Workshop on
“Constructal Theory of the Generation of Optimal Flow Configurations”

Roma, School of Engineering, “Aule del Chiostro”, 17th and 18th of March, 2005

Constructal Theory is a new research development aimed at the identification of the most efficient “structure” that a flow of mass, energy, or “anything that moves” can take. The theory is validated by comparisons with natural phenomena, since it is generally accepted that systems in nature evolve to the most “efficient” configuration under the prescribed constraints. Constructal theory is being considered for analysis of the thermal management of components and systems, for system analysis of intrinsically energy-efficient vehicles, for traffic management, for networks for distribution and collection, etc. The approach shows potential for significant applications in land, air, sea and space systems, its generality being founded on its theoretical basis of “thinking about design as a science”. The objective of the workshop is to assess the status and limitations of present technologies for developing the necessary systems of systems; current developments in new technologies; potential applications to future system developments; research directions for future investment.

The Workshop has been jointly organised by the University of Roma 1 “La Sapienza”, Duke University and the United States Air Force Research Laboratory. It has been sponsored and funded by the European Office of Aerospace Research and Development, Air Force Office of Scientific Research and by the University of Roma 1.

The Organising Committee:
Adrian Bejan (Duke University)
David Moorhouse (AFOSR)
Enrico Sciubba (U. of Roma I)
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<td>Professor Enrico Scuibba</td>
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**ABSTRACT (Maximum 200 words)**

Contains collected presentations from the Workshop on Constructal Theory Application to Aerospace advanced vehicles considered as Complex Energy Conversion Systems, 16-18 March 2005. This was an interdisciplinary workshop and was by invitation only. Topics include evaluating the possibility of applying ‘Constructal Theory’ (CT) to the optimization of complex energy conversion systems, with specific regard to advanced aerospace vehicles. The workshop included discussions of configuration optimization procedures that are possible with CT. The workshop goal was toward delivering a set of general guidelines as to the practical design topics in which CT can be applied in the near term.

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### Program

**Thursday, March 17 (Sala Grande del Chiostro)**

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<tr>
<td>9.30</td>
<td>Welcome address</td>
<td></td>
</tr>
<tr>
<td>10.00</td>
<td>Constructal Theory: Shape &amp; Structure, from engineering to nature</td>
<td>Adrian Bejan</td>
</tr>
<tr>
<td>10.45</td>
<td>Requirements on methods for system-level analysis of efficient Aerospace flight vehicles</td>
<td>David Moorhouse</td>
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<tr>
<td>11.30</td>
<td>Tree-shaped networks for distribution, collection &amp; cooling</td>
<td>Sylvie Lorente</td>
</tr>
<tr>
<td>12.15</td>
<td>Constructal Theory &amp; variational principles for the Navier-Stokes equations: is there a link?</td>
<td>Enrico Sciubba</td>
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<td>13.00</td>
<td>Lunch break</td>
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<tr>
<td>15.00</td>
<td>Aerosol particle deposition, and shape in inanimate/animate systems</td>
<td>Antonio F. Miguel</td>
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<tr>
<td>15.45</td>
<td>Flow architectures of lungs and river basins</td>
<td>A. Heitor Reis</td>
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<tr>
<td>16.30</td>
<td>Optimal geometry for a rectangular slab with internal convection cooling</td>
<td>Adrian Bejan, Mauro Robbe</td>
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<td>17.15</td>
<td>Discussion</td>
<td>A. Bejan, D. Moorhouse</td>
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<td>20.00</td>
<td>Workshop Dinner, Restaurant “La Piazzetta”</td>
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**Friday, March 18 (Sala Piccola del Chiostro)**

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<tr>
<th>Time</th>
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<tr>
<td>9.30</td>
<td>Performance improvement of a tubular Solid Oxide Fuel Cell stack through the study of its configurational design</td>
<td>Vittorio Verda</td>
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<td>10.15</td>
<td>Numerical simulations of the fluid &amp; heat flow in disc-shaped compact heat exchangers with dendritic flow structure</td>
<td>Mauro Robbe</td>
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<tr>
<td>11.00</td>
<td>Energy usage in Aircraft Equipment Systems</td>
<td>Lester Faleiro</td>
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<tr>
<td>11.45</td>
<td>On the synthesis of complex energy systems with several heat transfer interactions</td>
<td>Andrea Lazzaretto</td>
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<tr>
<td>12.30</td>
<td>Thermal Management of Aircraft Systems</td>
<td>Dave Pratt</td>
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<td>13.15</td>
<td>Lunch</td>
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<td>15.00</td>
<td>Panel discussion</td>
<td>E. Sciubba</td>
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<tr>
<td>15.45</td>
<td>General discussion, closure &amp; recommendations for future work</td>
<td>A. Bejan, D. Moorhouse, E. Sciubba</td>
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Constructal Theory: 
Shape and Structure, 
from Engineering to Nature 

Adrian Bejan 

J. A. Jones Distinguished Professor 
of Mechanical Engineering 
Duke University 
USA
1. Constructal theory: current progress

2. Aircraft research and design: needs, current work

3. Opportunities for constructal theory in aircraft development
Constructal theory (1996)

Internal spacings

Tree-shaped flow structures

Multiple scales, nonuniformly distributed

Multiple objectives

Complex engineering flow systems: flight

Transportation, economics

Engineering + Physics + Biology + Economics
Large resistance to fluid flow  Balanced resistances  Large resistance to heat flow

Optimal distribution of imperfection
Balanced resistances

Large horizontal resistance

Optimal distribution of imperfection

Large vertical resistance

Balanced resistances

Large horizontal resistance
Tree-shaped flow structure — deduced from principle
Round cross-sections, ducts not seen in nature first
Cross-sections of rivers not assumed
  · not ‘simulated’ based on an assumed (e.g., fractal) algorithm
  ·
  ·
  ·
Flow geometry deduced from principle
<table>
<thead>
<tr>
<th>Application</th>
<th>What flows</th>
<th>How</th>
<th>Channels low resistance at larger scales</th>
</tr>
</thead>
<tbody>
<tr>
<td>Electronics packages</td>
<td>Heat</td>
<td>Low-conductivity substrate</td>
<td>High-conductivity inserts (blades, needles)</td>
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<tr>
<td>River basins</td>
<td>Water</td>
<td>Darcy flow through porous media</td>
<td>Rivulets and rivers</td>
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<tr>
<td>Lungs</td>
<td>Air</td>
<td>Diffusion in alveoli, tissues</td>
<td>Bronchial passages</td>
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<tr>
<td>Circulatory systems</td>
<td>Blood</td>
<td>Diffusion in capillaries, tissues</td>
<td>Blood vessels, capillaries, arteries, veins</td>
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<tr>
<td>Turbulent flow</td>
<td>Momentum</td>
<td>Laminar, viscous diffusion</td>
<td>Streams, eddies</td>
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<tr>
<td>Urban traffic</td>
<td>People</td>
<td>Walking in urban structure</td>
<td>Street traffic</td>
</tr>
<tr>
<td>Economics</td>
<td>Goods</td>
<td>Hand delivery and collection</td>
<td>Freight, rail, truck, air, ship</td>
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</table>
Constructal law (1996):

“For a finite-size open system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the currents that flow through it”.

or the ‘physics’ behind:

Darwin’s observations
fractal algorithms
Average annual efficiency of all power stations in Great Britain

- Modern steam plants
- Open gas-turbine plants
- Large diesel engines
- Automobile engines

Heat engine efficiency

1920 1940 1960 1980
Constructal Multi-Scale Structures

Package of parallel plates with one spacing.
Multi-scale hierarchical structures
T. Bello-Ochende and A. Bejan,

Constructal multi-scale cylinders in cross flow,

A. K. da Silva, S. Lorente and A. Bejan,

Optimal distribution of discrete heat sources on a plate with laminar forced convection,

A. K. da Silva, S. Lorente, A. Bejan,

Constructal multi-scale tree-shaped heat exchangers,

View from above

View from the side
thermal resistance

one curve for one flow architecture

better flow architectures

increasing flow rate, fixed architecture

flow resistance
Exergy (fuel exergy) is destroyed during combustion and energy conversion. The environmental control system (ECS) is destroyed by the ECS. All the other flow systems on board are destroyed to overcome air friction. The weight of the aircraft is supported by power for flight.
Minimization of work/distance

induced friction
drag drag

\[
\frac{W}{L} \sim \frac{\rho_b g^2 D^4}{\rho_a V^2} + \rho_a D^2 V^2
\]

Balance of losses, optimal distribution of imperfection

\[
\left( \frac{W}{L} \right)_{\text{min}} \sim 2g\rho_b D^3 \sim 2gM
\]

\[
V_{\text{opt}} \sim \left( \frac{\rho_b}{\rho_a} gD \right)^{1/2}
\]

speed \sim (mass)^{1/6}
fuel mass, or destroyed exergy

- exergy destroyed by the system
- exergy destroyed for flying the system

system size

0

0
V (m/s)

Insects

- House fly
- Meat fly
- Hornet
- Bumblebee
- Dung beetle
- Honeybee
- Ruby-throated hummingbird
- Goldcrest
- Magnolia warbler
- Sand martin
- Skylark
- Blue jay
- Sparrow hawk
- Partridge
- Peregrine falcon
- Blue heron
- White stork
- Golden eagle
- Wandering albatross

Airplanes

- Piper Warrior
- Beech Bonanza
- Beech Baron
- Beech King Air
- Leerjet 31
- F-16
- Mig-23
- Fokker F-28
- F-14
- Boeing 737
- Airbus A310
- Douglas DC-10
- Boeing 747
**Fermat law**

point-point flow

\[ n_0, V_0 \]

\[ n_1, V_1 \]

\[ \alpha_0 \]

\[ \alpha_1 \]

**Constructal law**

volume-point flow

\[ L_0 \]

\[ H_0 \]

\[ V_0 \]

\[ V_1 \]
Hawaii, New Orleans, Panama Canal
Requirements on Methods for System-Level Analysis & Design of Efficient Aerospace Flight Vehicles ~ Exergy-Based or Constructal Theory or ???

Dr. David J Moorhouse
Air Force Research Laboratory
Wright-Patterson AFB USA
The MultiDisciplinary Engineer in a Concurrent Environment

TRADITIONAL DEVELOPMENT

- Aerodynamicist
  - Max L/D ??

- Structures
  - Min weight ?
  - Min cost ??

- Propulsion
  - Max T/W ?

- Subsystems
  - Max efficiency ??
  - Min O&S cost ?

SYSTEM DESIGN OPTIMIZATION

Integration of:
- Airframe & propulsion
  + fully adaptive structures
  + propulsion & power systems
  + aerodynamics & controls
  + etc, etc

Design for Synergistic Interactions

What is the Common Measure for Comparison ??
Why Develop New Methods ???

Technology Challenges:
- Accurate prediction/design tools
- Integration of airframe & propulsion
- Control of flow fields with plasma
- Energy extraction/power distribution


- 1st Law Principles → Energy conservation only
- 2nd Law Principles → Entropy can only be created

Minimizing Exergy Consumption Should Give an Efficient Design

What Will Constructal Theory Provide ??????????
Special Section, AIAA Journal of Aircraft, Jan-Feb 2003:

Moorhouse, “Proposed System-Level Multidisciplinary Analysis Technique Based on Exergy Methods”

Chandrasekaran et al, “Optimization of an Aircraft Power Distribution Subsystem”

Paulus & Gaggioli, “Rational Objective Functions for Vehicles”


Figliola & Tipton, “Exergy Approach to Decision-Based Design of Integrated Thermal Systems”

Integration Framework for System-Level Optimization of Aerospace Vehicles

A flight vehicle is a ‘device to do work’. The complete set of vehicle design requirements can be specified as an energy system. The mission can be stated in terms of work to be done by the exergy available from the fuel. The propulsion and every system are then components in minimizing the exergy consumed. This will provide the necessary understanding to allow decomposition into appropriate energy systems together with appropriate interactions. The exergy-based framework will support the design of a truly energy-efficient vehicle, although a realistic flight vehicle will also be subject to other constraints.
• Define weight specific energy as kinetic + potential energies per unit weight:

\[ E = h + \frac{1}{2g} U^2 \]

• **Customer Work** – includes generating the specific energy of the payload and overcoming drag and power requirements:

\[ \frac{dw_c}{dt} = W_p \frac{dE}{dt} + P_p + D_p U \]

\[ \Rightarrow \text{REQUIRED} \]

• **Overhead Work** – Sum of the work consumed and drag caused by every component of the system which has to carry the payload:

\[ \frac{dw_o}{dt} = \sum \left( W_i \frac{dE}{dt} + D_i U \right) \]

\[ \Rightarrow \text{TO BE MINIMIZED} \]
System Level Efficiency

The customer and overhead work integrated over the entire mission represents the total work that has to be supplied by the exergy of the fuel.

Conservation of Energy in time interval $dt$ is:

$$-\eta H \frac{dW}{dt} = \frac{dw_c}{dt} + \frac{dw_o}{dt}$$

$H$ is energy content of the fuel/weight, $\eta$ is overall efficiency, $dW/dt < 0$.

Design problem is to minimize overhead work and maximize overall efficiency.
Design Mission Stated in Terms of Work to be Done
How precise does this need to be?

Comes From the Exergy of the Fuel Consumed

Propulsion System Converts Fuel Into:
  Mission Work, Including Power to Drive Mission Equipment
  +
  Mission Overhead
    - Power to Lift Itself and Required Fuel
    - Overcome Vehicle Drag
    - Power for Other Subsystems
  +
  Waste due to Inefficiencies in Operation & Thermal Performance
# Exergy-Based Design Methods

## Potential System Energy Balance

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<tr>
<th>OVERHEAD</th>
<th>positives</th>
<th>negatives *</th>
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<tr>
<td>Actuation</td>
<td>essential functions</td>
<td>weight + power used</td>
</tr>
<tr>
<td>Structure</td>
<td>carries payload</td>
<td>weight</td>
</tr>
<tr>
<td>Adaptive structure</td>
<td>minimize drag, provide control</td>
<td>weight + power used</td>
</tr>
<tr>
<td>Controls</td>
<td>trim/stability/control</td>
<td>weight + cause drag + power used</td>
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<tr>
<td>Flow control</td>
<td>reduce drag</td>
<td>weight + power used + waste</td>
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<tr>
<td>ECS</td>
<td>essential cooling</td>
<td>weight + engine bleed + waste</td>
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<tr>
<td>Propulsion</td>
<td>supplies thrust + power</td>
<td>weight + waste</td>
</tr>
<tr>
<td>Fuel</td>
<td>supplies exergy + cooling</td>
<td>weight</td>
</tr>
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</table>

* weight now reflects work that needs to be done
Conventional high-speed vehicles have a requirement to protect the structure from elevated temperatures or provide active cooling, BUT:

**Structures as an Energy System**

Current efforts are utilizing structural deformation requiring actuation power as a flight control effector, i.e. wing twist for roll control. In high-speed vehicles, the structure will have to be designed as a fully-integrated part of the system. In these vehicles the structure may distribute energy between other systems, it may store energy for use as and when required, and it may be a user of exergy for tailoring different characteristics. For the future, with the advent of active structures technology, there is a need to develop an understanding of the vehicle structure as an energy system.
Adaptive Structure Design

**Approach**

- Develop a theoretical framework to identify energy flow inside of the body (input energy, transferred energy, stored energy and etc.) for efficiency calculation.

- Exergy-based framework to facilitate the design & system optimization of efficient systems.

\[
\eta_{\text{loaded}} = \frac{\text{Useful energy}}{\text{Input energy}}
\]

Total input energy = stored energy + transferred energy.
Energy Based Design
Structure as an Energy System

Exergy = \((u - u_o) - T_o (s - s_o) + \frac{P_o}{J} (v - v_o) + \frac{V^2}{2gJ} + \frac{g}{g_cJ} (z - z_o) + \sum_c (\mu - \mu_o) N_c + \ldots\)

Energy Harvesting
Basic Research

Shape Control
Basic Research
Design Example

High Temp. Reservoir at \(T_1\)
Low Temp. Reservoir at \(T_{N+1}\)

Fuel
Internal systems and sub-systems
Systems interacting with external environment

Energy harvesting / control systems

Waste energy

Actual shape

\(Y(n)\)
\(-5\)
\(-5\)

\(-20\)
\(-10\)
\(0\)
\(10\)
\(20\)
\(30\)
\(40\)
In current aircraft, the engine obviously supplies the direct power, cooling air, etc for all the aircraft subsystems. The definition of a turbine engine as an exergy component with airframe weight and drag has been done.

**Propulsion as an Integrated Energy System**

A high-speed vehicle will require new forms of propulsion and airframe system integration. It may power a device to reduce airframe drag as opposed to providing thrust to overcome the drag. There must be a credible way to perform the design trades. We need to develop the understanding of how the hypersonic propulsion system and all the other required systems can be designed to common metrics and optimization criteria.
**Sub-system Coupling Functions**

<table>
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<th>$u_{15}$</th>
<th>Weight of the PS</th>
<th>$\bar{u}_{35}$</th>
<th>VC/PAOS weight</th>
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<tr>
<td>$\bar{u}_{21}$</td>
<td>Momentum drag, FLS weight, power</td>
<td>$\bar{u}_{41}$</td>
<td>Momentum drag, ECS weight, bleed air</td>
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<tr>
<td>$\bar{u}_{25}$</td>
<td>FLS weight</td>
<td>$\bar{u}_{45}$</td>
<td>ECS weight</td>
</tr>
<tr>
<td>$\bar{u}_{31}$</td>
<td>Momentum drag, VC/PAOS weight, power</td>
<td>$\bar{u}_{51}$</td>
<td>Drag, weight of the AFS</td>
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<tr>
<td>$\bar{u}_{32}$</td>
<td>Heat rejection</td>
<td>$\bar{u}_{61}$</td>
<td>PPAY, EPAY, EG weight</td>
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<tr>
<td>$\bar{u}_{34}$</td>
<td>Heat rejection</td>
<td>$\bar{u}_{65}$</td>
<td>PPAY, EPAY, EG weight</td>
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A conventional flight control system is a user of exergy in the form of actuation power, and can impact vehicle drag which consumes exergy.

**Future task required: Control Strategies for Energy Systems**

- The control design problem is magnified when we consider the control requirements and associated subsystems of future vehicles.

*Develop an Understanding of How the Total Vehicle Control System can be Designed to Exergy-Based Principles.*

*Develop an Understanding of How to Control the Distribution or Transfer of Energy Between the Different Systems in Order to Produce an Optimum Vehicle Control System.*
Scaling Laws for Energy Systems

There is a need to understand how the results of a laboratory test will have to be applied in the design of a full-scale vehicle. This will become increasingly critical as we attempt to design a full-scale hypersonic flight vehicle using components with no previous flight experience. Need to integrate mechanical work, thermal energy, rate of change of momentum, etc, etc. This requires research to develop an understanding of the appropriate scaling laws for energy systems.
A common problem area with flight vehicle development involves the full-scale characteristics not matching the predicted models. In many areas of design it is common practice to design for uncertainties, e.g. sensitivity analyses, robust control theories.

**Uncertainty Analysis of Energy Systems**
Currently, there is no database on the expected accuracy or errors in modeling some of the new energy systems that will be used in a hypersonic flight vehicle. It is probable that each system will interact with every other one. The propagation of uncertainties through an interactive system must also be addressed. There is a need to develop an understanding of the uncertainties in modeling these systems and the ramifications of using the models in full-scale design.
Optimization Techniques for a System of Energy Systems

It is not guaranteed that conventional optimization techniques are the most appropriate for the envisioned system of energy systems. It is also possible that a system of energy systems will be subject to new and unforeseen constraints.

The understanding must be developed of how to minimize total entropy generation in a complex integrated system, plus the appropriate constraints.
Exergy-Based Design Methods

Summary

Energy Consideration is NOT New, BUT The Uses Have Been Implicit, Disconnected and Indirect
e.g. the Breguet range equation ~ work against drag
e.g. propulsion system design

We Considered Explicit Use of Entropy & Exergy Principles to Develop System-Level Design/Analysis Methods Based on the Laws of Thermodynamics:
• Mission Requirements in Terms of Work to be Accomplished
• Energy/Exergy Terms Flow Down to Every Component
• Computation of Entropy Generation Rate -> Exergy Destroyed
• Minimize Available Work that is Wasted, i.e. Fuel Required
System-Level Design Methods

Summary

Is Constructal Theory the Natural Energy Flow for Mechanical Systems?

Does Natural Selection Apply for Constrained Optimization??

How do We Need to Model Flight Vehicles & Systems???

What are the Constraints for Efficient Energy Usage????

Will This Approach Give Us Better Practical Solutions????

What Further Research Is Required?
Tree-Shaped Networks for Distribution, Collection & Cooling

Sylvie Lorente
Lab. Materials and Durability of Constructions
National Institute of Applied Sciences (INSA),
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“For a finite-size open system to persist in time (to survive) it must evolve in such a way that it provides easier and easier access to the currents that flow through it”.

Adrian Bejan, Duke University, North Carolina, USA
1996
Tree architectures

everywhere in nature and engineering
Introduction

Purpose to make a flow connection between one point and an infinity of points

Complex flow structures with multiple scales: tree-shaped networks

The flow resistances cannot be eliminated. They can be rearranged, assembled ...

To minimize their influence on performance = optimization

Method

Integration of an increasing number of smaller and smaller flow components

Elemental area → Constructs

**Shape** (or geometry) is the result of the minimization of imperfections, or the optimal distribution of imperfections.
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Networks at human scale

Tree-shaped networks of insulated pipes for the distribution of hot water uniformly over a given territory (area A)

Objectives:
- Maximum delivered temperature
- Minimum pumping power

Constraints:
- Total amount of insulation
- Total volume of the pipes
Construction of the network

Elemental System

- $r_i$: inner radius of the pipe
- $r_o$: outer radius of the (pipe + insulation)
- $L_0$:
- $L_0/2$:
- $T_{end}$:
- $T_0$, $m_0$, $\Delta P_0$:

Different configurations

Case 1: every construct covers a square area

Case 2: the new construct is obtained by pairing the previous ones

\[ A_1 = 2L_0^2 \]

\[ A_2 = 2^2L_0^2 \]

\[ A_4 = 2^4L_0^2 \]
Constraints

- Total volume occupied by the ducts
  - **Fluid Mechanics aspect**
    - objective: minimization of the pressure drops
    - minimization of pumping power

- Total amount of insulation wrapped around the pipes
  - **Heat Transfer aspect**
    - objective: minimization of the heat losses
    - maximization of the temperature of the hot water received by the end user
**METHOD**

Pressure drops and heat losses are minimized at every step

Set of equations:

*Fluid Mechanics aspect*

- \[ \Delta P = C \frac{L \dot{m}^2}{r_i^5} \] fully turbulent flow (rough)

- elemental system \[ \Delta P_0 = C \frac{L_0 \dot{m}_0^2}{2 \frac{r_{i_0}^5}{r_{i_0}^5}} \]

example of case 1:

\[ \Delta P_1 = C \frac{L_0}{2} \left( \frac{4m_0}{r_{i_1}} \right)^2 + CL_0 \left( \frac{2m_0}{r_{i_1}} \right)^2 + C \frac{L_0}{2} \frac{m_0^2}{r_{i_0}^5} \]

\[ \Delta P_1 = CL_0 m_0^2 \left[ \frac{12}{r_{i_1}^5} + \frac{1}{2r_{i_0}^5} \right] \]
total volume occupied by the ducts

\[
\frac{3}{2} L_0 \pi r_i^2 + 2 L_0 \pi r_{i0}^2 = \text{constant}
\]

minimization \( \frac{\partial}{\partial r_{i0}} \Delta P_1 = 0 \) \( \frac{r_{i1}}{r_{i0}} \)_{\text{opt}} = 2^{5/7} \)
Heat Transfer aspect

- heat losses to the ambient (per unit of pipe length)

\[ q' = \frac{2\pi k}{\ln\left(\frac{r_o}{r_i}\right)} (T - T_\infty) \]

the pipe thickness is neglected
k: thermal conductivity of the insulating material
• energy conservation \( \dot{m}C_p dT = q'dx \)

\[
\frac{T_{\text{end}} - T_\infty}{T_{\text{initial}} - T_\infty} = \exp\left(-\frac{N}{\ln R}\right)
\]

with \( R = \frac{r_o}{r_i} \) and \( N = \frac{2\pi kL}{mC_p} \) "number of heat loss units"

\[ N = \frac{2\pi k L}{m C_p} \]

"number of heat loss units"

a low \( N \):
- high performance materials (low k)
- high mass flow rate

\[ N_0 = \frac{2\pi k L_0}{2 m_0 C_p} \]

- elemental system
example of case 1:

\[ \theta_1 = \frac{T_{\text{end}} - T_\infty}{T_1 - T_\infty} = \exp\left( -\frac{N_0}{\ln R_0} - \frac{5N_0}{4 \ln R_1} \right) \]
total amount of insulation

\[ \tilde{V}_1 = \frac{V_1}{\pi L_0 r_{i_0}^2} = \frac{3}{2} \left( \frac{r_{i_1}}{r_{i_0}} \right)^2 (R_1^2 - 1) + 2(R_0^2 - 1) \]

maximization of \( \theta_1 \)

\[ \frac{R_1 \ln R_1}{R_0 \ln R_0} = \left( \frac{5}{6} \right)^{1/2} \left( \frac{r_{i_0}}{r_{i_1}} \right)_{\text{opt}} \]

\[ R = \frac{r_0}{r_i} \]

The insulation shell is relatively thicker over the smaller ducts

Comparison of the different systems

Which network serves the farthest user better?
\[ q' = \frac{2\pi k}{\ln(\frac{r_0}{r_i})} (T(x) - T_\infty) \]

\[
\frac{T(x) - T_\infty}{T_i - T_\infty} = \exp \left( -N \int_x^{L} \frac{dx}{x \ln (\frac{r_o}{r_i})} \right)
\]
2 levels of comparison: square areas $4L_0^2$ and $16L_0^2$

the same surface covered by the network amount of insulating material

"Case 2" performs best

$T_{\text{end}}$ is higher.

The difference decreases as the Structure becomes more complex

"Case 2" each user receives hot water at the same temperature
The addition of new users to an existing network

« one-by-one » design less performant than the constructal one. But, the gap shrinks for larger structures.
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Tree-shaped networks in a disc-shaped body

objectives
To deliver a fluid from a source to a given number of outlets (users)

constraints
Disc-shaped area
Number of outlets
Total volume of the tubes

2 levels of pairing
N = 12 outlets

Flow resistance (Hagen-Poiseuille regime)

\[ \frac{\Delta P_i}{m_i} = \frac{128 \nu L_i}{\pi D_i^4} \]

When the total volume of the tubes is constrained

Optimal tube diameters follow Murray’s law

\[ \frac{D_{i+1}}{D_i} = 2^{-1/3} \]
First step

\[ \Delta P = \Delta P_0 + \Delta P_1 \]

\( L_0 \) and \( L_1 \) are expressed using the angles \( \alpha \) and \( \beta \)

\[ \frac{\partial \Delta P}{\partial \beta} = 0 \quad \text{Leads to} \]

\[ \beta = 37.47^\circ \]

Finally

\[ \frac{\Delta P}{\dot{m}} = 8\pi \nu \frac{R^3}{V^2} f \]

With \( f \) a dimensionless resistance factor depending on tube lengths

We obtain the connecting angles

The shape of the network is the result. It is «given» by the angles.

(48 outlets)
Resistance factor

Details of the previous figure

When optimized complexity is beneficial

\[ N = \text{constant} \]

pairing is a useful feature if \( N \) sufficiently large

\[ N \text{ increases} \rightarrow \text{the level of pairing increases} \]

\[ \text{complexity increases} \]

By complexity, the dendritic flow assures its minimal resistance.

The designer can choose between two structures ≈ same resistance.
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Concluding remarks
• Every duct has a fixed area allocated to it
• On its area, the duct must have the shortest length possible (smallest $\Delta P$)

optimal rectangle shape
Minimization of flow path lengths

Elemental curvilinear rectangle

Area of the element fixed

\[ A = \frac{1}{2} \theta (2r\delta - \delta^2) \]

Minimize \( L \)

\[ L^2 = a^2 + b^2 \]

Result:

\[ \frac{x}{\theta} = \frac{(1 - x)^2 \sin \frac{\theta}{2}}{(2 - x)(x - 1 + \cos \frac{\theta}{2})} \]

with \( x = \frac{\delta}{r} \)

Construction of the minimal path length

Outer circle

\[ \theta_0 = \frac{d}{r_0} \ll 1 \]

Selection of \( \theta_0 \): 0.1

Next

\[ \theta_1 = 2\theta_0 \]

Stop \( \theta_i > 2\pi \) or \( r_i < 0 \)

Note: rectangle approximation poor when \( r \to 0 \)

Every geometric detail is optimized

Minimization of flow path lengths
Global flow resistance

Construct obtained by optimizing every geometric detail

Construct obtained by the length-minimization algorithm

Comparison

Performance of minimal length structures resemble closely the performance of fully optimized structure → Effective shortcut
Every geometric detail is optimized

*Global optimization*

The optimized design is generated step by step

*Local optimization*

Assembling locally optimized pieces does not lead to fully optimized structures

Comparison: fractal-like structure

- $f = 36.89$
  - four pairings, assumed
  - $N = 192$
  - $n_0 = 12$

- $f = 33.16$
  - three pairings, optimized
  - $N = 192$
  - $n_0 = 24$

- $f = 23.61$
  - four pairings, optimized
  - $N = 192$
  - $n_0 = 12$
Complexity does not mean performance

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Limits of the previous approach

In case of accident along the ducts, interruption of the flow.

Solution: look at nature

At a certain scale, loops appear
The ginkgo leaf is the exception
Designed networks mimic nature:
The biggest networks have loops for the sake of security.

• Is an optimized network with loops much less efficient than a dendritic one?

• Does the result on the comparison between the two depend on the complexity level?
Constraints:
- total surface (radius of the circle fixed)
- total pipes volumes, $V$

Degrees of freedom
- number of tubes connected to the center
- tubes diameter and lengths
- pairing level
- number of outlets
Laminar flow
Local losses are neglected

If $N < 5$, optimal solution: radial network

$N > 5$, the « one loop size » can be optimized

very high flow resistances
N > 10, the « two loop size »
can be optimized

even higher flow resistances

Conclusion:
The price is heavy in terms of fluid mechanics
Other possibility

Again, the flow resistances increase with security

Robustness of optimized complex flow structures

Impact of having loops decreases when complexity increases

Optimal diameter ratio (one tube is cut) # insensitive to the increase of complexity
Concluding remark

Several applications at different scales
Urban hydraulics
Cooling of electronics

Complexity is an optimization result.

Optimized complexity must not be confused with maximized complexity.
Workshop on
U. of Roma 1, 17&18 march 2005

Constructal Theory & variational principles for
the Navier-Stokes equations: is there a link?

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To practice Constructal Theory means “to construct complex structures by connecting simple elements and to optimise each connection” [Bejan 2000]

• For fluid flow, “constructing structures” means to assemble flow paths
• Question: what is the objective function of the optimisation?
For laminar liquid flow in pipes & channels, constructalists basically contend that:

a) There is an “optimal” diameter ratio at each bifurcation, given by biology-based “optimal volume” considerations [Murray 1926]

b) If no heat transfer is involved, the “structure” is optimised under the “minimal flow resistance” criterion (minimal $\Delta p_{\text{tot}}$)

c) If thermal transport is involved, the “maximum heat flow” becomes a suitable optimising criterion (there may be others)
My contention here is that the optimising criterion is always the minimum entropy generation compatible with the prescribed constraints, and that “Constructal Theory” ought to be viewed as an “entropy-based optimisation tool”

In this first attempt to put things in this particular perspective, I will limit my considerations to fluid flow only (no heat transfer)
1) The standard derivation of the velocity profile in plane Poiseuille flow

- Navier-Stokes equations

\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right)
\]

- Physical assumptions: steady flow (\( \frac{\partial}{\partial t} = 0 \));
  fully developed (v=0, \( \frac{\partial}{\partial x} = 0 \), \( \frac{\partial p}{\partial x} = \frac{\Delta p}{L} \));
  laminar

- Boundary conditions: u=0 at y=0
  u=0 at y=d

\[
\Rightarrow \quad u = \frac{d^2 \Delta p}{L} \frac{y^2}{2\mu} \left( \frac{y}{d^2} - \frac{y}{d} \right)
\]
Once the velocity profile is known, we can compute the entropy generation:

\[
s(y) = \frac{\mu}{T} \left( \frac{du}{dy} \right) = \frac{d^2}{L^2} \frac{(\Delta p)^2}{\mu T} \left( \frac{y^2}{d^2} - \frac{y}{d} + \frac{1}{4} \right)
\]

- And the entropy generation per unit channel length

\[
S_{L,d} = \left( \frac{\Delta p}{L} \right)^2 \frac{L d}{12 \mu T}
\]
The pressure gradient is linked to the mass flow rate by:

$$\frac{\Delta p}{L} = \frac{12 \nu \dot{m}}{d^3}$$

So that the entropy generation per unit channel length becomes:

$$s_{L,d} = \frac{12 \nu \dot{m}^2}{\rho T} \frac{L}{d^5} = K_f \frac{m^2 L}{d^5}$$

\(s_{L,d}\) grows with \(L\)
\(s_{L,d}\) grows with decreasing \(d\)
2) Alternative derivation of the velocity profile of plane Poiseuille flow

- Assume steady, laminar, fully developed flow
- Assume that the velocity profile is such that it minimises the viscous entropy generation in the flow

\[ s_v = \frac{\mu}{T} \left( \frac{du}{dy} \right)^2 \]
n unknown function of y
The relevant variables in the optimisation are $u$, $\frac{du}{dx}=0$, $\frac{du}{dy}$. The Euler-Lagrange equations for this variation reduce to:

\[
\left( \frac{\partial s_v}{\partial u} \right) - \frac{\partial}{\partial x} \left( \frac{\partial u}{\partial x} \right) - \frac{\partial}{\partial y} \left( \frac{\partial u}{\partial y} \right) = \ldots
\]

\[
= - \frac{\partial}{\partial y} \frac{\partial s_v}{\partial \left( \frac{\partial u}{\partial y} \right)} = 0
\]

The boundary conditions on $u$ and the additional constraint that the flow must transport a prescribed mass flow rate provide:

\[
u = \frac{d^2}{L} \frac{\Delta p}{2\mu} \left( \frac{y^2}{d^2} - \frac{y}{d} \right)
\]
This result is indeed general: for viscous steady flow the Navier-Stokes equations admit of a variational formulation whose Lagrangian is the exergy destruction ($T_0^*ds/dt$). There is no definitive proof yet that this is also the Lagrangian for thermal transport by convection, but it would seem reasonable to assume so.

The question is: since the entropy generation “governs” the structure of the flow in a channel, why should it not be the “Lagrangian” for the structure of more complex flow domains?

I shall now try to derive an “optimal” bifurcation structure from a “minimum $ds/dt$” principle
The relevant variables are: \( m_0, m_1, L_0, L_1, d_0, d_1 \), and the fluid properties.

a) \( m_0 \) is supposed prescribed
b) \( m_1 = 0.5 m_0 \)
c) The flow is isothermal

\[
\dot{s}_{tot} = K_f \left( \frac{m_0^2 L_0}{d_0^5} + 2 \frac{m_1^2 L_1}{d_1^5} \right) = \\
= K_f \frac{m_0^2 L_0}{d_0^5} \left( 1 + \frac{L_1 d_0^5}{L_0 d_1^5} \right)
\]

We want to minimise \( s_{tot} \) w.r.t. \( L_0, L_1 \) and to the ratio \( d_1/d_0 \)
In general, for a prescribed global geometry, $L_0$ and $L_1$ will be functionally linked:

$$L_1 = f(L_0)$$

It follows:

$$\dot{s}_{\text{tot}} = K_f \frac{m_0^2 L_0}{d_0^5} \left( L_0 + \frac{f(L_0)}{\delta^5} \right)$$

where $\delta = d_1/d_0$

which has non-trivial minima that depend on $f$:

$$\frac{\partial \dot{s}_{\text{tot}}}{\partial L_0} = K_f \frac{m_0^2}{d_0^5} \left( 1 + \frac{f'(L_0)}{\delta^5} \right)$$

$$L_{0,\text{opt}} = -\frac{b + \delta^5}{a}$$ if $f = aL_0^2 + bL_0 + c$;

and

$$L_{0,\text{opt}} = -\frac{\ln(\delta^5)}{a \ln(a)}$$ if $f = \exp(-aL_0)$
If the link between \( L_0 \) and \( L_1 \) is one of pure proportionality, \( L_1 = \alpha L_0 \):

\[
\frac{\partial \dot{s}_{\text{tot}}}{\partial L_0} = K_f \frac{m_0^2}{d_0^5} \left( 1 + \frac{\alpha}{\delta^5} \right)
\]

and no minimum is possible

But if the link between \( L_0 \) and \( L_1 \) is dictated by a conservation of the total volume occupied by the channels,

\[
L_1 = \frac{V_{\text{tot}}}{2d_1} - \frac{L_0 d_0}{2d_1}
\]

it turns out that there is a minimum for

\[
\delta_{\text{opt}} = \left( \frac{d_1}{d_0} \right)_{\text{opt}} = \frac{1}{\sqrt[6]{2}}
\]
In conclusion, there is a set of different minima, each one corresponding to a proper (i.e., mathematically consistent and physically realistic) link between $L_0$ and $L_1$

Let us consider an application to a small set of bifurcational geometries: which one is “optimal”? 

![Diagram](image.png)
<table>
<thead>
<tr>
<th>( \delta ) ( \Rightarrow )</th>
<th>A, 0.891</th>
<th>A, 0.794</th>
<th>B, 0.891</th>
<th>B, 0.794</th>
<th>C, 0.891</th>
<th>C, 0.794</th>
<th>D, 0.891</th>
<th>D, 0.794</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_0 )</td>
<td>( \sqrt{3} \ell / 4 )</td>
<td>( \sqrt{3} \ell / 2 )</td>
<td>0</td>
<td>( \ell / \sqrt{3} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( L_1 )</td>
<td>( \sqrt{7} \ell / 4 )</td>
<td>( \ell / 2 )</td>
<td>( \ell )</td>
<td>( \ell / \sqrt{3} )</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( s_{\text{tot,adim}} )</td>
<td>1.022</td>
<td>1.483</td>
<td>1.311</td>
<td>1.659</td>
<td>0.891</td>
<td>1.587</td>
<td>1.09</td>
<td>1.493</td>
</tr>
<tr>
<td>( v_{\text{tot,adim}} )</td>
<td>1.611</td>
<td>1.483</td>
<td>1.757</td>
<td>1.659</td>
<td>1.782</td>
<td>1.587</td>
<td>1.606</td>
<td>1.493</td>
</tr>
<tr>
<td>( \Delta p_{\text{tot,adim}} )</td>
<td>0.9</td>
<td>1.09</td>
<td>1.22</td>
<td>1.366</td>
<td>0.707</td>
<td>1</td>
<td>0.985</td>
<td>1.15</td>
</tr>
<tr>
<td>Time through ( L_0 + L_1 )</td>
<td>1.611</td>
<td>1.483</td>
<td>1.757</td>
<td>1.659</td>
<td>1.782</td>
<td>1.587</td>
<td>1.606</td>
<td>1.494</td>
</tr>
</tbody>
</table>
These results -albeit grossly preliminary- indicate that:

1) “minimum $\Delta p$” and “minimum $s$” generate the same optima
2) “minimum volume” consistently indicates different optima
3) “minimum delivery time” leads to the same optima as the minimum volume

(1) was to be expected: under the present model, $\Delta p \propto s_{tot}$
(2) the reason for which minimum $V$ and minimum $t$ lead to the same optimum, remains to be investigated
(3) biological scientists generally assume that “minimum $V$” is the objective function for the optimisation of dendritic structures, but it is more likely that Nature may prefer something like:
Suggestion) “Natural” structures are generated under the following conditions:

1) A weighted sum of the exergy destruction (friction, thermal gradients...) and of the embodied exergy (material & energy to “build” the channels) is minimised;

2) External boundary conditions dictate the details
$$\mathcal{L} = \alpha L_0, d_0 \dot{S}_{\text{tot}} + \beta L_0, d_0 V_{\text{tot}}$$

For instance, for the “constructor” of the bifurcations discussed above:

1) If the pumping power required to make up for the entropy dissipation is much smaller than the exergy needed to build the channels, $\alpha \ll \beta$, and the optimal structure is C1;

2) If exergy is abundant, and $\beta \ll \alpha$, the optimal structure is A2;

3) If $\alpha = \beta = 0.5$, the optimal structure is D1
Constructal Theory & Exergy Analysis have the same goal: are they using the same tools?

There is an enormous incentive to investigate this field, for the applications in engineering, physical & biological sciences are of substantial importance (heat transfer, filters, chemical reactors, composite structures, allocation of resources, neuronal science, vascular systems...)

\[ \mathcal{L} = \alpha_{\text{I}} L_0, d_0 \dot{S}_{\text{tot}} + \beta_{\text{II}} L_0, d_0 V_{\text{tot}} \]
Conclusion:

"Science...is good for the scientist: whether it is good for the rest of mankind, it is arguable"

Erwin Chargaff, 1905-2002
Proceedings of the Symposium

Bejan’s Constructal Theory of Shape and Structure
Particle deposition, and shape in some inanimate/animate systems

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A set of inanimate particles

coaugulation = agglomeration

Katzer et al. 2001

Deposition

Smith et al. 2002

Kanaoka et al., 2001
A set of living organisms

Coral colony association of polyps

- Tentacles to trap food
- Cells on the lower sides and bottom produce the limestone that builds the colony

Kaandorp and Sloot (2001)
Why agglomeration/deposition is important?

*At USA (1996)*

- 134,109 people were diagnosed with lung diseases *(7,202 people died)*
- Medical care for people with chronic respiratory diseases cost 45.3 billion USD


**Inhalation therapy** (drug delivery via the respiratory system easier than delivery via ingestion and the transdermal route)
Air pollution control devices

- **Electrostatic precipitators** (expansive, fail to filtrate fine particles and high-resistivity dust)  
  Dietz (1977), Shapiro (1998)

- **Space-charged precipitators** (only effective for high density aerosols)  
  Shapiro & Brenner (1990)

- **Cyclones** (only effective for coarse particles > 5 µm)  
  Shapiro & Brenner (1990)

- **Diffusion batteries** (very efficient for particles < 0.1 µm)  
  Knutson (1999)

- **Filters** (simple, highly efficient for both micrometer and submicrometer particles, coast-saving process)  
Outline

Constructal view

- Particle coagulation (agglomeration)
- Colony association of polyps
- Design of air pollution control devices
Design of air pollution control devices

- Operate at very low Re (laminar flow)
- Very important for removal of submicrometer particles
What are the rules to obtain the best design?

**Constructal idea:**
The ultimate purpose is to bring the system into equilibrium through capturing all the particles in the air.

**Global constrain**
Due to practical and economical reasons, the device should fit into a volume $V$.

\[
\phi = \frac{H}{d}
\]

Similar to optimal geometry of plates in forced convection heat transfer
(i) finding of two $d$-value limits
(ii) identifying the $d$-value that maximizes the particle deposition

Bejan (2000); Bejan (2004)
(i) d (small) φ (large)

Outlet concentration of particles \( \text{(minimum)} \)

Steady-state particle transfer density \( m_p = \frac{HWU}{HLW} (C_{in} - C_o) \)

Fully developed (Hagen-Poiseuille) flow
\[
U = \frac{d^2}{12\mu L} \Delta p
\]

\[
m_p = \frac{d^2}{12\mu} \frac{\Delta p}{L^2} (C_{in} - C_o)
\]
steady-state particle transfer density for the boundary layers

\[
m_p = \frac{2 \phi \lambda LW}{HLW} (C_{in} - C_o)
\]

Sh = 0.664 Re\(^{1/2}\) Sc\(^{1/3}\)

Sherwood number \((\lambda/LD)\) --- Nu
Schmidt number \((\nu/D)\) --- Pr

free-stream velocity \(U_o\)


\[
U_o^{1.5} = \frac{\Delta pH}{1.328L^{1.5}(\rho \mu)^{0.5}}
\]
(ii) optimal number of plates  

**method of intersecting asymptotes**

\[
\frac{d_{\text{opt}}}{L} \approx 2.73 \text{Be}_{m}^{-1/4}
\]

\[
\text{Be}_{m} = \frac{L^2 \Delta p}{\mu D}
\]

Best elementary construct

\( C_{\text{in}} \)

\( U \)

\( L_{\text{dev}} > L \) flow fluid developed particles interact with plate

\( L_{\text{dev}} < L \) air in the core duct not interact with plate

Maximum particle transfer \( L_{\text{dev}} = L_{\text{opt}} \)

\[
\left( \frac{d}{L} \right)_{\text{opt}} = 1.86 \text{Sh}^{1/4} \text{Be}_{m}^{-1/4}
\]

Bhattacharjee & Grosshandler (1988)
Device with a porous structure

(i) finding of two extremes

\[ \varepsilon \sim 0 \]

- outlet concentration of particles (minimum)

- steady-state particle transfer density

- Darcy flow

\[ U = \frac{K}{\mu} \frac{\Delta p}{L} \]

Carman-Kozeny equation

\[ K = \zeta_K \frac{\varepsilon^3}{(1-\varepsilon)^2} d^2 \]

\[ m_p = \frac{\zeta_K}{\mu} \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{d^2}{L^2} \Delta p (C_{in} - C_o) \]
ε \sim 1

steady-state particle transfer density to N collectors

\[ m_p = N \zeta_{m} D \frac{d}{V} (C_{in} - C_{o}) \]

ε and N

\[ \varepsilon = 1 - \zeta_{\varepsilon} \frac{Nd^3}{V} \]

\[ m_p = \zeta_{m\varepsilon} \left(1 - \varepsilon\right) \frac{D}{d^2} (C_{in} - C_{o}) \]
(ii) identifying the $\varepsilon$-value that maximizes the particle deposition

particles transferred from air stream to $V$

$\frac{\kappa}{\mu} \frac{\varepsilon^3}{(1-\varepsilon)^2} \frac{d^2}{L^2} \Delta p (C_{in} - C_o)$

particles transferred from air stream to collectors of porous material

$\zeta_m (1 - \varepsilon) \frac{D}{d^2} (C_{in} - C_o)$

$$\varepsilon_{opt} = \left( 1 + \zeta \left( \frac{d}{L} \right)^{4/3} Be_m^{1/3} \right)^{-1}$$
**Constructal view of aerosol agglomeration/deposition process**

What determines the shape of in these processes?

**Constructal law**

- architecture optimized
- accumulation of particles maximized

aerosol particles >100 nm carry an electrostatic charge

Vincent (1986); Kerasev et al. (2001)

aerosols travel under the influence of Coulomb forces
**Electric field from a point charge**
\[ E = \frac{kq}{r^2} \]

**Coulomb Force**
\[ F = qE = \frac{kqq}{r^2} \]

**Aerosol migrates Stokes flow velocity**
\[ U = \frac{dr}{dt} \]

**Aerosol** migrates Stokes flow velocity
\[ U = \frac{3\pi \mu d}{c_c} \]

**“flying” time**
\[ t_x = \frac{\pi}{c_c k} x^3 \frac{qq}{\mu d} \]

The flow of particles that lands on a spherical annulus of radius \( x \) and thickness \( \Delta x \)
\[ \Delta N = n \cdot 4\pi x^2 \Delta x \]

\[ \Delta N/\Delta t \sim d(D^3)/dt \]

\[ D = \left( 2kc_c n \frac{\mu d}{qq} t \right)^{1/3} \]

- **Cluster diameter grows fast early, and slow late**
- **Mass accumulated (~D^3) increases linear with t**
Constructal question

There is any another configuration (shape) that maximizes the mass accumulated?

Needle body of revolution with radius $R = f x^{1/3}$

$x$ instantaneous needle length (Ut)

$$f = \left( \frac{2kc_c n \mu d}{Uqq} \right)^{1/3}$$
\[ V_\text{sphere} = \frac{4}{3} \pi \left( \frac{D}{2} \right)^3 = \frac{4}{3} \pi f^3 x \]

\[ V_\text{needle} = \int_0^x \pi R^2 \, dx = \frac{3\pi}{5} f^2 x^{5/3} \]

\[ x = Ut \]
Constructal view of coral colony association of polyps

- Tentacles to trap food
- Cells on the lower sides and bottom produce the limestone that builds the colony

Porites spp. (stony coral)

<table>
<thead>
<tr>
<th>Rounded and Compact</th>
<th>Dendritic</th>
</tr>
</thead>
<tbody>
<tr>
<td>cm/year</td>
<td>cm/year</td>
</tr>
<tr>
<td>1.2 ± 0.5</td>
<td>0.2 to 4.8</td>
</tr>
</tbody>
</table>

Constructal view of coral growing process

Why rounded and compact forms? Why branches? Why are the branches necessary?

Food particles very small  \[ \text{convective-dispersive transport} \]

Growth process starts at birth \((t=0)\)  \[ L_c = U_c t \]

\[ \text{Sheltered growth site} \]

\[ \text{Pe} \left( = \frac{U_l}{D^*} \right) < 1 \]

food particles near colony  \[ C_c \left( < C_{\text{bulk}} \right) \]

food “wave”  \[ L_p \approx (D \times t)^{0.5} \]

Food particles velocity  \[ U_p = \frac{D^*}{t^{0.5}} \]

fast early and slow late
How can the access to nutrients by the colony of coral be optimized?

\[ U_c \sim \text{const} \quad U_p = \frac{D^*}{t^{0.5}} \]

fast early and slow late
Does the nutrient size influence the form of coral colony?

\[ D^* = \frac{c_{cn} k_B T}{6\pi \mu r_p} \]

\[ U_p = \frac{D^*}{t^{0.5}} \]

\[ U_c \sim \text{const} \]
Colony more exposed to water current

\[ \text{Colony more exposed to water current} \]

\[ \text{Pe} \left( \frac{U}{D^*} \right) > 1 \]

\[ U_c \quad 1.2 \pm 0.5 \, \text{cm/year} \quad \text{(Lough and Barnes (1997))} \]

\[ 0.2 \text{ to } 4.8 \, \text{cm/year} \quad \text{(Harriott (1999))} \]

\[ U \gg U_c \]

\[ t_c \]
What is the constructal view of stony coral growth?

Branches generation allows a faster growth of the coral colony because it provides internal paths such that the entire substrate volume approaches internal equilibrium faster (depletion of nutrients).

Bacteria colony & plant roots

Miguel (2003)
Final remarks

Constructal Theory

• design optimization of air-cleaning devices

• cluster of particles grows fast early, and slow late

• at small times spherical agglomeration; at long times needle (dendrite) more effective

• in stony coral branches generation provides the best internal path to reach the food (i.e., internal paths such that the entire substrate volume approaches internal equilibrium faster).
Constructal theory of flow architectures of the lungs

(Reis, Miguel and Aydin, 2004)

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Outline

• A fluid tree with purpose
• Bronchial tree resistance and alveolar resistance
• Optimization of the respiratory tree based on the Constructal Principle
• Constructal laws of the lung airways
• Conclusions
The respiratory tree

- Starts at the trachea;
- Channels bifurcate 23 times before reaching the alveolar sac.

Has this special flow architecture been developed by chance or does it represent the optimum structure for the lung’s purpose, which is the oxygenation of the blood?
The purpose of the respiratory tree

- Oxygenation of blood;
- Removal of carbon dioxide.

The trade-off

- High alveolar surface promotes better oxygenation, but requires increased access to the external air (higher flow resistance).
- Lower level of channel branching reduces flow resistance, but reduces also the number of alveoli and therefore decreases the area allocated for blood oxygenation.
The asymptotic leeways

I) A duct system that ends with an alveolar volume from which the oxygen diffuses onto the tissues, where it meets the blood, and in which the carbon dioxide diffuses after being released from the blood;

\[ t_{\text{Flow}} = \pi \eta L^2/(d^2 \Delta P) \sim 1 \text{ s} \]

II) A unique volume open to the external air, in which the oxygen reaches the blood in the tissues, and removes the carbon dioxide rejected from the blood, only by diffusion through the internal air.

\[ t_{\text{Diff}} = L^2/D \sim 10^4 \text{ s} \]
Bronchial tree resistance

Assumptions

• Airflow within the bronchial tree is assumed to be laminar (Re<1000), isothermal and incompressible

• The difference in chemical potential drives oxygen into the lungs and carbon dioxide out of the lungs $\Delta \mu = \rho^{-1} \Delta P + \Delta \varepsilon$

• The bronchial tree is composed of cylindrical channels with Hagen-Poiseuille flow,
Airflow rate: \[ \dot{m}_n = \frac{\pi \rho D_n^4}{128\nu L_n} \Delta\mu_{cn} \]

Minimum flow resistance at a bifurcation

Constructal rules:
\[ \frac{D_n}{D_{n-1}} = 2^{-1/3} \quad \frac{L_n}{L_{n-1}} = 2^{-1/3} \]

Resistance posed by the \(n^{th}\) bronchial tube
\[ r_{cn} = \frac{\rho \Delta\mu_{cn}}{\dot{m}_n} = 2^n \frac{128\nu L_0}{\pi \rho D_0^4} \]
Resistance due to bifurcation

\[ \dot{m}_n = \frac{\Delta \mu_{bn}}{r_{bn}} = -\frac{\Delta \varepsilon_{bn}}{r_{bn}} \]

Resistance of the \( n^{th} \) level (\( 2^n \) bronchial tubes)

\[ r_{bn} = \frac{\dot{m}_0}{8\pi \rho^2 D_0^4} 2^{n/3} \]

Overall convective resistance

\[ R_b = \sum_{n=0}^{N-1} r_n = \frac{128\nu L_0}{\pi \rho D_0^4} \left[ N + \frac{\dot{m}_0 \left(1-2^{-2N/3}\right)}{379\pi \rho \nu L_0} \right] \]
Diffusive Resistance

\[ \dot{m}_{ox} = \frac{1}{2} (\phi_{ox})_0 - \phi_{ox} \dot{m}_b = \frac{\Delta \mu_b}{(R_{ox})_b} \]

\[ \phi - \text{concentration} \]

\[ \dot{m}_{ox} = 2^N \frac{2\pi \rho D_{ox} (\Delta \mu_{ox})_a}{R_{ox} T} \]

\[ d = L - \sum_{i=1}^{N} L_i \quad \Rightarrow \quad d = 4.85 \times 2^{-(N+1)/3} L_0 \]

resistance of the \(N^{th}\) level of bifurcation

\[ (R_{ox})_N = \frac{128 \nu L_0}{\pi \rho D_0} + 0.13 (R_g)_{ox} T \frac{2^{-2N/3}}{\pi L_0 D_{ox}} \]}
Global resistance of the constructal respiratory tree ($\text{kg}^2\text{J}^{-1}\text{s}^{-1}$)

$$R_{ox} \approx \frac{256\nu L_0}{\pi D_0^4 ((\phi_{ox})_0 - \phi_{ox}) \rho} (N+1) + \frac{0.13 \times 2^{-2N/3} (R_g)_{ox} T}{\pi L_0 D \phi_{ox} \rho}$$

Air, oxygen and carbon dioxide properties were taken at 36º C

$\phi_{ox}$ is evaluated from the alveolar air equation: $(\phi_{ox})_0 - \phi_{ox} Q - S = 0$,

$Q$ is the tidal airflow and $S$ is the rate of oxygen consumption.

$(\phi_{ox})_{air} = 0.2095$; $(\phi_{cd})_{air} \sim 0.315 \times 10^{-3}$; $Q \sim 6 \text{ dm}^3/\text{min}$ and $S \sim 0.3 \text{ dm}^3/\text{min}$.

trachea length $L_0 \sim 15 \text{ cm}$; trachea diameter, $D_0 \sim 1.5 \text{ cm}$. 
Constructal law (principle) the flow architectures evolve in time in order to maximize the flow access under the constraints posed to the flow

\[ N_{opt} = 2.164 \ln \left[ \frac{2.35 \times 10^{-4} D_0^4 (R_g)_{ox} T (\phi_{ox})_0}{L_0^2 v D_{ox}} \left( \frac{\phi_{ox}}{\phi_{ox}} - 1 \right) \right] \]

Minimization of the global resistance to oxygen access with respect to level of bifurcation

\[ N_{opt} = 23 \quad (23.4) \]

Minimization of the global resistance to carbon dioxide removal with respect to level of bifurcation

\[ N_{opt} = 23 \quad (23.2) \]
Total resistance to oxygen and carbon dioxide transport between the entrance of the trachea and the alveolar surface is plotted as function of the level of bifurcation (N).
Constructal laws of the human respiratory tree

(I)

If the number $N_{opt} = 23$ is common to mankind then a constructal rule emerges:

"the ratio between the square of the trachea diameter and its length is constant and a length characteristic of mankind:"

$$\lambda = \frac{D_0^2}{L_0} = \text{const.} = 1.6 \times 10^{-3} \text{ m}$$
V – Global volume of the respiratory tree; L – average length
A – Area allocated to gas (O₂ and CO₂) exchange

\[
\frac{D_0^2}{L_0} = 8.63 \frac{A L}{V} \left( \frac{\nu D_{\text{ox}} \phi_{\text{ox}}}{(R_g)_{\text{ox}} T \left( (\phi_{\text{ox}})_0 - \phi_{\text{ox}} \right)} \right)^{1/2}
\]

(II)

The non-dimensional number AL/V, determines the characteristic length \( \lambda = D_0^2/L_0 \), which determines the number of bifurcations of the respiratory tree by:

“The alveolar area required for gas exchange, A, the volume allocated to the respiratory system, V, and the length of the respiratory tree, L, which are constraints posed to the respiratory process determine univocally the structure of the lungs, namely the bifurcation level of the bronchial tree.”
Global volume and average length of the respiratory tree, and area allocated to gas (O_2 and CO_2) exchange

Average length of the respiratory tree: \( L = 4.85L_0 = 0.68\text{m} \); Observed \( \sim 1\text{m} \)

Alveolar diameter: \( d = 6V/A = 2.67 \times 10^{-3}\text{m} \); Observed \( \sim 10^{-3}\text{m} \)

Alveolar surface area as \( A = 2^{23}\pi d^2 = 187.7\text{m}^2 \); Observed 100 - 150\text{m}^2

Total alveolar volume \( V = 2^{23}(\pi/6)d^3 = 83.5\text{dm}^3 \); Observed 8 - 10\text{m}^3
CONCLUSIONS

• The best oxygen access to the tissues where it reaches the blood is performed by a flow structure composed of ducts with 23 levels of bifurcation.

• The same structure has been shown being optimized for carbon dioxide removal.

• The optimized number of bifurcation levels matches the 23 levels that the physiology literature indicates for the bronchial tree.

• Theory also predicts the dimension of the alveolar sac, the total alveolar surface area, the total alveolar volume, and the total length of the airways. These values agree, at least in an order of magnitude sense with the values found in the physiology literature.
It was shown that the length $\lambda$ (defined as the ratio between the square of the first airway diameter and its length) is constant for every individual of the same species and related to the characteristics of the space allocated for the respiratory process.

The length $\lambda$ is univocally determined by a non-dimensional number, $AL/V$, which involves the characteristics of the space allocated to the respiratory system, namely the total alveolar area, $A$, the total volume $V$, and the total length of the airways, $L$.

In general, we conclude, that for every species whose respiratory tree is optimized the same rule must hold, and exhibit the respective characteristic length, $\lambda$.

The application of the Constructal Principle to the generation of the optimal configuration of the respiratory tree was based on the view that Nature has optimized the living flow structures, in time.
Fluid trees – from Constructal theory to actual river basins

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Outline

• Natural flow architectures
• Scaling laws of river basins
• River networks as constructal fluid trees
• A constructal model of river basin development
• Conclusions
Tagus river basin (Tejo, in Portuguese)

Length (km): 1 100
Drainage Area (km²): 88 700
Discharge (m³/s): 600
Tevere river basin

Length (km): 405
Drainage Area (km$^2$): 17375
Discharge (m$^3$/s): 250
River networks are self-similar structures over a range of scales

Horton’s law of stream lengths: \( \frac{L_i}{L_{i-1}} = R_L \)
\[ 1.5 < R_L < 3.5 \]

Horton’s law of stream numbers: \( \frac{n_{i-1}}{n_i} = R_B \)
\[ 3 < R_B < 5 \]

Hack’s law: \( L_\omega = \alpha(A_\omega)^b \)
\[ b \sim 0.56 - 0.5 \]

Melton’s law: \( F_S = 0.694(D_\omega)^2 \)
\[ D_\omega = L_T/A; F_S = N_\omega/A \]
CONSTRUCTAL THEORY (Bejan)

River basins as area-to-point flows

First construct made of elemental areas, $A_0 = H_0L_0$.

A new channel of higher permeability collects flow from the elemental areas.
The optimised geometry of area-to-point flow (channels with Hagen-Poiseuille flow, Bejan)

\[ \hat{K} = \frac{K}{A_0} \quad (\tilde{H}_i, \tilde{L}_i) = \left(\frac{H_i}{A_0}\right)^{1/2} \quad \Phi_i = \frac{D_i}{H_i} \]

<table>
<thead>
<tr>
<th>$i$</th>
<th>$\tilde{H}_i$</th>
<th>$\tilde{L}_i$</th>
<th>$n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$\frac{2^{5/6}3^{1/6}\hat{K}^{1/6}}{\Phi_0^{1/2}}$</td>
<td>$\frac{\Phi_0^{1/2}}{2^{5/6}3^{1/6}\hat{K}^{1/6}}$</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>$\frac{2^{1/6}\Phi_0^{1/2}}{3^{1/6}\hat{K}^{1/6}}$</td>
<td>$\frac{\Phi_1^{3/2}}{2^{3/2}\hat{K}^{1/2}}$</td>
<td>$\frac{\Phi_1^{3/2}\Phi_0^{1/2}}{2^{4/3}3^{1/6}\hat{K}^{2/3}}$</td>
</tr>
<tr>
<td>2</td>
<td>$\frac{\Phi_1^{3/2}}{2^{1/2}\hat{K}^{1/2}}$</td>
<td>$\frac{3^{1/6}(\Phi_2\Phi_1)^{3/2}}{2^{5/3}\Phi_0^{1/2}\hat{K}^{5/6}}$</td>
<td>$\frac{3^{1/3}(\Phi_2\Phi)^{3/2}}{2^{5/6}\Phi_0\hat{K}^{2/3}}$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{3^{1/6}(\Phi_2\Phi_1)^{3/2}}{2^{2/3}\Phi_0^{1/2}\hat{K}^{5/6}}$</td>
<td>$\frac{3^{1/3}(\Phi_3\Phi_2\Phi_1)^{3/2}}{2^{4/3}\Phi_0\hat{K}^{7/6}}$</td>
<td>$\frac{2^{1/6}3^{1/3}(\Phi_3\Phi_2)^{3/2}}{\Phi_0\hat{K}^{2/3}}$</td>
</tr>
<tr>
<td>4</td>
<td>$\frac{3^{2/3}(\Phi_3\Phi_2\Phi_1)^{3/2}}{2^{1/3}\Phi_0\hat{K}^{7/6}}$</td>
<td>$\frac{3^{1/2}(\Phi_4\Phi_3\Phi_2\Phi_1)^{3/2}}{2^{1/2}\Phi_0^{3/2}\hat{K}^{9/6}}$</td>
<td>$\frac{2^{7/6}3^{2/3}(\Phi_4\Phi_3)^{3/2}}{\Phi_0\hat{K}^{2/3}}$</td>
</tr>
</tbody>
</table>
Void-allocation (channel) optimisation

\[ \Phi_i = \frac{D_i}{H_i} \quad \hat{K} = \frac{K}{A_0} \]

\[ \Phi_1 = \Phi_0 \quad \Phi_2 = (6/7)\Phi_0 \]

\[ \Phi_3 = (60/77)\Phi_0 \quad \Phi_4 = (8/11)\Phi_0 \]

<table>
<thead>
<tr>
<th>( \bar{L}_i )</th>
<th>( n_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 ( \Phi_0 \hat{K}^{-1/3} )</td>
<td>-</td>
</tr>
<tr>
<td>1 ( 0.331(\Phi_0 \hat{K}^{-1/3})^2 )</td>
<td>0.331((\Phi_0 \hat{K}^{-1/3}))^2</td>
</tr>
<tr>
<td>2 ( 0.300(\Phi_0 \hat{K}^{-1/3})^5/2 )</td>
<td>0.809((\Phi_0 \hat{K}^{-1/3}))^2</td>
</tr>
<tr>
<td>3 ( 0.312(\Phi_0 \hat{K}^{-1/3})^7/2 )</td>
<td>0.883((\Phi_0 \hat{K}^{-1/3}))^2</td>
</tr>
<tr>
<td>4 ( 0.414(\Phi_0 \hat{K}^{-1/3})^9/2 )</td>
<td>1.990((\Phi_0 \hat{K}^{-1/3}))^2</td>
</tr>
</tbody>
</table>
Assumption:

Channel hierarchy is understood in the Hortonian sense, i.e. all streams of order $i$ are tributaries of streams of order $i+1$.

Horton’s law of stream lengths

$$\frac{\sim L_{i-1}}{\sim L_i} \sim \Phi_0 \hat{K}^{-1/3} \sim \text{constant}$$

<table>
<thead>
<tr>
<th>$\sim L_i$</th>
<th>$n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.467$(\Phi_0 \hat{K}^{-1/3})^{1/2}$</td>
</tr>
<tr>
<td>1</td>
<td>0.357$(\Phi_0 \hat{K}^{-1/3})^{3/2}$</td>
</tr>
<tr>
<td>2</td>
<td>0.300$(\Phi_0 \hat{K}^{-1/3})^{5/2}$</td>
</tr>
<tr>
<td>3</td>
<td>0.312$(\Phi_0 \hat{K}^{-1/3})^{7/2}$</td>
</tr>
<tr>
<td>4</td>
<td>0.414$(\Phi_0 \hat{K}^{-1/3})^{9/2}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\frac{\sim L_1}{\sim L_0}$</th>
<th>$\frac{\sim L_2}{\sim L_1}$</th>
<th>$\frac{\sim L_3}{\sim L_2}$</th>
<th>$\frac{\sim L_4}{\sim L_3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.758$\Phi_0 \hat{K}^{-3/3}$</td>
<td>0.847$\Phi_0 \hat{K}^{-3/3}$</td>
<td>1.038$\Phi_0 \hat{K}^{-3/3}$</td>
<td>1.327$\Phi_0 \hat{K}^{-3/3}$</td>
</tr>
</tbody>
</table>
Number of streams of order $i$: $N_i = n_i \times n_{i-1} \times n_{i-2} \times \ldots \times n_1$

The ratio of the number of streams of order $i-1$ to the number of streams of order $i$ is $N_i / N_{i+1} = n_i$

<table>
<thead>
<tr>
<th>$i$</th>
<th>$L_i$</th>
<th>$n_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$0.467(\Phi_0 \hat{K}^{-1/3})^{1/2}$</td>
<td>-</td>
</tr>
<tr>
<td>1</td>
<td>$0.357(\Phi_0 \hat{K}^{-1/3})^{3/2}$</td>
<td>$0.331(\Phi_0 \hat{K}^{-1/3})^2$</td>
</tr>
<tr>
<td>2</td>
<td>$0.300(\Phi_0 \hat{K}^{-1/3})^{5/2}$</td>
<td>$0.809(\Phi_0 \hat{K}^{-1/3})^2$</td>
</tr>
<tr>
<td>3</td>
<td>$0.512(\Phi_0 \hat{K}^{-1/3})^{7/2}$</td>
<td>$0.882(\Phi_0 \hat{K}^{-1/3})^2$</td>
</tr>
<tr>
<td>4</td>
<td>$0.414(\Phi_0 \hat{K}^{-1/3})^{9/2}$</td>
<td>$1.990(\Phi_0 \hat{K}^{-1/3})^2$</td>
</tr>
</tbody>
</table>

Horton’s law of stream numbers

$$N_i / N_{i+1} \sim \left(\Phi_0 \hat{K}^{-1/3}\right)^2 \sim \text{constant}$$
Relationship between $R_L$ and $R_B$

Horton’s law of stream lengths:

$$1.5 < R_L < 3.5 \; ; \; R_L \sim 2$$

$$L_i / L_{i-1} = R_L$$

Horton’s law of stream numbers:

$$3 < R_B < 5 \; ; \; R_B \sim 4$$

$$n_{i-1} / n_i = R_B$$

$$\frac{\bar{L}_{i-1}}{\bar{L}_i} \sim \Phi_0 \hat{K}^{-1/3}$$

$$\frac{N_i}{N_{i+1}} \sim \left(\Phi_0 \hat{K}^{-1/3}\right)^2$$

$$R_B \sim (R_L)^2$$
Hack’s law

\[ L_\omega = \alpha (A_\omega)^b \]

\[ b \sim 0.56 - 0.5 \]

\[ A_i = H_i L_i \]

Constructal Hack’s exponent \( \beta \) for river basins up to order 4.

\[
\begin{array}{c|c|c|c}
\hline
n_i & \tilde{H}_i & \tilde{L}_i & \tilde{n}_i \\
\hline
0 & 2^{5/6} \beta_0^{1/6} K^{1/6} & \phi_0^{1/2} & - \\
1 & 2^{1/6} \phi_0^{1/2} & \Phi_1^{1/2} & \phi_1^{3/2} \phi_0^{1/2} \\
2 & \Phi_1^{3/2} & \Phi_1^{1/2} K^{1/2} & \Phi_1^{3/2} \phi_0^{1/2} K^{1/2} \\
3 & 3^{1/6} (\Phi_2 \Phi_1)^{1/2} & \Phi_2^{1/2} K^{1/2} & \Phi_2^{1/2} \phi_0 K^{1/2} \\
4 & 3^{2/3} (\Phi_3 \Phi_2 \Phi_1)^{1/2} & \Phi_3^{1/2} (\Phi_3 \Phi_2 \Phi_1)^{1/2} & \Phi_3^{1/2} \phi_0 K^{2/3} \\
\hline
\end{array}
\]

\[
\tilde{L}_1 \sim A_1^{0.750} \quad \tilde{L}_2 \sim A_2^{0.625} \quad \tilde{L}_3 \sim A_3^{0.583} \quad \tilde{L}_4 \sim A_4^{0.563}
\]

Constructal rule for the exponent \( b \)

\[
\beta_\omega = \frac{2 \omega + 1}{4 \omega}
\]

Muller reported that \( b \sim 0.6 \) for river basins less than 8000 mi², \( b \sim 0.5 \) for basins between 8000 and 100,000 mi² and \( b \sim 0.47 \) for basins larger than 100,000 mi².
Melton’s law: $F_S = 0.694(D_\omega)^2$

$D_\omega = \frac{L_T}{A} ; F_S = \frac{N_\omega}{A}$

$D_\omega = \sum_{i=1}^{\omega} n_i \bar{L}_i / \bar{H}_\omega \bar{L}_\omega$

$F_S = \sum_{i=1}^{\omega} N_i / \bar{H}_\omega \bar{L}_\omega$

River network with streams up to order 4

$D_4 = 0.182(\Phi_0 \hat{K}^{-1/3}) + 0.135(\Phi_0 \hat{K}^{-1/3})^{-0.5} +$

$+ 0.345(\Phi_0 \hat{K}^{-1/3})^{-1.5} + 0.443(\Phi_0 \hat{K}^{-1/3})^{-2.5}$

$F_4 = 1 + 0.381(\Phi_0 \hat{K}^{-1/3})^{-2} + 1.155(\Phi_0 \hat{K}^{-1/3})^{-4} +$

$+ 1.424(\Phi_0 \hat{K}^{-1/3})^{-6}$
$F_4 \sim (D_4)^{2.45}$

$Y = X^2$
A constructal model of river basin development
(Errera and Bejan)

Objective: providing minimal resistance to flow

Changes in the river channel are possible because finite blocks can be dislodged and entrained in the stream

Rule: every squared block of area $L^2$ and height $W$, is dislodged whenever the pressure difference $\Delta P$ across the block surpasses the critical force needed to dislodge it, i.e. $\Delta PLW > \tau L^2$, 

Flow resistance decreases with block removal. Further increase in the flow rate may create the conditions for the removal a second block and for the repetition of the process. A macroscopic dendrite-like structure emerges progressively while flow resistance decreases.
Conclusions

The scaling laws of geometric features of river basins can be anticipated based on Constructal Theory, which views the pathways by which drainage networks develop in a basin not as the result of chance but as flow architectures that originate naturally as the result of minimization of the overall resistance to flow (Constructal Law).

The ratios of constructal lengths of consecutive streams match Horton’s law for the same ratio.

The ratios of constructal lengths of consecutive streams match Horton’s law for the same ratio.
Agreement is also found with the number of consecutive streams that match Horton’s law of ratios of consecutive stream numbers.

Hack’s law is also correctly anticipated by Constructal Theory, which provides Hack’s exponent accurately.

Melton’s law is verified approximately by Constructal Theory, which indicates 2.45 instead of 2 for the Melton’s exponent.

It was also shown that a constructal model of erosion can generate dendrite like patterns of low resistance channels that reproduce the typical shape of a river basin.

As has been demonstrated with many other examples either from engineering or from animate structures, the development of flow architectures is governed by the Constructal Law.
Optimal geometry for a rectangular slab with internal convection cooling
Initial Assumptions

• The slab thickness is prescribed
• The geometry is made dimensionless w.r.t. the slab thickness
• Some characteristic length ratios between relevant structures are also prescribed
• Additional geometrical constraints are introduced to make the model as real as possible and as close as possible to the sections of internally cooled blades
• The boundary conditions are realistic and derived from real blade cooling cases
Slab configuration

Used relations:

$$ \phi_0 = \frac{\pi \cdot D_o^2}{4 \cdot S_0^2} $$

Corrected relations:

$$ \phi_0 = \frac{\pi \cdot D_o^2}{4 \cdot S_0 \cdot H} $$

$$ \phi_1 = \frac{\pi \cdot D_0^2 + 2 \cdot \pi \cdot D_1^2}{4 \cdot S_0 \cdot H} $$
Values of surface heat transfer coefficients

![Temperature Distribution](image1)

**STATIC TEMPERATURE**

![Temperature Graph](image2)

![Heat Transfer Coefficient](image3)

![Exch. Coeff. Graph](image4)
<table>
<thead>
<tr>
<th>fi1 value</th>
<th>fi2 value</th>
<th>fi3 value</th>
<th>Width S</th>
<th>Diameter H1</th>
<th>Diameter H2</th>
<th>Diameter H3</th>
<th>H ext</th>
<th>Hint</th>
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<th>Tint</th>
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<td>7,916</td>
<td>3,233</td>
<td>1,252</td>
<td>300,000</td>
<td>1500,000</td>
<td>900,000</td>
<td>700,000</td>
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<tr>
<td>0,300</td>
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<td>0,730</td>
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<td>300,000</td>
<td>1500,000</td>
<td>900,000</td>
<td>700,000</td>
</tr>
<tr>
<td>0,300</td>
<td>0,400</td>
<td>0,730</td>
<td>25,620</td>
<td>15,833</td>
<td>6,466</td>
<td>2,504</td>
<td>300,000</td>
<td>1500,000</td>
<td>900,000</td>
<td>700,000</td>
</tr>
</tbody>
</table>
First level holes
Second level holes

Contours of Static Temperature (1)
FLUENT 6.3 (2D, conjugated, temp)

Contours of Static Temperature (2)
FLUENT 6.3 (2D, conjugated, temp)
Third level holes
Position of high-temperature spots
The position of the hots spots is determined by:

- The relative ratios of the heat transfer coefficients of different tubes
- The distance of the smallest tubes from the slab surface
Next steps

• 3d case
• Entry length analysis
• Different heat transfer values for different holes diameters
Performance Improvement of Tubular Solid Oxide Fuel Cell Stack through Variation of its Configuration Design

Michele Calì, Chiara Ciano, Vittorio Verda
Politecnico di Torino – Dipartimento di Energetica
Some advantages of Solid Oxide Fuel Cell
- high efficiency,
- low NOx production,
- possible direct use of methane by operating an internal reforming,
- possibility of using SOFC in combination with bottoming cycles

This research aims at studying the configuration of the system integrating cells and internal reformers, by changing the cells geometry and disposition.
From a single cell to a bundle

A model accounting for thermal and electrochemical phenomena in a single cell

- to calculate the performances of the cell for varied boundary conditions
- to evaluate the effects produced by varying geometrical data, particularly the cell length.

A network approach to compute a bundle

- accounting for the electrical connections among the cells and the energy transfers inside the bundle and with the extern
- to show how the system evolve varying the shape of the bundle and the number and length of the cells.
System configuration

Three-in-parallel by eigth-in-series cell bundles
Electrochemical model

Each cell is represented as divided into resistances connected in series and in parallel.

The fuel feeding the cell flows for each layer of nodes in branches 1, 3, 6 and 8.

Branches 2 and 7 are the electrical connections.

By branches 4 and 5 the cell results electrically isolated from those beside it.
Electrochemical model

The model accounts for ohmic, activation and concentration resistances.

\[
\Delta V_{cell} = \Delta V_N - \eta_\Omega - \eta_{act} - \eta_{conc}
\]

\[
\eta_{ohm} = R \cdot I
\]

\[
\eta_{act} = \frac{2RT}{n_eF} \cdot \sinh^{-1} \left( \frac{i}{i_e} \right)
\]

\[
\eta_{concA} = -\frac{RT}{2F} \cdot \ln \left( 1 - \frac{i}{i_{Al}} \right) + \frac{RT}{2F} \cdot \ln \left( 1 + \frac{i}{i_{Al}} \cdot \frac{p_{H_2}}{p_{H_2O}} \right)
\]

\[
\eta_{concC} = -\frac{RT}{4F} \cdot \ln \left( 1 - \frac{i}{i_{Cl}} \right)
\]
Thermal model

Thermal flows are generated inside the cell both by voltage losses and the electrochemical reaction that takes place at the anode-electrolyte interface.

This reaction, consisting in the oxidation of hydrogen and carbon monoxide, is the cause of the electromotive force $E$:

$$ E = -\frac{\Delta G}{F} = -\frac{\Delta G^o}{F} + \frac{RT}{2F} \cdot \ln \left( \frac{P_{H_2} \cdot P_{O_2}^{0.5}}{P_{H_2O}} \right) $$
Thermal model

Heat is partially supplied to the reformers, where it is required for methane to produce hydrogen and carbon monoxide.

\[
\begin{align*}
CH_4 + H_2O & \rightarrow 3H_2 + CO \\
CO + H_2O & \rightarrow H_2 + CO_2
\end{align*}
\]

In actual design, reformers are located between the bundles.
A part of the thermal flow released in the electrochemical reaction is recuperated by heating the air flux flowing in the annular section.
Resistivity

Electronic and ionic resistivities depend on the local temperature.

\[
\rho_A = 0.00298 \cdot \exp\left(-\frac{1392}{T}\right)
\]

\[
\rho_E = 10 \cdot \exp\left[10092 \cdot \left(\frac{1}{T} - \frac{1}{1273.15}\right)\right]
\]

\[
\rho_C = 0.008114 \cdot \exp\left(\frac{500}{T}\right)
\]
Cell simulation

Single cell simulation
(Operating temperature ~ 900 °C)

DOE, FC Handbook (2002)
Boundary conditions:
- DV bundle = Design DV cell (0.72 V) · Number of cells in series
- Fuel temperature (550 °C)
- Air temperature (830 °C)
- Heat flow for internal reforming (30% of the requirement)

Present design parameters:
- Bundle shape
- Cell length

Constraints:
- Total active area
- Anode, cathode and electrolyte thickness
From constructal theory

$\Delta V_{\text{bundle}}$

Actual design

$\Delta V_{\text{bundle}}$

Optimal shape
Electronic/ionic resistance

Overall resistance decreases as the number of cells in series increases.
Geometrical parameters investigated/under investigation:

- Cell length ✓
- Cell length and diameter
- Cell shape
The voltage losses decreases as the cell length increases (design voltage).
Numerical simulations of the fluid & heat flow in disc-shaped compact heat exchangers with dendritic flow structure
Compact heat exchangers: rectangular box

actual geometries multi-level refinement

Global geometry

Tubes stage

Fin stage

Louver stage
Constructal compact heat exchanger

A. Bejan
four levels disc dimensions (mm) 1/2
Disc dimensions 2/2
Disc-shaped constructal compact heat exchangers model
Model improvement and CAD simplifications

Discontinuities simplifications

Angles smoothed at the junctions (15°)
Implications of the modification introduced

- Increased pressure drop for square sections

\[ \lambda_{\text{rect}} = K_1 \lambda \]

\[ (K_1 = 0.9) \]

- Reduced pressure drops for smoothed junctions

\[ \zeta = \zeta'(1 - F_0/F_1) \]
Boundary and Initial conditions

Fluid: WATER

Outflow

Pressure inlet = p₀ + 2500 Pa

Pressure outlet = p₀

Disc material: ALUMINIUM

Inlet water Temperature = 300 °K

Inlet hot water temperature = 370 °K

Velocity inlet = 1 m/s
Grid Description

Non conformal hybrid mesh:
Fluid cells: Hexahedra (62900+62890) – solid cells: tetrahedra

<table>
<thead>
<tr>
<th>Level</th>
<th>Cells</th>
<th>Faces</th>
<th>Nodes</th>
<th>Partitions</th>
</tr>
</thead>
<tbody>
<tr>
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<td>1539373</td>
<td>3326935</td>
<td>450871</td>
<td>1</td>
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</table>

![Diagram](image-url)
Values of Reynolds numbers at each level

<table>
<thead>
<tr>
<th>Level</th>
<th>Value</th>
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</thead>
<tbody>
<tr>
<td>Level 0</td>
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<tr>
<td>Level 1</td>
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<td>Level 2</td>
<td>1605</td>
</tr>
<tr>
<td>Level 3</td>
<td>1274</td>
</tr>
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</table>
Numerical model details

Solver: segregated
Time: steady
Viscous model: laminar
Velocity profiles - level 0 – In->out
Temperature profiles – level 0 – In->out
Zoom on the recirculation bubble (temperature)
Temperature profiles – level 0 – In->out
Temperature profiles – level 0 – In->out
Temperature profiles – out->in
Temperature profiles at the hot fluid inlet
Shells
performance

<table>
<thead>
<tr>
<th>Area-Weighted Average Static Pressure (pascal)</th>
<th>Area-Weighted Average Static Temperature (k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>outflow1          -836.25</td>
<td>outflow1          321.80</td>
</tr>
<tr>
<td>outflow2          -1018.80</td>
<td>outflow2          321.67</td>
</tr>
<tr>
<td>outflow3          -978.87</td>
<td>outflow3          320.00</td>
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<tr>
<td>outflow4          -748.10</td>
<td>outflow4          320.32</td>
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<tr>
<td>central_outflow   -</td>
<td>central_outflow  350.30</td>
</tr>
</tbody>
</table>
Proposal for an improved model:

Tubes slightly curved to obtain symmetrical junctions

Exploit to the utmost the boundary layer development along each tube

Splits optimized on the basis of pressure drops
Solution Hystograms

Velocity Magnitude (m/s)

Static Temperature (k)

Static Pressure (pascal)
Dendritic pattern

Pressure drop

Diameter ratio

Pressure drop factor

\[ \Delta P = \dot{m} \frac{8\pi V}{V^2} R^3 f(n_0) \]

\[ \frac{D_{i+1}}{D_i} = 2^{-1/3} \]

\[ f(n_0) = n_0 \left[ \cos \frac{\pi}{2n_0} + \sin \frac{\pi}{2n_0} \left( \frac{2^{1/3}}{\sin \beta} - \frac{1}{\tan \beta} \right) \right]^3 \]

<table>
<thead>
<tr>
<th>( n_0 )</th>
<th>( \beta ) deg</th>
<th>( \gamma ) deg</th>
<th>( L_0 )</th>
<th>( L_1 )</th>
<th>( L_2 )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( f )</th>
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<td>0.828</td>
<td>0.153</td>
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</table>
Performance Graphs

- thermal resistance vs. flow resistance
  - one curve for one flow architecture
  - better flow architectures
  - increasing flow rate, fixed architecture

- Graph showing resistance vs. flow rate for different configurations
  - legends indicating number of parallel sections

- Graph showing relationship between thermal resistance and flow rate
  - Radial ducts
  - Different number of parallel sections

- Graph with x-axis in log scale and y-axis in linear scale
Pressure drop on a diverging section

\[ \zeta = \frac{\Delta H}{\gamma w_0} - \frac{2g}{2g} \]

*Simulation and measurements of turbulent heat transfer in a channel with a surface mounted heated block* (chen-wang), heat and mass transfer, 31, 1996, 463-473- Springer Verlag
Energy usage in Aircraft Equipment Systems

A potential future, from the EC Power Optimised Aircraft project

Workshop on Constructual Theory
Rome, 17-18 March 2005

Lester Faleiro, Jacques Herzog
Liebherr-Aerospace, Germany
Project Goals

The target of POA is to validate, at aircraft level and both qualitatively and quantitatively, the ability of next generation aircraft equipment systems to enable the reduction in consumption of non-propulsive power

- **Drivers**
  - Safety Standards

- **Objectives**
  - Reduction of peak non-propulsive power by 25%
  - Reduction of total non-propulsive power
  - Reduction of fuel consumption by 5%
  - Reduction of total equipment weight

- **Constraints**
  - Maintenance Costs
  - Equipment production costs
  - Reliability
Conventional Aircraft Architecture

- Primary Controls
- Secondary Controls
- Engine systems
- Environmental Control
- Electrical Distribution
- Gearbox
- Engine
- APU
- Commercial Loads
- Generator
- Mechanical Power
- Pneumatic Power
- Hydraulic Power
- Electrical Power

Energy usage in Aircraft Equipment Systems
Energy usage in Aircraft Equipment Systems

Optimised Aircraft Architecture?

- Primary Controls
- Secondary Controls
- Engine systems
- Environmental Control
- Landing Gear
- Electrical Distribution
- Commercial Loads
- Starter Generator
- Wing Anti-ice
- Engine
- Cabin Expansion generator

- No Gearbox
- Local Compressor
- Reduced Engine Bleed
- Local Hydraulic source
- More Electrical Power
**Engine Electrical Systems**

**Electric Actuators**

- **Fan Shaft Generator**
  - ~150kW main Power Generation at Idle and Above
  - Emergency Power Generation from Windmilling Fan to Airframe (engine-out) or HP Machine for Assisted Windmill Relight.

- **Active Magnetic Bearing**
  - Investigate Potential for Removal of Oil System
  - Monitoring of Shaft Rotordynamics

- **DC Power Bus on Engine**
  - Simplified Airframe/Engine Interface
  - Each Machine will have a Power Electronic Drive
  - Each Drive will Appear as a Node in a Distributed Control System

- **Electric Oil Pump/Scavenge System**
  - Optimise Oil Flow to Bearings over Engine Cycle

- **Electric Fuel Metering Unit**
  - ~100kW Motor
  - Simplified Fuel System
  - Lower Heat Input to Fuel

- **Electric Oil Breather Model**

**High Pressure Starter/Generator**
- ~200kW Motor for Engine Starting.
- Will Generate Power after Engine Start
- LP to HP Power Transfer may Improve Fuel Burn and Reduce Thrust During Descent.

**Fan Shaft Generator**
- ~150kW main Power Generation at Idle and Above
- Emergency Power Generation from Windmilling Fan to Airframe (engine-out) or HP Machine for Assisted Windmill Relight.

**Active Magnetic Bearing**
- Investigate Potential for Removal of Oil System
- Monitoring of Shaft Rotordynamics

**Electric Oil Pump/Scavenge System**
- Optimise Oil Flow to Bearings over Engine Cycle
Actuation Systems

The Objective is to achieve lower life cycle costs, through power optimisation, reduced weight and maintenance costs

Electro-Hydraulic Actuation (EHA)
- Derivation of Standards
- Hardware and model verification

Electro-Back-up Hydraulic Actuation (EBHA)
- Novel and versatile hybrid actuation

Trimmable Horizontal Stabiliser Actuation (THSA)
- Proof of concept
- More electrical actuation with innovative mechanical technologies

Large wide-body thrust reverser ball and screw EMA

Nacelle Systems
- More Electrical Thrust Reversal
- Alternative to pneumatic and hydraulic thrust reversal systems
- Mechanical and data bus synchronisation

2.5 kW EHA

More electrical actuation with innovative mechanical technologies

Wide-body aileron actuator ~2 kW

Typical wide-body stabiliser actuator

Hurel-Hispano test facility

Spoiler actuator ~25 kW

Electro-Mechanical Actuation (EMA)
- Distributed High-Lift systems
- Comparison of hinge line versus rotary technologies

More Electrical Actuation for Main Gear

Landing Gear Systems
- Landing Gear system integration
- Decentralised actuation for Nose Wheel
- More Electrical Actuation for Main Gear
- More Electrical Wheel Braking

Wide-body aileron actuator ~2 kW

Airbus A300/A310
The Objective is to reduce and optimise the effect of bleed air off-take on power usage, as this is a large consumer of non- propulsive power. Bleed Air Off-Takes are mainly used for the Environment Control System and for Wing Ice Protection.

**Air Conditioning System (ACS)**

To increase the efficiency of the ACS

The main innovation is the combination of a Vapour Cycle (containing an environmentally neutral fluid) with an electrical driven Air Cycle. A variable speed motor for the re-circulation fan and a Cabin Energy Recovery Device will be used.

The main outputs are a Model of a complete ACS and the Test of a Hybrid ACS (Vapour +Air Cycle)

**Wing Ice Protection (WIP)**

To Increase the efficiency of WIP Systems

The main innovation is the use of ultrasonic surface ice sensors (Anti Ice on demand) and hybrid wing heating (electrical and hot air)

The main outputs are Models of WIP Systems and Test of an innovative WIP System

Wing heat distribution using: Ultrasonic sensors, Electro-thermal devices, On demand active intelligence control and Monitoring of unprotected surfaces

**Fuel Cells (FC)**

To Increase the efficiency of electrical power generation for pneumatic systems

The main innovation is the validation of a Solid Oxide Fuel Cell (SOFC) with its reformer for use with kerosene

The main outputs are a Model of FC System and the Test of a 5 kW Fuel Cell System
Project Process so far

- The A330-300 was chosen as a reference for all comparison
- More Electrical systems chosen as technology solutions for POA
  - Perception of more efficiency at the system level
  - Higher synergy when integrated into a more electrical aircraft (MEA)
- Systems combined into various POA (mostly MEA) architectures
  - Each system has been designed and many are being manufactured

- Note: This is NOT optimisation – this is a pre-selection of a few possible combinations that the industrial partners together feel have the greatest potential for application in the near to medium future
Energy related trends at the Systems Level

- MESys tend to waste less energy
  - No hydraulic leakage
  - No large losses from engine bleed
  - Power on demand

- Many technologies can be demonstrated today
  - MEE technologies
  - High density power generation
  - Reliable electrical distribution architectures
  - Better aeronautical cabling
  - Electro-hydrostatic and electro-mechanical flight control actuation
  - Electrical braking
  - Electrical Environmental Control System
One challenge for Constructal Theory – The ECS
Tasks of the Environmental Control System (ECS)

- **Pressurisation** (at altitude Pressure not lower than 752 hPa)
- **Ventilation** (Fresh flow per passenger 0.55 lb/min)
- **Temperature control** (Cabin temperature around 24°C)

- Engine Bleed Air and Pre-cooling with Engine fan air
- **Bleed Air with Ram Air**
- Turbomachines and Water separation loops
- Mixing chamber and distribution

*Fig. 1*
Principle of the current ECS on large civil Aircraft

- 2 bleed ports: LP port at high engine power and HP port at low engine port.
- Pre-cooler limits the temperature to 200°C using Fan air
- PRV regulates the pressure delivery

Flow controlled by FCV
- Temperature controlled by Ram air flow (RAI) and Bypass flow (BPV)
- Water extraction realised by high pressure water separation loop (HPWS)
Energy Impacts of the current ECS on a large civil Aircraft

- Impact on the Engine - High Specific Fuel Consumption
- Losses – high temperature and pressure air has to be cooled and de-pressurised before it is useable
  - Weight of components included in order to induce energy losses
- Drag for the cooling of the Ram Heat Exchangers
- And many more....
Currently the Engines provide (pneumatic) power to the ECS. This supplied power is strongly dependent on the flight phase.

Two bleed ports needed to supply the required power and to limit the impact of Bleed Air on Engine fuel consumption.

Nevertheless a lot of losses are induced by the divergence of needs and supply.
ONE solution to energy “optimisation”: the electrical ECS (Bleed-less)

- Engines produce mechanical (kinetic) energy from fuel
- Generators convert the mechanical energy into electrical energy
- ECS use the electrical energy to perform its tasks:
  - Pressurisation
  - Ventilation
  - Temperature control
Adaptability of the electric ECS

- The power consumption of the electrical ECS is independent of the engine speed (no dissipation of excessive engine power)
- Adaptation of the ECS to the cabin needs
  - Fresh flow can be adapted to the number of Cabin occupants
  - Cooling power can be adapted to the heat loads
  - If needed ECS power consumption can be reduced for a short time so as to allow other systems to use the released power

![Diagram showing pressure changes over time for different flight phases.](Fig. 10)
Impact on engine of pneumatic and electrical power source

- Mechanical Off-takes (to produce electrical power) and Pneumatic Off-takes have different impacts on the Specific Fuel Consumption of an engine
  - to cover the ECS needs, the impact of electrical power on the engine specific fuel consumption is not so strong as the impact of the pneumatic off-takes

Potential to reduce the Fuel consumption (energy) by using electrical power – this pseudo-optimisation is what goes on at the equipment systems level

Fig. 11
A role for Constructal Theory?

- A Bejan has published 2 papers to address the issues in an aircraft ECS
  - The method is structured and makes sense, but the problem to solve is complex and multi-faceted.
    - It cannot be solved by only thermodynamic considerations, as the systems involved are multi-physics.
    - The inputs required to address the problem fully are difficult to obtain, as many organisational entities may be involved (aircraft, engine and systems manufacturers)
    - It is difficult to solve problems for one component without addressing the impacts on the other components
    - It is difficult to solve problems for the ECS without addressing the impacts on engine and rest of the aircraft (drag, etc.)

- Could Constructal Theory produce a technological solution such as bleedless ECS, or is something else needed to complement it, such as a palette of potential technological solutions and a palette of constraints?
Further Challenges for Energy Based Methods

- Constraints are not easy to include in such methods
  - Weight limits
  - Volume limits
  - Design limitations on the performance and function of Aircraft Equipment Systems
  - Safety considerations

- Every divergence from the ideal model can reduce the degree of freedom of the method
  - The idea of creating an ideal energy based model and then systematically altering the model to include design constraints is good, but the complexity of such a model is enormous
Energy Based Methods are probably needed

- For a really Power Optimised Aircraft (not just MEA), thinking must start to occur at the level of **functions**, rather than **systems**
  - Design Process Re-engineering for equipment systems integration may be necessary – energy based methods might be useful
  - e.g. ECS is currently used to provide pressurisation and cooling. In POA, various architectures that provided these two functions in various numbers of systems and sub-systems were examined

- Decreasing Systems Autonomy – energy is the common denominator
  - More Electrical Engine (MEE) and MEA go hand-in-hand
  - Availability of MEA engine thrust is connected to generation system reliability through the use of electrical fuel pumps
  - This trend on interdependency is most visible on Boeing B787: bleedless engine/electrical ECS/electrical generation and distribution – can energy based methods, such as Constructual Theory, come up with such solutions?
New Challenges

- Cooling in a MEA
- Thermal management of Power Electronics is a challenge
- MOET Project – European University to look at this?
Thank you!
Energy usage in Aircraft Equipment Systems
ON THE SYNTHESIS
OF COMPLEX ENERGY SYSTEMS
WITH SEVERAL HEAT TRANSFER INTERACTIONS

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• SYNTHESIS is concerned with putting separate elements into a whole, i.e. the particular equipment items making up the overall thermal system and their interconnections are specified

• The objectives of the ANALYSIS and OPTIMIZATION are to identify the preferred configuration among those synthesized.
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**Synthesis, Analysis and Optimization**

- **ANALYSIS** generally entails thermal analysis (mass, energy and exergy analysis), costing and sizing of equipment on at least a preliminary basis.

- **OPTIMIZATION** can take two general forms:
  1) **PARAMETER** and 2) **STRUCTURAL** optimization.

1) pressures, temperatures, mass flow rates and chemical composition at various points and/or other system key variables are determined, at least approximately, with the aim of satisfying some desired objectives.

2) the equipment inventory and/or interconnections among components are altered iteratively to achieve a superior design.
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**Synthesis, Analysis and Optimization**

- Concept Generation and Preliminary Screening
- Synthesis
- Alternative Concepts
- Analysis
  - Thermal Analysis
  - Sizing
  - Costing
  - Reliability, Safety, etc.
- Parameter Optimization
- Structural Optimization
- Base Case Flowsheet
- Detailed Design
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Other criteria to improve thermal system structure

- Definition of a **superstructure** including all feasible structures and optimization of design parameters to “cut” useless branches of the superstructure

- **Criterion applied here** is based on a “**basic plant configuration**” which is defined by subdividing the plant into two zones:
  - One including the “**basic components**” that are always included and connected in the same way in the plant structure;
  - The **heat transfer zone**, considered as a “**black box**” in which heat is transferred between hot and cold thermal flows (made available from the components in the first zone) independently of the number, type and interconnections among heat exchangers, which are not defined.
Optimization of the basic plant configuration

• **General problem**: optimal synthesis of the components, i.e. how to interconnect them in order to reach the optimal value of the objective function (here the exergetic efficiency)

• **Idea**: fix a part of the structure (basic components and their interconnections) and keep the definition of the heat exchanger structure independent of the structure of the overall system

• **Suggested solution**: parameter optimization of the basic plant configuration + construction of the HEN which fulfill the optimal thermodynamic conditions at the black-box boundaries and guarantees the feasibility of the heat transfer

• **Final solution**: find a criterion to generate all possible basic plant configurations and then choose and optimize the best one according to the objective of the analysis
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HAT cycle  “ Basic Plant Configuration “

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HAT cycle “Basic Plant Configuration”
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Heat transfer section (black-box)

Hot side                  Cold side
HAT cycle “Basic Plant Configuration”

- Basic components: compressors (c1, c2), combustion chamber (cc), turbine (t), saturator (s)

- Order:
  - air/gas loop: (c1, c2), (cc), (t)
  - water loop (s), (cc), (t)

- Temperature links between basic components are “virtually” cut, i.e. the temperature at the inlet of a component is left independent of the temperature at the outlet of the preceding component (with the exceptions of the links between hot source (cc) and turbine (t) and between cold source (ambient) and compressor (c1) where additional heat transfer is not meaningful)
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Structure of complex energy systems

- One or more thermodynamic cycles are involved
- Each cycle involves a loop of one or more materials
- In each cycle the order of the basic components is defined
- Additional basic components can be added to improve the system performance (e.g. compressor and/or turbine stages, combustion stages)
Parameter optimization (thermodynamic) of the basic plant configuration

- Objective function: total plant exergetic efficiency

- Decision variables are the degrees of freedom needed to set the thermodynamic conditions at the inlet/outlet of the basic plant components -> once their values are fixed, inlet and outlet temperatures and mass flow rates at the black-box boundaries are fixed

- Optimization problem is subject to the heat transfer feasibility constraint, simply expressed by the condition that the minimum temperature difference between composite curves should be greater than a minimum $\Delta T$
Composite Curves

- Thermal capacities of flows which transfer heat in the same direction (hot to cold or vice versa) and in the same temperature interval are summed up.

- A Hot and Cold Composite Curves (HCC, CCC) are so drawn in a heat-temperature diagram, showing the sequence of the heat transferred at increasing temperature values according to the thermal capacities calculated for hot and cold flows, respectively.

- No external hot sources exist -> HCC and CCC have the same ending abscissa.

- The black-box may release heat to the ambient -> the projection of the HCC on the abscissa is longer than that of the CCC.
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Composite Curves

Temperature [°C]
Heat transfer in the black-box

- In the search for the optimum total plant exergetic efficiency, the optimization algorithm *finds implicitly* the condition of most efficient heat transfer in the black-box (optimum matching between hot and cold thermal flows)

-> optimum matching is meant as the set of inlet and outlet temperatures and mass flow rates of the hot and cold streams deriving from the independent variables set which minimizes the distance between the CC
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Temperature Profiles

Temperature [°C]

1200
1100
1000
900
800
700
600
500
400
300
200
100
0

0 1 2 3 4 5 6 7 8 9 10

component 1
intercooling
component 2
aftercooling
saturator
regenerator
combustor
turbine
exhaust gases

MWH
SWH
FH
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HAT cycle heat exchanger network (1)
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HAT cycle heat exchanger network (2)
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Heat exchanger network (3)
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HAT cycle plant
Building the heat exchanger network (HEN)

Need of fulfilling black-box optimal boundary conditions (corresponding to achieving the maximum total plant exergetic efficiency) and minimum pinch point ($\Delta T_{pp}$)

HEN can be built in two different ways:

1) No mixing at different temperatures
   -> same composite curves;
2) Mixing at different temperatures
   -> different composite curves ($\Delta T_{pp}$ may or not be fulfilled)

Other solutions in which $\Delta T_{pp}$ is respected but boundary conditions are not fulfilled -> lower exergetic efficiency - different composite curves
Basic Plant Configuration
Regenerative gas turbine cycle
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Air preheating - composite curves
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CRCC “Basic Plant Configuration”
Fig. 2 Black-box representing the heat transfer section in the basic plant configuration.
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**Interactions between HRSG and black-box**

*Input water = Recovered H₂O + Ext. Make Up – Saturator Make Up*
Design improvement procedure for the “CRCC Basic Plant Configuration”

- **Problem**: high number of free variables in the design improvement procedure:
  1) decision variables in the components outside the heat transfer section
  2) variables at the boundaries of the heat transfer section (inlet/outlet temperatures and mass flow rates of the thermal flows in the black-box)

  -> need of reduction of the degrees of freedom at the heat transfer section boundaries

  -> Because of the system complexity, system is simulated into parts (sections), no optimization algorithms are used
Design improvement procedure for the “CRCC Basic Plant Configuration“

- Preliminary thermodynamic analyses are performed to understand whether some of the decision variables can be fixed as parameters or show predictable trends.

- Series of simulation runs of the total system are carried out by varying the remaining decision variables one at a time.

- Basic importance of a correct choice of the reformer operating variables.
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Preliminary thermodynamic analysis

![Graphs showing thermodynamic analysis results including Unconverted Methane Ratio, Efficiency, Water Temperature at the saturator inlet, and HTS Temperature.](image-url)
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Independent variables of the “basic plant configuration”

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>TARGET</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reformer Pressure</td>
<td>Optimize (10 - 22- 30 bar)</td>
</tr>
<tr>
<td>Gas Temperature at the reformer outlet</td>
<td></td>
</tr>
<tr>
<td>Steam / Natural Gas Ratio</td>
<td>T_REF and SGR are functions of p_REF and they are related if methane conversion ratio is fixed (95%)</td>
</tr>
<tr>
<td>HTS Temperature</td>
<td>Optimize (range: 350-475°C)</td>
</tr>
<tr>
<td>LTS Temperature</td>
<td>Optimize (range: 180-250°C)</td>
</tr>
<tr>
<td>Water temperature at the saturator inlet</td>
<td>Optimize</td>
</tr>
<tr>
<td>Water mass flow rate at the saturator inlet</td>
<td>Fix at constant value (mole frac. H₂)</td>
</tr>
<tr>
<td>Exhaust gas mass flow rate to reformer</td>
<td>Optimize (range: 35-100% GT outlet)</td>
</tr>
<tr>
<td>Reformer reactive mixture temperature</td>
<td>Optimize (450 - 500 - 565°C)</td>
</tr>
<tr>
<td>Exhaust gas temperature at the reformer outlet</td>
<td>Optimize (range: 480 - 540°C)</td>
</tr>
<tr>
<td>Preheating of SYNGAS</td>
<td>Minimize</td>
</tr>
<tr>
<td>Preheating of natural gas</td>
<td>Optimize (range: 500-800°C)</td>
</tr>
<tr>
<td>IP steam mass flow rate superheated with heat recovered in the black-box</td>
<td>Optimize (range: 10-20 kg/s)</td>
</tr>
<tr>
<td>HP water mass flow rate heated, evaporated and superheated with heat recovered in the black-box</td>
<td>Maximize</td>
</tr>
</tbody>
</table>
Heat transfer maximization

- In order to have the maximum feasible heat transfer in the “black-box”, a Fortran routine is included in the plant simulation model to set the last of the decision variables ($m_{HP}$) while keeping the other variables as constant parameters varied in selected ranges.

As in previous examples:
- the heat transfer feasibility constraint applies, i.e. the minimum temperature difference between composite curves should be greater than a minimum $\Delta T$ (it is directly applied to solve for $m_{HP}$)
- no external hot sources exist -> HCC and CCC have the same ending abscissa
- the black-box may release heat to the ambient -> the projection of the HCC on the abscissa is longer than that of the CCC
Heat transfer maximization

• The only HP water mass flow rate re-circulated outside the HRSG ($m_{\text{HP}}$) is considered as decision variable in the black-box sub-optimization (heat transfer maximization) while keeping the other variables as constant parameters (varied in selected ranges)

• Each $m_{\text{HP}}$ value corresponds to a specific shape and position of the Cold Composite Curve.

Fig. 9 Cold composite curves obtained using values of $m_{\text{HP}}$ lower than (CCC1), greater than (CCC2) and equal to (CCC) the maximum one.
Decisions about thermal flows to be included in the black-box

- In general, to have maximum freedom in the definition of the system structure and parameters, all thermal flows should be included in the heat transfer section (black-box)

Thermal flows not “directly” included in the black-box of the CRCC plant:
- structure of the HRSG is considered to be fixed (is part of the basic components of the system), but HP and IP steam can be produced also outside the HRSG using heat available from the black-box (p and T levels decided in advance)
- p and T levels of thermal flows taken from the black-box and used in the Section for Absorption/Stripping/Compression of the CO$_2$ decided in advance
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Influence of variations of the design variables on the composite curves

Fig. 12 “Composite Curves” at maximum total plant efficiency for three different thermal profiles (numbered according to the order in Figure 10) of the reformer reactor operating at 22 bar.
Influence of variations of the design variables on the black-box composite curves

Fig. 11 “Composite Curves” for two values of the water temperature at the saturator inlet and reformer pressure fixed at 22 bar.
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Priority order in the variation of the decision variables in the First Series of simulation runs

**Reformer Pressure**
- 10, 22, 30 bar

**Outlet temperature, SGR and thermal profile for the 3 reformer sections (95% CH₄ conversion):**
- (a) 840°C / 3.5 / 840-720-570
- (b) 800°C / 5 / 800-680-530
- (c) 775°C / 6 / 775-655-505

**HTS temperature:** 400°C

**LTS temperature:** 181°C

**Water temperature at the inlet of saturator:** it is function of the saturation temperature at the defined pressure → \( t_{min} = 140°C \)
- 140, 160, 180, 200, 220°C

**Exhaust gases mass flow rate to reformer combustor:** 35-75%
- Fixed at 50%

**Reactive mixture temperature**
- 450°C, 500°C, 565°C
- Fixed at 450°C

**Exhaust gases temperature at the reformer outlet**
- 480°C, 500°C, 520°C, 540°C
- Fixed at 480°C

**SYNREF temperature**
- 500°C, 600°C, 700°C, 800°C
- Fixed at 500°C

**Natural gas preheating temp.**
- 200°C, 300°C, 400°C, 500°C

**Water temperature at the inlet of saturator:** it is function of the saturation temperature at the defined pressure → \( t_{min} = 175°C \)
- 175, 195, 215, 235°C

**Exhaust gases mass flow rate to reformer combustor:** 35-75%
- Fixed at 50%

**Reactive mixture temperature**
- 450°C, 500°C, 565°C
- Fixed at 450°C

**Exhaust gases temperature at the reformer outlet**
- 480°C, 500°C, 520°C, 540°C
- Fixed at 480°C

**SYNREF temperature**
- 500°C, 600°C, 700°C, 800°C
- Fixed at 500°C

**Natural gas preheating temp.**
- 200°C, 300°C, 400°C, 500°C

**Water temperature at the inlet of saturator:** it is function of the saturation temperature at the defined pressure → \( t_{min} = 185°C \)
- 185, 205, 225, 245, 265°C

**Outlet temperature, SGR and thermal profile for the 3 reformer sections (95% CH₄ conversion):**
- (a) 920°C / 3.5 / 920-800-650
- (b) 900°C / 4 / 900-780-630
- (c) 850°C / 6 / 850-730-580

**HTS temperature:** 400°C

**LTS temperature:** 181°C

**Exhaust gases mass flow rate to reformer combustor:** 35-75%
- Fixed at 50%

**Reactive mixture temperature**
- 450°C, 500°C, 565°C
- Fixed at 450°C

**Exhaust gases temperature at the reformer outlet**
- 480°C, 500°C, 520°C, 540°C
- Fixed at 480°C

**SYNREF temperature**
- 500°C, 600°C, 700°C, 800°C
- Fixed at 500°C

**Natural gas preheating temp.**
- 200°C, 300°C, 400°C, 500°C

**Water temperature at the inlet of saturator:** it is function of the saturation temperature at the defined pressure → \( t_{min} = 195°C \)
- 175, 195, 215, 235°C

**HTS temperature:** 400°C

**LTS temperature:** 181°C

**Exhaust gases mass flow rate to reformer combustor:** 35-75%
- Fixed at 50%

**Reactive mixture temperature**
- 450°C, 500°C, 565°C
- Fixed at 450°C

**Exhaust gases temperature at the reformer outlet**
- 480°C, 500°C, 520°C, 540°C
- Fixed at 480°C

**SYNREF temperature**
- 500°C, 600°C, 700°C, 800°C
- Fixed at 500°C

**Natural gas preheating temp.**
- 200°C, 300°C, 400°C, 500°C

**Figure 10**
Design improvement procedure: First series of simulation runs

Design variables are varied in the ranges of Figure 10, and

• the intermediate pressure mass flow rate re-circulated outside the HRSG \( m_{IP} = 20 \text{ kg/s} \), and

• the H\(_2\) molar fraction at the saturator outlet equal to 65%

The high pressure mass flow rate re-circulated outside the HRSG calculated by the Fortran routine in order to maximize the heat transfer in the black box is \( m_{HP} = 115 \text{ kg/s} \) and the maximum thermal efficiency of the total plant is 50.96%
**Indications for design improvement from the composite curves**

Fig. 13 Possible actions on the cold composite curve (CCC) for improving the performances of the best plant configuration obtained in the first series of simulation runs (reformer pressure equal to 30 bar).
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Second Series of simulation runs

- In the second series:
  - The intermediate pressure mass flow rate \( m_{1P} \) re-circulated outside the HRSG is decreased from 20 to 10 kg/s;
  - The \( \text{H}_2 \) molar fraction at the saturator outlet is increased from 65% to 70%.

Among the variables indicated in Figure 10:
- The preheating temperature of the syngas to the reformer before expansion is set to 250°C, instead of 500°C
- The temperature of the exhaust gases from the reformer is set at 400°C, instead of 480°C

- Using these values, the high pressure mass flow rate re-circulated outside the HRSG obtained by the Fortran routine to maximize the heat transfer in the black box is \( m_{HP} = 120 \text{ kg/s} \) instead of 115 kg/s and the maximum thermal efficiency of the total plant is 52.32%.
### Results of the design improvement procedure

<table>
<thead>
<tr>
<th>Series of Simul. Runs</th>
<th>P_{ref} [bar]</th>
<th>SGR</th>
<th>Temperature Profile [°C]</th>
<th>T_{SAT} [°C]</th>
<th>P_{TOT} [MW]</th>
<th>P_{spec} [kJ/kg]</th>
<th>CO_{2} Spec. Emissions [kgCO_{2}/kWh]</th>
<th>η [%]</th>
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<tr>
<td>Reference Simple Combined Cycle Configuration</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>/</td>
<td>396</td>
<td>717.4</td>
<td>0.363</td>
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<td><strong>Optimization Steps</strong></td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I 10</td>
<td>3.5</td>
<td>570-720-840</td>
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<td></td>
<td>5</td>
<td>530-680-800</td>
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<td>505-655-775</td>
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<td>22</td>
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<td>650-800-920</td>
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<td>4</td>
<td>630-780-900</td>
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<td>404</td>
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<td>0.054</td>
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<td></td>
<td>6</td>
<td>580-730-850</td>
<td>215</td>
<td>395</td>
<td>833.6</td>
<td>0.051</td>
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<td>3.5</td>
<td>680-830-950</td>
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<td>0.058</td>
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<tr>
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<td>5</td>
<td>630-780-900</td>
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<td>423</td>
<td>893.7</td>
<td>0.0514</td>
<td>50.96</td>
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<tr>
<td>Final Optimized Configuration</td>
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<td></td>
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<tr>
<td>II 30</td>
<td>5</td>
<td>630-780-900</td>
<td>245</td>
<td>434.16</td>
<td>900</td>
<td>0.05</td>
<td>52.32</td>
<td></td>
</tr>
</tbody>
</table>

**Tab. 4 Results obtained by the first (I) and second (II) series of simulation runs.**
Comparison between the composite curves of the two series of simulations runs

Fig. 14 Comparison between the CCs obtained in the two series of simulations with a reformer pressure of 30 bar.
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CONCLUSIONS

• Method for the design improvement of a plant configuration (named “basic”) in which heat is transferred in a “black-box” without having the heat exchanger structure defined

• The method keeps the problem of the structural optimization of the system independent of the structure of the HEN

• When an optimization problem is set up for the basic plant configuration, the most efficient heat transfer in the black-box is directly found by the optimization algorithm

• If, conversely, a design improvement procedure is performed through a series of simulation runs, the optimum matching between thermal flows in the black-box is found at each step of the procedure using a sub-optimization procedure that maximizes the heat transfer between hot and cold flows
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CONCLUSIONS

• The method improves the comprehension of the heat transfer internal processes
  -> it helps locate cold thermal flows to be heated and properly match them with all the hot thermal flows available

  -> it helps minimize temperature differences in the heat transfer processes, and heat rejected to the environment -> maximize plant efficiency

  -> it helps find directions for design improvement (new values for the design variables to be fixed as parameters)
CONCLUSIONS

APPLICATION TO THE CRCC PLANT

- High reformer pressure (30 bar) with high methane conversion ratio (95%)

- Expander to recover the pressure content of the hydrogen sent to the reformer

- Saturator to lower NOx production and improve thermal recovery at low temperature

- Further advantages by the water recovery section because of the heat recovered in the black-box at low temperature
Need For Energy and Thermal Management Program Planning

- Current And Future Air Force Weapons Systems Lack The Necessary Power and Cooling Capacity To Provide Full Systems Level Capability
- Cooling Capacity Of Fuel Already Fully Utilized Leaving Little Room For Additional Cooling Needs
- Increasing Speed, Power, And Miniaturization Of Future Systems Continue To Stress Any Thermal Management Capability We Can Now Deliver

Focused, High Level Initiative Is Needed To Support Today’s And Tomorrow’s Weapon Systems
Need For Energy and Thermal Management Program Planning

• Define Focused Technology Transition Capability
  – Enhanced Systems Engineering Process Implementation
  – Better “Rules And Tools” for System Level Assessments
  – Understand Impact Of Technologies on System Performance

• Apply Systems Engineering Principles -- Thermal Management Considered Early in System Design
  – Assure Thermally Efficient Designs
  – Understand Power and Heat Generation Impacts

• Generate “Game Changing” Approaches To Thermal Management

• Develop Innovative and More Effective Material Systems
  – Fuels, Lubes And Seals
  – Power, Electronics, Actuation

Solution Requires Both Better Assessment Capabilities and More Robust Technologies
Workshop Goals and Expectations

Address the Big Questions – How Much, How Bad

- What are the stressing drivers
- Focus first on directed energy on airborne platforms

Guide Technology Development

- From your perspective, what are some key energy and thermal management technologies for challenges such as directed energy weapons on airborne platforms?

Provide Information to Guide Resource Allocation, Technology Choices, Technology Designs, and Research Needs

- What are highest payoff areas for energy and thermal management research investments?
- What systems-of-systems analysis methods are needed?
- How do we measure system level effectiveness?
Capablity Focused Tech Investment
A Process for Integrating Solutions

Col Michael B. Leahy, Jr.
Material Group Director
Air Vehicles Directorate
Air Force Research Laboratory

“We give the Air Force its Wings”
Air Vehicles Directorate

Technology Developer and System Integrator

AFOSR – Basic Research

Structures | Control Sciences | Aero Sciences

Aerospace Systems Integration

Develop and transition superior air vehicle technology solutions that enable dominant military aerospace vehicles
What we bring to the fight

Superior Technologies for Future Aerospace Dominance

Strike

Mobility

ISR

Force Projection

Operationally Responsive Space Access

Agile Combat Support

Force Application

Agile Combat Support

Long Range Strike

Prompt Global Strike

Command & Control

Cooperative Airspace Ops

Persistent ISR

Surveillance & Reconnaissance
**Top Down Capability Driven Process to Support Strategic System Tech Investment Decisions**

**Attributes**
- High Availability
- Rapid Turn/Low Cost Ops
- Rapid Turn/Low Cost Ops
- Operationally Responsive Spacelift
- Technology Planning Process
- Integration CONOPS
- SBIR

**Product (s)**
- RLV Integration Technologies
- Int RLV Eng Sys
- Ops & Cost Tools
- Many Programs!

**Capability**
- Rapid turn 24 hrs
- Low sortie cost $1M
- Vehicle reliability 0.999
- All Wx availability 95%
- 500 Sortie Propulsion & Systems

**What do we have to do to deliver it?**
- What do we have to do to deliver it?
- When?

**Effects needed? When?**
- Effects needed? When?

**S&T needed? When?**
- S&T needed? When?

**Operationally Responsive Spacelift Attribute Availability Forecast**

<table>
<thead>
<tr>
<th>Year</th>
<th>Near Term</th>
<th>Mid Term</th>
<th>Far Term</th>
</tr>
</thead>
<tbody>
<tr>
<td>2009</td>
<td>Rapid turn 24 hrs</td>
<td>Low sortie cost $1M</td>
<td>Low sortie cost $1M</td>
</tr>
<tr>
<td>2010</td>
<td>Vehicle reliability 0.999</td>
<td>Vehicle reliability 0.999</td>
<td>Vehicle reliability 0.999</td>
</tr>
<tr>
<td>2011</td>
<td>All Wx availability 95%</td>
<td>All Wx availability 95%</td>
<td>All Wx availability 95%</td>
</tr>
<tr>
<td>2012</td>
<td>500 Sortie Airframe</td>
<td>500 Sortie Airframe</td>
<td>500 Sortie Airframe</td>
</tr>
<tr>
<td>2013</td>
<td>250 Sortie Propulsion &amp; Systems</td>
<td>100 Sortie Propulsion &amp; Systems</td>
<td>250 Sortie Propulsion &amp; Systems</td>
</tr>
<tr>
<td>2014</td>
<td>100 Sortie Propulsion &amp; Systems</td>
<td>250 Sortie Propulsion &amp; Systems</td>
<td>100 Sortie Propulsion &amp; Systems</td>
</tr>
<tr>
<td>2015</td>
<td>250 Sortie Propulsion &amp; Systems</td>
<td>100 Sortie Propulsion &amp; Systems</td>
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</table>

**Total Capability Cost:** $214,315K based on $200K class demonstrator + $275,000K extended flight tests

**Technology Planning Process**

<table>
<thead>
<tr>
<th>Phase</th>
<th>Description</th>
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<tbody>
<tr>
<td>AF CONOPS Construct</td>
<td>Technology at a glance</td>
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**Integration CONOPS**

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<td>Far Term</td>
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**SBIR**

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<td>Far Term</td>
<td>Technology at a glance</td>
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**Total Funding**

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<td>Far Term</td>
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**Extended Flight Test**

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<tr>
<td>Far Term</td>
<td>Technology at a glance</td>
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**Directly links tech programs to capability gaps/desires**
Capability Focused Tech Investment
Long Range Strike

“The capability to achieve a desired effect(s) rapidly and/or persistently, on any target, in any environment, anywhere, at any time.”

- Regional to Intercontinental Reach
- Hold Targets at Risk for Extended Time
- Survive & Operate in Anti-Access Environment
- High Volume and Flexible Payload
- Network Centric Operability

Based on LRS AoA FY07 and Multiple LRS Studies FY01-03
Thermal Structures for Responsive Global Strike

- High Temperature Fuel Tanks
- Aeroheating
- Propulsion Heat Loads
- Hot bays
- High Temp Inlets and Nozzles
- Thin wings
Responsive Global Strike

- Flight vehicle system heat loads engine, environmental control systems, hydraulic systems, and electrical systems
- High temperature stealth structures - Analytical capabilities to optimize and integrate energy systems - Innovational concept advancement in actively cooled structures - Both of these capabilities and concepts are in their infancy.

A system level approach to thermal management is required
Persistent ISR

Detect and track every target in the battlespace…

*anywhere, at any time*

- 360 degree coverage
- On Station 24/7
- Multi-function/Multi-sensor
- Foliage penetration
- Integrated/Fused data
- Penetrating

Based On ICRRA and GH ORD/Annex
### Sensor Mission Power Requirement

<table>
<thead>
<tr>
<th></th>
<th>Vol. ft³</th>
<th>Wgt lbs</th>
<th>Pow. Kw</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radar</td>
<td>0.73</td>
<td>820</td>
<td>62</td>
</tr>
<tr>
<td>Radar</td>
<td>0.02</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Radar</td>
<td>0.01</td>
<td>100</td>
<td>1.2</td>
</tr>
<tr>
<td>IFF</td>
<td>0.00</td>
<td>50</td>
<td>1</td>
</tr>
<tr>
<td>ELINT</td>
<td>0.00</td>
<td>500</td>
<td>0</td>
</tr>
<tr>
<td>COMINT</td>
<td>0.00</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>E/O</td>
<td>0.12</td>
<td>450</td>
<td>3.5</td>
</tr>
<tr>
<td>Comms</td>
<td>0.80</td>
<td>52</td>
<td>1</td>
</tr>
<tr>
<td>Receiver Subsystem</td>
<td>0.18</td>
<td>533</td>
<td>10</td>
</tr>
<tr>
<td>Processor Subsystem</td>
<td>0.78</td>
<td>192</td>
<td>4.8</td>
</tr>
<tr>
<td><strong>TOTAL</strong></td>
<td><strong>39.33</strong></td>
<td><strong>5000</strong></td>
<td><strong>160-200</strong></td>
</tr>
</tbody>
</table>

For comparison, Global Hawk Integrated Sensor Suite (ISS) weighs about 900 lbs, and requires 6.5 kW of power.
SensorCraft Profile

- Climbing: 50,000 ft to 60,000 ft ≤ 200 nm
- Cruise/Climb Ingress: 70,000 ft
- Loiter: 3000 nm for 30+ hours
- Egress: 50,000 ft to 50,000 ft ≤ 200 nm
- Loiter/Land: 1 hour at SL

States:
- Idle/Takeoff
- Loiter/Land

Flight Levels:
- 50,000 ft
- 60,000 ft
- 70,000 ft
Capability Focused Tech Investment
Persistent Precision Strike

Persistent delivery of lethal and non-lethal precision effects

- Long Duration
- Rapid Deployment
- Survivable
- Interoperable (Force Integration)
- Affordable
- Lethal/Non-Lethal Precision Effects
- Unmanned/Manned
AADS System

• 5-10 MW electrical
  – 4 kW Input Power for Cryogenic Cooler (30K) ~1 kW
    Input Power for Cathode Heater
  <50% efficient
  
  Idea duty cycle is 100% availability

• 1-2 kHp in
• ~10% efficient
• 1000 W/cm²

• Seconds between shots
Stressing Scenario Focus
Energy and Thermal Management Executive IPT
and Technical Advisory Group

ETM Workshop Outbrief

...Enabling Airborne Directed Energy Weapons...
Major Themes

• **System Level Assessments** *(Must Do Now)*

  – Focus: Get arms around three concepts – and hold true to them
    
    • 50 kw SSL on a Mid-Sized Airborne Platform
    • 100 kw SSL on a Tactical Platform
    • Airborne Active Denial on a Mid-Sized Airborne Platform

  – Bound Problem to create realm of possible, not preselect point solution
Sim Based R&D
The Engine in Capability Planning

- Conceptual Studies
  - Vehicle Concepts
  - Technology Assessment

- Sensitivities Studies
  - Automated Iterative
  - Archived

- Campaign Analysis
- Mission Effectiveness

- Capability
- Attributes
- Products

- Tech Programs

- Common Synthetic Battlespace
- Impact of Key Performance Parameters
- Campaign Analysis
- High Fidelity Analysis
- Technology Trades
- Requirements
- Metrics

- Tech Programs
Current Status

Thermal and Energy Management Being Worked At Component And Subsystem Level

Shortfall

• Methods And Capabilities To Rapidly Understand The Impacts Of Technologies At The System/Capability Level
Assessment

- Identified Key Technology Development Drivers
- What is Required to...
  - Fly how Long
  - Shoot so Far
  - Carry How Much
- Power Generation
- Device
- Thermal Tech Trades
- Power and Thermal Storage
- High Fidelity Cooling Analysis
- Structural Analysis
- Sophisticated Device Modeling

- Mission Definition and Analysis
- Effectiveness and LCC Assessments
- Assessment of Technology Impacts
- Linkage of KPPs to Capability
- Link to Technology Demonstrations
Major Themes (Con’t)

• Theme: Increasing Weapon Efficiency will only help the thermal management issues, not solve the issue
• Theme: Need for advanced fluids and materials
  • Theme: Mature Two-Phase Flow
  • Theme: Energy Harvesting (Way Cool If…)
• Theme: Energy Storage Systems
  • Phase Change Materials (PCM), Adsorption, Chemical reaction, …
Energy and Thermal Management Executive IPT Vision

To develop/promote & establish a robust energy and thermal management research & development program that is:

• Integrated across AFRL -- Maximizes use of other organizations’ efforts
• Focused on capabilities at the system level
• Provides for a balance of 6.1, 6.2 and 6.3 research & development

... to unleash the power of innovative technology...

Cross Cutting Executive IPT Formed and Working Integrated Approach
Defending America by Unleashing the Power of Innovative Aerospace Technology

Energy and Thermal Management Stressing Scenarios
An Air Vehicles Directorate Point of View

David M Pratt, PhD
Airborne Active Denial System

- non-lethal antipersonnel
- millimeter-wave
- electromagnetic energy
Tactical Laser
Mid-Sized Airborne Platform

Solid state laser
TMS Requirements

- 1-2 kHp in
- ~10% efficient
- 1000 W/cm²
- Seconds between shots