Nonlinear Acoustics in Cicada Mating Calls Enhance Sound Propagation

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An analysis of cicada mating calls, measured in field experiments, indicates that the very high levels of acoustic energy radiated by this relatively small insect are mainly attributed to the nonlinear characteristics of the signal. The cicada emits one of the loudest sounds in all of the insect population with a sound production system occupying a physical space typically less than 3 cc. The sounds made by tymbals are amplified by the hollow abdomen, functioning as a tuned resonator, but models of the signal based solely on linear techniques do not fully account for a sound radiation capability that is so disproportionate to the insect’s size. The nonlinear behavior of the cicada signal is demonstrated by combining the mutual information and surrogate data techniques; the results obtained indicate decorrelation when the phase-randomized and non-phase-randomized data separate. The Volterra expansion technique is used to fit the nonlinearity in the insect’s call. The second-order Volterra estimate provides further evidence that the cicada mating calls are dominated by nonlinear characteristics and also suggests that the medium contributes to the cicada’s efficient sound propagation. Application of the same principles has the potential to improve radiated sound levels for sonar applications. [DOI: 10.1121/1.3050258]

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I. INTRODUCTION

A. Background

The objective of this research is to begin to understand how the cicada, a small insect, emits one of the loudest sounds in all of the insect population despite its relatively small size. Detailed knowledge of the characteristics of the cicada’s acoustic signature is a necessary step toward the ultimate goal of transferring this biotechnological feat of nature to a manmade transduction system of similar proportions. The cicada’s highly effective sound production system occupies a physical space typically less than 3 cc. Cicadas are sexually dimorphic, and only males possess the structures necessary for making loud audible sounds. Male sounds are broadcast advertisements for attracting females, and they are typically loud, rhythmic, and easily distinguished from background noise. Males create sound by flexing a pair of ridged abdominal membranes called tymbals. The sounds made by these tymbals are amplified by the hollow abdomen functioning as a tuned resonator, as described by current research. Nevertheless, the tuned resonator explanation in the current literature does not account for the sound radiation capabilities of the cicada.

Studying the sound production system of the cicada in captivity has some inherent difficulties: cicadas vocalize only on sunny days and do not respond as well to indoor lighting. However, the recent discovery of a female response to the mating call of the male has made it possible to conduct experiments on cicadas in the field. In many species, the females answer loud male signals with quiet wing flick responses. Males perceiving such responses will approach that signal and continue to call even if disturbed. These female signals are easily imitated, which provides an important tool for collecting and manipulating cicadas: in an acoustical duet with a female, a male will become sexually excited and continue to sing even if disturbed or manipulated. Similar manipulations without a duet may cause a male to stop calling.

B. Current experimental opportunities with periodic cicadas

Opportunities to collect and study cicadas of the midwestern and eastern United States are limited by the insects’ periodical cycles. The insects have a 13 or 17 year life cycle and emerge in mass numbers, known as broods, at predictable times in predictable locations. Since the sounds and behaviors of the cicadas are not as yet fully characterized, an upcoming emergence of the insects will provide an opportunity to test and report on the tymbal and abdomen structural dynamics that generate the cicada’s mating call. Measuring the cicada’s two most important anatomic structures for producing sound would add to the scientific understanding of the extent of the nonlinear nature of the cicada signals and would also help to explain the high sound levels produced by this small insect.

C. Current state of cicada research

A comprehensive review of the prominent journal articles on cicada sound production and mechanisms and a survey of a number of subject area expert textbooks in this field revealed that the mechanisms underlying the cicada’s sound levels and efficient sound propagation are not...
fully understood. The cicada song has been classically modeled using linear mathematical methods. However, these linear methods are insufficient for a true model of the system because the buckling tymbals within the cicada sound production system are essential to the acoustic level and propagation of this mating call. Inelastic buckling is well recognized as a nonlinear phenomenon among researchers.

II. QUANTIFYING NONLINEARITY IN CICADA SIGNALS

A. Technical approach

A signal processing repertoire includes methods to test for (a) Gaussianity, (b) non-Gaussianity, (c) linearity, and (d) nonlinearity. The basic analytical tools used to perform these tests are the temporal power spectrum, the average mutual information (MI) (i.e., an information theory technique), and surrogate data hypothesis testing (i.e., phase randomization). A preliminary determination of the degree and influence of these effects in the sound production system of the cicada is explored using the Volterra expansion.

B. Nonlinear signal processing with mutual information

In order to substantiate the existence of nonlinearity, a quantitative method must be established. This study used MI (Ref. 30) and a surrogate data method to confirm nonlinearity. Equation (1), which defines the general MI, is a probabilistic equation used quantitatively to assess information between two random variables A and B.

\[ I_{A,B} = \sum_{i,j} P_{AB}(a_i,b_j) \log_2 \frac{P_{AB}(a_i,b_j)}{P_A(a_i)P_B(b_j)}, \]  

where \( P_{AB}(a_i,b_j) \) is the joint probability of events from sets \( A=\{a_i\} \) and \( B=\{b_j\} \), and \( P_A(a_i) \) and \( P_B(b_j) \) are the marginal individual probabilities associated with sets A and B, respectively. For example, set B can be taken as a collection of events that are time-delayed versions of the events in set A, which is the case for this research.

The surrogate data method consists of randomizing the phase of a signal spectrum as shown in

\[ s(n) = \sum_{k=0}^{N-1} x(k)e^{-i2\pi kn/N} \quad \mbox{for } n = 0;N-1. \]  

\( S(k) = \sum_{n=0}^{N-1/2-1} s(n)e^{i\phi(n)+i2\pi kn/N} \]

\[ + \sum_{n=N/2}^{N-1} s(n)e^{-i\phi(n)+i2\pi kn/N} \quad \mbox{for } k = 0;N-1. \]

\( S(k) \) is the overall phase-randomized discrete Fourier transform (DFT) of signal \( s(k) \), where phases \( \{\phi(n)\} \) are independent, uniform, and randomly distributed over a 2\( \pi \) range. \( s(n) \) is the complex amplitude spectrum of the DFT of the original time series, which is altered by a random phase \( \phi(n) \). In the surrogate method, the inverse DFT, \( S(k) \), is calculated from \( s(n) \), and this transformed time series is called the surrogate data and used for a comparison with the original signal \( x(k) \). Nonlinearity exists if the randomized signal’s MI diverges from the original signal’s MI, thereby signifying that nonlinearity must be present in the time series.

Linear calculations are plotted with the nonlinear signal results in order to display the contrast between linearity and nonlinearity. The Gaussian rule in Eq. (3) is presented to indicate how the correlation coefficient and MI are linked for the special case of a pair of Gaussian random variables with normalized correlation coefficient \( \rho \).

\[ I_{A,B} = -\frac{1}{2} \log_2 (1 - \rho^2). \]  

Note that the MI \( I_{A,B} \) is 0 when the correlation coefficient \( \rho \) is 0, while the MI goes to infinity as the correlation coefficient goes to positive or negative 1. This holds for all joint Gaussian processes, which are considered linear processes. The frequency domain equivalent of the correlation coefficient for measuring linearity is the subject of many published papers on coherence in the 1993 reprint text by Carter.\(^{31}\) Papers on coherence include how to estimate coherence, how to minimize bias and variance, and how to determine confidence bounds for estimates of magnitude-squared coherence. In general, for stationary random process, proper averaging of large time segment improves estimation.

C. Higher-order spectral techniques using magnitude-squared bicoherence

Furthermore, multispectral techniques exist, such as bicoherence and tricoherence, which could provide additional understanding of a non-Gaussian process. If the cicada signals are non-Gaussian, the bicoherence could determine if a process is a mixed-phased process or a nonlinear process. Cumulants are higher-order statistical information about any data series. For example, the first-order cumulant for a stationary process is its mean value. The second-order cumulant for a zero-lag process is the covariance, the third is skewness, and the fourth is kurtosis. As the first step beyond first-order spectral analysis, this study analyzes the bicoherence. Equation (4) is the magnitude-squared bicoherence (MSB), which is used to provide evidence on whether a data sequence is linear or nonlinear:

\[ B(f_1,f_2) = \frac{|S_2(f_1,f_2)|^2}{S(f_1)S(f_2)S(f_1 + f_2)}. \]  

D. Nonlinear signal processing using a Volterra expansion

A nonlinear fit is performed on the cicada data by means of the Volterra expansion, which describes the first-order and second-order signal dynamics present within the cicada time series. The data acquired on the cicada consist of two laser measurements, denoted by sequences \( \{a(n)\} \) and \( \{b(n)\} \), as well as a simultaneously sampled microphone sequence \( \{x(n)\} \). This situation will be considered to be a two-input, one-output, nonlinear “cicada system” with mathematical memory, instead of the traditional system of one input and one output.\(^{34}\) This formulation lends itself to a Volterra ex-
pansion, which can be used to determine, quantitatively, the extent of nonlinearity present between the two inputs and the one output.

1. Second-order Volterra formulation

The standard Volterra expansion for a one-input system takes the form

\[ y(n) = h_0 + \sum_{k=0}^{K_1-1} h_1(k)a(n-k) \]

\[ + \sum_{k=0}^{K_2-1} \sum_{j=0}^{K_2-1} h_2(k,j)a(n-k)a(n-j), \tag{5} \]

when carried to the second order, where \( y(n) \) is the Volterra fit. The three functions \( h_0, h_1(k), \) and \( h_2(k,j) \) are the zeroth-order, first-order, and second-order kernels, respectively. The first-order terms are carried out to length \( K_1 \), while the second-order terms are carried out to length \( K_2 \). Because of the symmetry inherent to \( h_2 \) in Eq. (5), the summation index \( j \) can be limited to value \( k \) and above. The unknown kernels appear linearly in model Eq. (5), whereas the known excitation \( \{a(n)\} \) appears nonlinearly through a product of delayed versions.

With a two-input model, a generalization is necessary, namely,

\[ y(n) = y_0 + y_1(n) + y_2(n), \tag{6} \]

where components

\[ y_1(n) = \sum_{k=0}^{K_1-1} h_1(k)a(n-k) + \sum_{k=0}^{K_2-1} h_1(k)b(n-k), \tag{7} \]

\[ y_2(n) = \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_2-1} h_2(k,j)a(n-k)a(n-j) \]

\[ + \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_2-1} h_2(k,j)b(n-k)b(n-j) \]

\[ + \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_2-1} h_2(k,j)a(n-k)b(n-j). \]

There are two linear components in \( \{y_1(n)\} \) each of length \( K_1 \), and three nonlinear (second-order) components in model output \( \{y_2(n)\} \). The advantage of the inherent symmetry in the two auto components in \( y_2(n) \) reduces the number of kernel values that have to be determined. However, the cross component \( h_2(k,j) \) in \( y_2(n) \) has no such symmetry and therefore requires a full \( K_2 \) by \( K_2 \) expansion. The total number of unknown kernel coefficients in Eqs. (6) and (7) is

\[ K = 1 + 2K_1 + K_2(2K_2 + 1). \tag{8} \]

It is desired to choose these coefficients \( h \) so that the total model output, Eq. (6), fits the measured microphone output \( z(n) \) as well as possible using least squares, so that \( y(n) = z(n) \). The least-squares approach is adopted because the simultaneous equations for the optimum kernel coefficients will then all be linear.

Although the laser and microphone data have been sampled at frequency \( f_s = 96 \text{ kHz} \), the crucial frequency content of the cicada system itself is not believed to extend above 10 kHz. Therefore, reductions in the memory lengths \( K_1 \) and \( K_2 \) for the first-order and second-order kernels in the model will not alias the cicada statistical information. Also, the memory lengths are decreased to minimize the computer random access memory required to calculate Eq. (6). Consequently, a decimation factor of \( M \) is applied to the kernels. Thus, the model to be fitted is a modification of Eq. (7), namely,

\[ y_1(n) = \sum_{k=0}^{K_1-1} h_a(k)a(n-Mk) + \sum_{k=0}^{K_1-1} h_b(k)b(n-Mk) \]

\[ = y_a(n) + y_b(n), \tag{9} \]

\[ y_2(n) = \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_1-1} h_{ab}(k,j)a(n-Mk)a(n-Mj) \]

\[ + \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_1-1} h_{bb}(k,j)b(n-Mk)b(n-Mj) \]

\[ + \sum_{k=0}^{K_1-1} \sum_{j=0}^{K_1-1} h_{ab}(k,j)a(n-Mk)b(n-Mj) \]

\[ = y_{ab}(n) + y_{bb}(n) + y_{ab}(n). \]

By this means, the memory length of the first-order kernels is \( MK_1 \) units of the sampling increment \( 1/f_s \), while that of the second-order kernels is \( MK_2 \) units. Notice that the measured data \( \{a(n), b(n), z(n)\} \) are not decimated, thereby retaining any harmonic and intermodulation products that might have been created by the cicada system itself.

2. Least-squares considerations

The details of a first-order fitting procedure will be presented; this formulation can then be extended to include all the terms in Eq. (9). The pertinent equation that governs the least-squares approach is to make

\[ y_a(n) = \sum_{k=0}^{K_1-1} h_a(k)a(n-Mk) \]

\[ = h_a(0)a(n) + h_a(1)a(n-M) \]

\[ + \cdots + h_a(K_1-1)a(n-M(K_1-1)) \tag{10} \]

approximate \( z(n) \) for

\[ N_1 \leq n \leq N_1, \quad N_1 = M(K_1-1) + 1. \tag{11} \]

where \( N_1 \) is the common data length of the three available data sequences. The particular starting value \( N_1 \) for \( n \) arises so that the inherent buildup transient of the first-order kernel \( h_a(k) \) will be excluded from the fitting procedure using Eq. (10). Trying to fit the transient can only degrade the procedure; confining the error minimization to the steady-state model output is the best approach.

Equations (7) and (9) can be put into a matrix formulation as follows:

Hughes et al.: Nonlinear characteristics of cicada sound generation

Hughes et al.: Nonlinear characteristics of cicada sound generation

961

measurement was made with dual Polytec OFV-508 optical measurement heads controlled by the Polytec electronic signal processor OFV-2802. The dual optical sensor heads allowed simultaneous measurement of the motion of both tymbals and the tymbal-to-abdomen motion. Meanwhile, the parabolic microphone was used to continuously record either the output of the two tymbals or a tymbal-abdomen experimental setup. The measurements provide the quantitative velocity from the laser, which can be transformed into displacement (position versus time) information. The tymbal motion is proportional to acoustic mechanical vibration and thus can be used in conjunction with the microphone acoustic output data. Thus, the data gathered in this field experiment, in which the output and input signals were obtained simultaneously, allows a unique opportunity to gain further scientific understanding of the cicada sound production system. The input signal is from the tymbal, and the output signal is recorded via the microphone. These data sets allow the application of nonlinear mathematical techniques, such as the Volterra expansion, to real-world signals.

Two channels were designated for collecting dual laser measurements of the motion of both tymbals and the tymbal-abdomen simultaneously. Most collections consisted of live wingless insects, otherwise intact. A third channel was used to record the microphone output. A fourth channel was used to time tag all signals. Two multichannel recorders were used to record and back up the data.

The experimental setup used a multichannel digital recorder set to a 96 kHz sampling rate, a laser and its controller, a backup digital recorder, and a digital timekeeper. The digital recorder facilitated the simultaneous acquisition of the tymbal motion, a microphone, and time stamp. One advantage afforded by this experimental arrangement was the opportunity to analyze quantitatively both synchronous and asynchronous tymbal motion and how such motion might impact the vocalization output.

IV. NONLINEAR SIGNAL PROCESSING RESULTS

A. Nonlinear and linear modeling of cicada signal results

A linear representation of a simulated time series of the cicada vocalization based on the superposition of a Gaussian white-noise signal passed through three parallel independent narrowband filters and the power spectral representation of this simulated time series representation is shown in Fig. 1(a). Figure 1(b) shows the temporal power spectrum of the cicada vocalization measured in the field. Given the striking similarity between these two spectra, the immediate, but erroneous, conclusion might be that the modeled spectra are a good representation of the insect’s actual acoustic signature. This is the classical error made by analysts using strictly linear techniques to model behavior in a physical system. In fact, linear techniques are insufficient for a correct analysis of the underlying signal processing mechanics of the cicada’s sound production system.

The simulated signal’s Mi (i.e., white Gaussian excitation passed through three parallel narrowband filters) and the Gaussian M1 model of the simulated signal [from Eq. (3)] are
FIG. 1. (Color online) Power spectra of simulation vs field measurement. (a) Power spectrum of simulated cicada call estimated with linear model. (b) Temporal power spectrum of cicada call as measured in the field. The spectra appear similar, but the linear model does not accurately represent the measured acoustic signature.

compared, which produce an identical plot to Fig. 2 where the solid line is the \( M_i \) and the "X" or cross symbol indicates the Gaussian rule. However, Fig. 2 compares a plot of the \( M_i \) obtained from the non-phase-randomized simulated date with the phase-randomized simulated data. As expected, the phase-randomized signal and the non-phase-randomized signal \( M_i \) generate the same values since phase randomization implies near Gaussianity (according to the central limit theorem), which in turn implies linearity. In Fig. 3, the Gaussian curve fit to the probability density function (PDF) is very similar to the simulated data curve based on its histogram. These graphs are like textbook examples of what the results should be for a linear Gaussian process. The graphs also serve as a baseline with which to compare the field measurements of cicada signals. A field recording of the cicada signal is displayed in Fig. 4(a), with a zoomed-in version of the same signal in Fig. 4(b). The expanded view illustrates the complexity of the cicada call. In Fig. 5, the cicada data (as \( M_i \) of the cicada vocalization) are compared with the surrogate data (the phase-randomized cicada data) to show how the phase-randomized cicada signal approaches the Gaussian rule. The separation between \( M_i \) plots of the cicada signal and the phase-randomized version of the same signal corroborate its non-Gaussian probability distribution. Also, the separation between the \( M_i \) of the cicada vocalization and the simulated data using the Gaussian rule is identical to Fig. 5, which is consistent with strong non-Gaussianity [i.e., Eq. (3)].

Additional evidence of the non-Gaussian behavior of the cicada vocalization is illustrated in Fig. 6. This figure compares the Gaussian fit with the curve based on the histogram associated with the cicada's non-Gaussian signal. Note that the non-Gaussian behavior in the cicada call was not reflected in the modeled spectra shown earlier in Fig. 1. Thus, Fig. 6 reinforces the point made earlier against relying solely on power spectral techniques to analyze cicada time waveforms.

FIG. 2. (Color online) \( M_i \) obtained from the phase-randomized simulated signal as a function of the time lag \( \tau \) (X) compared with the non-phase-randomized simulated data (solid line plot). The marker (X) is indistinguishable from the solid line, as expected, since phase randomization implies near Gaussianity (according to the central limit theorem), which in turn implies linearity. If plotted, the simulated signal's \( M_i \)—white Gaussian excitation passed through three narrowband filters—as a function of the time lag \( \tau \) (solid line) is compared with the Gaussian rule model of the simulated signal from Eq. (3) (X). The plots are identical to what is shown.

FIG. 3. (Color online) The Gaussian curve fit to the probability density function (dashed line) is very similar to the simulated data curve based on its histogram (solid line)—textbook example of results for a linear Gaussian process and baseline for the field measurements of cicada signals.

FIG. 4. (Color online) Time series of measured cicada vocalization: a 0.5 s field recording (a) and an expanded segment illustrating the complexity of the cicada call (b).

Hughes et al.: Nonlinear characteristics of cicada sound generation
FIG. 5. (Color online) Mi of the cicada vocalization as a function of the time lag \( t \) (solid line) compared with the surrogate data, i.e., Mi of the phase-randomized cicada data (X). The separation between Mi plots of the cicada signal and the phase-randomized version of the same signal confirm its non-Gaussian probability distribution. If plotted, the cicada signal's Mi as a function of the time lag \( t \) (solid line) is compared with the Gaussian rule model of the signal from Eq. (3). (X) is identical to this plot.

B. Results of higher-order spectral technique using MSB

The results from Eq. (4) are shown in this section of the report. Namely, the bicoherence indicates whether the time series data is linear or nonlinear. The theoretical MSB \( B(f_1, f_2) \) of a linear process is flat throughout the \( f_1-f_2 \) plane. Therefore, the example shown in Fig. 7 with an exact solution is utilized to ascertain if the bicoherence will always predict a flat surface for this linear process, where the center frequency \( f_c \) is zero. Here is a zero-mean, unit-variance Gaussian process. Consider the entire expression of Eq. (4) with the denominator terms containing the individual frequencies as well as the addition of both frequencies. Figure 8 illustrates that the denominator terms do not completely flatten the conical peak when the numerator (bispectrum) is divided by the denominator shown in Eq. (4). Note that the MSB plot is not flat suggesting that the process is not linear. Figure 9 illustrates that the PDF does have an effect on the random amplitudes generated in the bicoherence computation. However, the amplitudes of the sample MSB in Fig. 9 also suggest that averaging may help reduce the peaks and valleys of the plots. Figure 10 confirms that averaging does reduce the amplitude variation of the sample MSB. Such observations are consistent with the bias and variance reduction observed in coherence estimation. Nonetheless, the amplitude variation is not completely removed from the estimate. Hence, averaging the MSB only scales down the problem of a fluctuating surface. Consequently, a flatness factor could yield a meaningful parameter to employ on the variations calculated for the sample MSB.

\[
\bar{x} = \frac{1}{N} \sum_{i=1}^{N} x_i, \\
\sigma = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (x_i - \bar{x})^2}, \\
\sigma_{MSB} = \sqrt{\frac{1}{N-1} \sum_{i=1}^{N} (\sigma_1 - \sigma_2)^2}.
\]

Equation (16) defines the "flatness factor," which derives from the sample standard deviation of the fluctuations in the \( f_1-f_2 \) plane. The variable \( \sigma \) represents the calculation of the sample standard deviation for a known sample set \( \{x_i\} \) with a total sample size \( N \), while the parameter \( \bar{x} \) is the sample mean.
mean of the data set. Therefore, the sample standard deviation \( \sigma_{\text{MSB}} \) represents the variation for the entire plane.

Since the PDF of a given data set affects the standard derivation, a determination must be made to show how the PDF affects the MSB. A cumulative distribution function (CDF) is a convenient approach to determine if a PDF has Gaussian characteristics, which is the case for the exponential and normal distributions. If the exponential and the normal CDFs differ, the implication is that the PDF influences the MSB significantly. In Fig. 11, the CDF for the exponential signal is shown. Compared to Fig. 12, there is a difference in value of the accumulation of bins in the MSB for the exponential and normal distributions of 0.5-0.95, respectively. This difference is considerable and thus indicates that the PDF of the data contributes notably to the MSB. Consequently, a prerequisite algorithm must account for the effect of the PDF on the MSB, and PDFs could vary between individual cicadas and even more so with different species. Because the MSB significantly depends on the PDF, additional work beyond the scope of the present research is required for quantifying how statistically useful the sample MSB is for analyzing cicada signals.\(^{15,16}\)

C. Volterra expansion method applied to cicada experimental results

A sample of 100,000 data points from the laser and microphone measurements is selected for fitting purposes, using a decimation factor \( M=4 \). The number of coefficients employed for the first-order fit is \( K_1=100 \), and the number used for the second order is \( K_2=50 \), which results in solving for a total of \( K=5251 \) coefficients. The execution time required simply to fill the \( \mathbf{D}'\mathbf{D} \) matrix, using segment length \( L=1000 \), is 1670 s on a 2.4 GHz computer. The solution time for the optimum kernel is 22 s, and the time required to compute all five individual component waveforms \( \mathbf{y}_a(n), \mathbf{y}_b(n), \mathbf{y}_{ab}(n), \mathbf{y}_{aa}(n), \mathbf{y}_{bb}(n) \) in Eq. (9) is 220 s. The singular value decomposition of matrix \( \mathbf{D}'\mathbf{D} \) took 720 s, and its condition number is \( 7.2 \times 10^9 \). Thus, approximately 8 decimal digits (out of 15) of significance remain in the numerical results obtained. The total execution time is almost 44 min. The ratio of the power in the total model output, per Eq. (6), to the power in the measured microphone output \( z(n) \) is 0.41. Thus, the Volterra fitting procedure of the second order is capable of representing 41\% of the power of the microphone waveform.

![FIG. 9. The MSB is calculated for a normalized random noise signal and the mesh plot exposes that surface undulations are present in the \( f_1-f_2 \) plane, which would give the false conclusion that Gaussian random noise is not a linear process.](image)

![FIG. 10. The mesh plot of MSB for a normalized random noise averaged over 1000 trials still has ripples on the \( f_1-f_2 \) plane. However, this averaging scales the fluctuating surface but determining nonlinearity remains difficult to quantify.](image)

![FIG. 11. (Color online) A CDF for an exponential random noise averaged over 1000 trials indicates the Gaussian characteristics of the MSB.](image)

![FIG. 12. (Color online) CDF for a normal random noise averaged over 1000 trials is also Gaussian but has a significant different PDF, which affects the MSB when compared to the exponential random noise in Fig. 11.](image)
FIG. 13. (Color online) Short time segments of simultaneous laser measurements of the motion of the two tymbals and the sound output at the microphone in the field are shown. The repeated bursts of narrowband energy are clear in the laser A and B measurements in (a) and (b), respectively. The microphone output (c) tends to capture environmental noise, which blurs the individual bursts seen in (a) and (b).

As a check case, an unrelated random white-noise process replaces the microphone output \( z(n) \), and the fitting procedure is repeated with identical parameters. The power ratio of the final fit is reduced to 0.053. An additional run with a different random sequence for \( z(n) \) yields a comparable value for the power ratio. Thus, the fitting ratio 0.41 that is actually attained is a significant value and indicates that nonlinearities are present in the cicada system between laser inputs and microphone output. In fact, the power in the second-order component \( y_2(n) \) in Eq. (9) is almost three times greater than the power in the linear component \( y_1(n) \). Also, the power in the cross component \( y_{ab}(n) \) is greater than the powers in the two autocorrelations \( y_{aa}(n) \) and \( y_{bb}(n) \).

D. Volterra graphical results

A short time segment of the three simultaneous field measurements is displayed in Fig. 13. Each of the lasers—A and B—measured the narrowband buckling of a single cicada tymbal, shown in Figs. 13(a) and 13(b), respectively, while the microphone [see Fig. 13(c)] tended to capture all the acoustic noise in that segment of the field measurement. The corresponding (unsmoothed) spectral estimates of the complete data segments used in the Volterra fit are plotted in Fig. 14. A segment of the microphone data \( z(n) \) is compared with the total fitted waveform \( y(n) \) in Fig. 15(a). The fit is rather good in some time intervals but poorer in others—a manifestation of the actual fitted power ratio of 0.41. The plots in Figs. 15(b) and 15(c) show the individual total first-order and total second-order fits, respectively. The two first-order time-domain kernels are plotted in Fig. 16, while their complex frequency-domain transfer functions are displayed in Fig. 17. The decimation factor \( M=4 \) is the reason for the upper frequency limit of 12 kHz for these kernels.

An alternative Volterra expansion is computed that measures the nonlinearity in the cicada's mating call. Table I describes each tymbal’s contribution to the first- and second-order components in the cicada signal by using a Volterra expansion with \( M=3 \) and \( K_2=70 \). The Volterra technique

FIG. 14. (Color online) Unsmoothed spectral estimates of the data measured at lasers A (a) and B (b) and at the microphone (c). These complete data segments, corresponding to both tymbal and microphone outputs in Fig. 13, are used in the Volterra fit.

FIG. 15. (Color online) Least-squares fit for Volterra expansion. (a) A segment of the microphone data (solid line) [see Fig. 13(c)] is compared with the total fitted waveform (dashed line). The fit is rather good in some time intervals, but poorer in others—a manifestation of the fitted power ratio of 0.41. (b) and (c) are the individual total first- and second-order fits, respectively.

FIG. 16. (Color online) First-order time-domain kernels of tymbal motion measured by lasers A (a) and B (b).
FIG. 17. (Color online) Transfer functions of first-order time-domain kernels of tymbal motion: power (a) and phase (b) for laser A data, and power (c) and phase (d) for laser B data.

TABLE I. Volterra expansion to quantify cicada signal nonlinearity for laser A (a), laser B (b), and the microphone (zz).

<table>
<thead>
<tr>
<th>Ya</th>
<th>Yb</th>
<th>Yaa</th>
<th>ybb</th>
<th>yab</th>
<th>Y1</th>
<th>Y2</th>
<th>Y</th>
<th>zz</th>
<th>Error</th>
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<tr>
<td>(6.4677 \times 10^{-8})</td>
<td>(3.2900 \times 10^{-7})</td>
<td>(2.5461 \times 10^{-7})</td>
<td>(1.0602 \times 10^{-6})</td>
<td>(6.0081 \times 10^{-7})</td>
<td>(3.9690 \times 10^{-7})</td>
<td>(1.6650 \times 10^{-6})</td>
<td>(1.9100 \times 10^{-4})</td>
<td>(4.8180 \times 10^{-6})</td>
<td>(2.9090 \times 10^{-6})</td>
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</table>
indicate nonlinear behavior. Although the curse of dimensionality pertains to the Volterra expansion, the current second-order solution is satisfactory for initiating the quantification of the nonlinear parameters for modeling and simulating purposes.

Mi and surrogate hypothesis testing techniques can be combined with the Volterra or Wiener expansion to parametrize the cicada’s abdominal cavity motion and tymbal excitation and create a model to simulate the insect’s sound production mechanism. The combined and individual contributions of the motion of the abdomen and tymbals can be evaluated and quantified using the nonlinear least-squares technique combined with a Wiener expansion. The Wiener method allows the computation of second- and third-order solutions with fewer coefficients. Reducing the coefficient size for each order makes the computational limitation less of a concern, and consideration can be given to calculating higher-order kernels. Determining these parameters will aid in developing a device that generates sound propagation with the efficiency of the cicada vocalizations. Such efficient sound wave propagation, if viable in water, would markedly enhance source radiation efficiency for a variety of sonar applications.

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