1. INTRODUCTION

Quantitative reconstruction of climatic parameters from both oceanic and continental records shows several time scales of the successive glacial-interglacial episodes that have characterized the Earth's climate about the past million years. For example, after analyzing data from the 2,083 m ice core recovered by the Soviet Antarctic Expeditions at Vostok (East Antarctic), Jouzel et al. (1982) found several peaks (25.3, 45.7, and 107.5 kyr) in the variation spectra of the Vostok isotope temperature, and related these peaks to the astronomical forcing, i.e., the obliquity of the Earth's axis (period at 41 kyr) and the precession of the equinox (periods at 23 and 19 kyr). Is the climatic variation purely caused by the external forcing? If so, the climate prediction would be relatively simple because it becomes the forecast of the Earth's orbit tilt and precession. In fact, the climate system is much more complicated. The climatic variation is caused by both external and internal sources (Fig.1).

The internal forcings are processes and mechanisms within the Earth's system, giving rise to self-excited climate fluctuations. The temperature-albedo feedback mechanism associated with long and short wave radiation fluxes at the Earth's surface are given considerable attention in glacial and climate models (e.g., Kallen et al., 1979; Oerlemans and van der Veen, 1984). However, not until recently (Chu, 1990), the role of hydrological cycle on the climatic variation on glacial and interglacial time-scales is given less attention although it is realized that ice freeze/melt and evaporation/precipitation are contributors to the change of air and ocean temperatures. Taking hydrological cycle into consideration, several intrinsic time scales of the atmosphere-ocean-cryosphere climate system will be discussed in this paper.

2. ATMOSPHERE-OCEAN-CRYOSPHERE CLIMATE SYSTEM

The total surface, ice covered, and open ocean areas are set to be $1, N_s$, and $1 - N_s$, respectively (Fig.2). The interactions among the atmosphere, ocean, and ice is proposed by Chu (1990) as follows. From the global point of view, ice advance isolates the ocean from the atmosphere and reduces the heat loss from the ocean surface, which in turn decreases the air temperature and increases the ocean temperature. The air and ocean feed back into the ice through two different kinds of mechanisms (Fig.3): (1) thermodynamical feedback; the in-
# Intrinsic Thermodynamical Time-Scales of the Atmosphere-Ocean-Cryosphere Climate System

**Abstract:**

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crease of ocean temperature melts the ice and causes the ice retreat (negative feedback between ice and ocean), however, the decrease of air temperature causes freezing and makes the ice further advance (positive feedback between ice and air); (2) mechanical feedback; the modification of the atmospheric and oceanic temperatures varies the surface evaporation rate, which in turn changes the sea-level height and the ice accumulation rate. Both effects lead to a further change of the ice flow.

Fig. 2 Physical processes in the coupled atmosphere, ocean, and cryosphere climate system (after Chu, 1990).

Fig. 3 Feedback mechanisms among air, ice, and ocean.

2.1 Atmosphere

The atmosphere is heated (or cooled) by radiation, and heat fluxes (including latent heat flux) from the air-ocean and air-ice interfaces, i.e.,

$$\frac{dT_a}{dt} = \frac{1}{\rho_a c_p H_a} \left[ -N_t Q_{at} + (1 - N_t) Q_{wa} + R_a \right]$$

(1a)

where $T_a$ is the air temperature, $c_p$ is the air specific heat, $Q_{at}, Q_{wa}$ are, respectively, the heat fluxes across air-ice and air-ocean interfaces, $R_a$ is the net radiation absorbed by the atmosphere, and $\rho_a$ and $H_a$ are the characteristic values of atmospheric density and thickness.

In the present study, it is assumed that the water in the atmosphere is always in the vapor form. As soon as condensation happens, the condensed water droplets are assumed to be precipitation right away. Therefore, the time rate change of specific humidity $q_a$ is caused by the excessive surface evaporation over precipitation and ice accumulation

$$\frac{dq_a}{dt} = \frac{1}{\rho_a H_a} \left[ (1 - N_t) p_w (E - Pr) - N_t \rho_p c_v \right]$$

(1b)

where $E, Pr, c_v$ are rates of ocean surface evaporation, precipitation, and ice accumulation. $\rho_p$ and $\rho_w$ are the characteristic values of ocean and ice densities.

2.2 Ocean

Similar to the atmosphere, the ocean thermodynamics can be depicted by

$$\frac{dT_o}{dt} = -\frac{1}{\rho_w c_p H_w} \left[ N_t Q_{ot} + (1 - N_t) Q_{wt} + R_t \right]$$

(2)
where $T_o$ is the ocean temperature, $c_w$ is the ocean specific heat, $Q_{al}$ is the heat fluxes across ice-ocean interfaces, $R$ is the net radiation absorbed by the ocean, and $H_o$ is the ocean thickness.

From a thermodynamical point of view, the ocean thickness is reduced by the excessive surface evaporation rate $E$ over precipitation rate $P_R$, and by the ice edge freezing rate $F$,

$$\frac{\partial}{\partial t} [(1 - N_i) \rho_w H_o] = -(1 - N_i) \rho_w (E - P_R)$$

$$- (1 - N_i) \rho_i \Lambda(F) F - N_i \rho_i \Lambda(-F) F$$

(3)

where $\Lambda$ is the Heaviside function, i.e., $\Lambda(x) = 1$, as $x \geq 0$, and $\Lambda(x) = 0$, as $x < 0$. $F > 0$, is taken as ice freezing; and $F < 0$, is taken as ice melting.

### 2.3 Ice-shelf

The ice is assumed to be isotropic with thickness $H_i$ (Fig.2), and the ice spreading rate is computed by (Weertman, 1957)

$$< e_h > = \Lambda \left[ \frac{1}{4} \rho_g H_i \right]^n$$

(4)

where $H_i$ is the height between the ice top and the sea-level (Fig.2). Values of $n$ vary from about 1.5 to 4.2 with a mean of about 3, and for randomly oriented polycrystalline ice at $-10^\circ C$ and $n = 3$, a value $A = 3 \times 10^4$yr$^{-1}$kPa$^{-3}$ is reasonable for ice stress. The angle brackets denote values averaged over ice thickness.

The change of ice coverage $N_i$ is due to ice spreading and ice freezing/melting:

$$\frac{\partial N_i}{\partial t} = N_i < e_h > + \frac{(1 - N_i) \Lambda(F) F}{H_i}$$

$$+ \frac{N_i \Lambda(-F) F}{H_i}$$

(5)

Following Killworth (1979), the heat flux is accomplished through an exchange coefficient $K$ and is of the form $K(T - T_o)$, where $T_i$ represents the freezing point of the sea water. The freezing rate is therefore computed by

$$F = - \frac{1}{\rho_i L_i} \left[ \rho_w c_{pw} K_{al}(T_w - T_o) + \rho_u c_{pu} K_{ul}(T_u - T_i) \right]$$

(6)

where $K_{al}$, $K_{ul}$ are the heat exchange coefficients for water-ice and air-ice, respectively. $L_i$ is the latent heat of ice. If $T_o$, $T_u$ are greater than $T_i$, $F$ is taken as a negative value, which indicates ice melting.

Based on the continuity, Shumskiy (1965) proposed a method to determine the change of ice thickness by comparing the accumulation rate with horizontal strain-rates. Neglecting nonlinear inertial terms the time rate change of ice thickness becomes

$$\frac{\partial H_i}{\partial t} = c_i - H_i < e_h >$$

(7)

The atmosphere-ocean-cryosphere climate system is depicted by (1), (2), (3), (5), and (7).

### 2.4 Interfacial Heat Fluxes

The heat fluxes across the air-ocean, air-ice, ice-ocean interfaces change the thermal features of each component of the atmosphere-ocean- cryosphere climate system. There are many different methods to compute these fluxes, among them, the simplest form was proposed by Killworth (1979):

$$Q_{al} = \rho_u c_{pu} K_{al}(T_u - T_i)$$

$$Q_{wa} = \rho_u c_{pu} K_{wa}(T_w - T_u)$$

$$Q_{ul} = \rho_u c_{pu} K_{ul}(T_u - T_i)$$

(8)
3. ROLE OF THE HYDROLOGICAL CYCLE

Summation of (1b), (3), (5) multiplied by $H_a$ and (7) multiplied by $N$, leads to
\[
\frac{d}{dt}[(1 - N_i)\rho_w H_w + \rho_i H_i N_i + \rho_e H_e g_e] = 0
\]
which shows the conservation of the water mass in the Earth system.

The water vapor evaporated from the global ocean can be estimated by
\[
E = C_D |V_a| \frac{\rho_a}{\rho_w} [q_s(T_w) - q_a]
\]
where $C_D$ is the drag coefficient, $V_a$ is the surface wind, $q_s$ is the saturated mixing ratio, and $q_a$ is the mixing ratio near the ocean surface. If the Earth surface is assumed to be covered by ocean and ice only, the water vapor evaporated from the ocean surface is transported by the atmospheric circulation, condensed and precipitated. The precipitation over ice leads to ice accumulation, therefore, if the total water vapor evaporated from the ocean surface is balanced by the precipitation over ocean ($Pr$) and the ice accumulation rate over ice ($c'_i$), the relationship among surface evaporation rate, precipitation rate (over ocean), and ice accumulation rate should be
\[
(1 - N_i)\rho_w(E - Pr) = N_i \rho_e c'_i
\]

4. TIME SCALES OF THE CLIMATE SYSTEM

In explaining the climatic variability, one may first ask: is climatic variability caused by external or internal forcings? The external forcings are agencies outside the Earth system, such as astronomically caused changes in the Earth's orbital parameters. In the atmosphere-ocean-cryosphere climate system discussed in this paper, the external forcings mainly change the radiation absorbed by atmosphere ($R_a$) and ocean ($R_o$).

On the other hand, the internal forcings are processes and mechanisms within the Earth system, giving rise to self-excited climatic fluctuations. Substitution of (8) into (1) and (2), leads to thermal relaxation time scales for the atmosphere
\[
\tau_a = \left[\frac{K_{al}}{H_a} N_i + \frac{K_{wa}}{H_a} (1 - N_i)\right]^{-1}
\]
and for the ocean
\[
\tau_w = \left[\frac{K_{wl}}{H_w} N_i + \frac{\rho_a c_{pa} K_{wa}}{\rho_w c_{pw} H_a} (1 - N_i)\right]^{-1}
\]
The ice-spreading time scale can be defined from (5),
\[
\tau_i = \left(<\epsilon_i>\right)^{-1}
\]
Horizontal ice growth (or ice edge freezing/melting) time scale is obtained from (5):
\[
\tau_f = \frac{H_i}{[\Lambda(F) - \Lambda(-F)]}
\]
The perturbed ice accumulation $c'_i$, is caused by the perturbed evaporation, precipitation, and the ice advancement:
\[
c'_i = \frac{(1 - N_i)\rho_w(E' - Pr') - [\rho_w(E - Pr) + \rho_e c_i]N'_i}{N_i \rho_i}
\]
The perturbations of evaporation and precipitation rates $E'$ and $Pr'$ vary on a much shorter time scale than the time-scale for ice advance/retreat. Another time scale for the ice growth in vertical direction can be defined by
\[
\tau_i = \frac{\rho_i H_i}{[\rho_w(E - Pr) + \rho_e c_i]}
\]
Here the variables without "'" means the basic states. By the use of the values of the parameters listed in Table I, these time scales are estimated as:

$$\tau_d \approx 11.6d, \quad \tau_w \approx 66 \text{ yr},$$
$$\tau_i \approx 400 \text{ yr}, \quad \tau_j \approx 3000 \text{ yr}$$

Since the atmospheric and oceanic thermal relaxation timescales are so much shorter than the ice-spreading time scale, the atmospheric and oceanic temperature perturbations $T_a$ and $T_w$ almost "instantaneously" follow the ice advance/retreat processes in the atmosphere-ocean-cryosphere climate system. From (1) and (2), we may obtain

$$T_a' = -A_a\tau_a N_t' \quad T_w' = A_w\tau_w N_t'$$  \hspace{1cm} (17)

where

$$A_a = (Q_{wa} + Q_{da})/(\rho_a c_{pa} H_a), \quad A_w = Q_{wa}/(\rho_w c_{pw} H_w)$$

Equation (17) clearly shows the cooling of the atmosphere ($T_a < 0$) and warming of the ocean ($T_w > 0$) due to the ice advance. The perturbed ice freezing rate becomes

$$F = -H_0(\sigma_a A_w \tau_w' - \sigma_w A_a \tau_a')N_t'$$  \hspace{1cm} (18)

where

$$\sigma_a = \rho_a c_{pa} K_a/(\rho_i L_i H_i), \quad \sigma_w = \rho_w c_{pw} K_w/(\rho_i L_i H_i)$$

The perturbed ice spreading rate is computed by

$$\epsilon_{h}' = \tau_i^{-1} \left( \frac{h_i' - h_w'}{H_i} \right)$$  \hspace{1cm} (19)

6. LOW FREQUENCY MODES GENERATED BY THE FEEDBACK MECHANISM

All the time scales presented in the last section are shorter than the time scales of the successive glacial-interglacial episodes (10-100 kyr) shown in Fig.1. If we consider the atmosphere-ocean-cryosphere as a one system, the low frequency modes will be generated. If the coupled system perturbed from its equilibrium states, after mathematical manipulation, the perturbation equations are

$$[\frac{\partial}{\partial t} + (1 - \sigma)\tau_f^{-1} + N_1\mu_1\mu_2\tau_j^{-1}]N_t' = \mu_2 N_1\tau_j^{-1} \frac{h_i'}{H_i}$$  \hspace{1cm} (20)

$$[\frac{\partial}{\partial t} + (1 + \mu_2)\tau_1^{-1}] h_i' = \mu_1\mu_2 N_1^{-1} - N_1^{-1} \tau_1^{-1}]N_t'$$  \hspace{1cm} (21)

where

$$\mu_1 = n \frac{\rho_w H_w - \rho_i H_i}{\rho_w H_w (1 - N_1)} \approx 2.431$$

$$\mu_2 = n \left[ 1 + \frac{N_1\rho_i}{(1 - N_1)\rho_w} \right] \approx 3.66$$

$$\mu_3 = \frac{H_w}{H_i} \approx 2.5$$

and

$$\sigma = \tau_f(\sigma_a A_a \tau_a' - \sigma_w A_w \tau_w')$$

The general solutions of equations (20) and (21) have the following forms:

$$(h_i'/H_i, N_t') \sim e^{-i\omega t}$$  \hspace{1cm} (22)

Substitution of (22) into (20) and (21) leads to a second order algebraic equation for $\omega$

$$\omega^2 + \omega [(1 - \sigma)\tau_f^{-1} + (1 + \mu_2 + N_1\mu_1\mu_2)\tau_j^{-1}] - [(1 - \sigma)(1 + \mu_2)\tau_1^{-1} + \mu_1\mu_2 N_1^{-1}\tau_1^{-1}] = 0$$  \hspace{1cm} (23)

The frequency here is a complex number, i.e., $\omega = \omega_r + i\omega_i$, $\omega_i$ is a growth rate, and $\omega_r$ is the frequency.
7. FREQUENCIES FOR NEUTRAL STABILITY MODES

The modes with zero growth rate ($\omega = 0$) are called neutrally stable modes. The solution of (23) with $\omega = 0$ is

$$\omega_r = \tau_r^{-1} \left( \mu_2 v - \left[ (1 + \mu_2)^2 + N \mu_1 \mu_2 \mu_3 \right]^{1/2} \right)$$

where

$$v \equiv \frac{\tau_r}{\tau_i}$$

The time scale for the atmosphere-ocean-cryosphere climate system depends on the ratio of the two shorter time scales: $\tau_i$ and $\tau_r$. Fig. 4 shows the dependence of periods ($2\pi/\omega_r$) of the neutrally stable modes on the ratio $v$. When $v = 7.2$, the period is 43.3 kyr, which indicates that the feedback mechanisms among atmosphere, ocean, and cryosphere can also generate climate change on 10-100 kyr time scales.

DISCUSSION

The conventional theories refer the climate change on 10-100 kyr time scales to the concentration of variance near the Earth's orbit tilt and precession frequencies (Jouzel et al., 1987). However, the conceptual atmosphere-ocean-cryosphere model described in this paper shows a possible positive/negative feedback mechanism, induced by the hydrological cycle.

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