
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<td>Reduced Chi-Squared $\chi^2$</td>
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Conclusions

• Goodness-of-fit statistics indicated that the model’s solutions were acceptable in 20 out of 24 cases
  – Acceptable for 100% of TLP cases and 67% of XLP cases
  – The solution emulated the multi-state count collapse (Cases 2 and 8)
  – Capability overreach was observed, including one double overreach (TLP Case 6)
  – Capability under-reach (opposite of overreach) was discovered (Cases 4 and 9)
  – Inverted a capability under-reach in the TLP into an overreach in the XLP (Case 12)
Practical Recommendation

• Apply the model to other real configurations of ISR assets
  – Assess goodness of fit
  – Empirical parameters would enable quantitative predictions of the time-evolution of live ISR operations
    • In other words, the model would aid MDA/ISR force planning & development
Questions?