4. TITLE AND SUBTITLE
COLLABORATIVE RESEARCH:
AN OPTIMIZATION FRAMEWORK BASED ON DOMAIN DECOMPOSITION
AND MODEL REDUCTION

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14. ABSTRACT
This collaborative research has developed rigorous mathematical and computational frameworks for reduced order model (ROM) generation and
the use of ROMs in real-time, design, control, and probabilistic applications of relevance to the Air Force.
This research has provided theoretical analyses and numerical studies for several new/extensions of existing ROM approaches, such as
goal-oriented, model-constrained approach, balanced truncation model reduction (BTMR) of descriptor systems, and the integration of domain
decomposition and BTMR for systems with localized nonlinearities. Additionally, several important questions related to the design, analysis,
efficient computation, and application of ROM were studied. The use of ROMs was demonstrated on example applications, including optimal flow
control of linearized Navier-Stokes equations, linearized flow control of a supersonic diffuser, subsonic compressor blade row unsteady
aerodynamics and geometric mistuning, a thermal design problem, and nonlinear combustor model dynamics.

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1. Introduction

1.1. Research Objectives

As part of this research program we proposed the development of an energy-based thermodynamic stabilization framework for hybrid control design of large-scale aerospace systems. In particular, we concentrate on hybrid control, hierarchical control, impulsive dynamical systems, nonnegative dynamical systems, compartmental systems, large-scale systems, nonlinear switching control, cooperative control, and adaptive control. Application areas include large flexible interconnected space structures, spacecraft stabilization, cooperative control of unmanned air vehicles, network systems, swarms of air and space vehicle formations, and pharmacological systems.

1.2. Overview of Research

Controls research by the Principal Investigator [1-74] has concentrated on an energy-based thermodynamic stabilization framework for hybrid control design of large-scale aerospace systems. This framework provides a rigorous foundation for developing a unified energy-based (network thermodynamic) analysis and synthesis methodology for large-scale aerospace systems possessing hybrid, hierarchical, and feedback structures. This framework additionally provides a rigorous alternative to designing gain scheduled controllers for general nonlinear dynamical systems by constructing minimal complexity logic-based nonlinear controllers consisting of a number of subcontrollers situated in levels (protocol layers of hierarchies) such that each subcontroller can coordinate lower-level controllers. Correspondingly, one of the main goals of this research has been to make progress towards the development of analysis and hierarchical hybrid nonlinear control law tools for nonlinear large-scale dynamical systems. This framework provides the basis for developing control-system partitioning/embedding using concepts of energy-based thermodynamic hybrid stabilization for complex, large-scale aerospace systems.

A thermodynamic stabilization framework for Eulerian swarm models is also developed. Specifically, we present a distributed boundary controller architecture involving the exchange of information between uniformly distributed swarms that guarantee that the closed-loop system is consistent with basic thermodynamic principles. Robustness of individual agent failures and unplanned individual agent behavior is also addressed. In addition, a general framework for designing semistable protocols in dynamical networks with switching topologies is also developed. Specifically, we develop a distributed nonlinear controller architecture for multilateral network consensus with time-dependent and state-dependent communication topologies. Correspondingly, the main goal of this research over the past year has been to make progress towards the development of analysis and control for nonlinear multi-agent systems.

1.3. Goals of this Report

The main goal of this report is to summarize the progress achieved under the program during the past three years. Since most of the technical results appeared or will soon appear in over 74 archival journal and conference publications, we shall only summarize these results and remark on their significance and interrelationship.

2. Description of Work Accomplished

The following partial research accomplishments have been completed over the past three years.

2.1. Hybrid Decentralized Maximum Entropy Control for Large-Scale Dynamical Systems

Modern complex dynamical systems\(^1\) are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication network constraints. The sheer size (i.e., dimensionality) and complexity of these large-scale dynamical systems often necessitates a decentralized architecture for analyzing and controlling these systems. Specifically, in the control-system design of complex large-scale dynamical systems it is often desirable to treat the overall system as a collection of interconnected subsystems. The behavior of the composite (i.e., large-scale) system can then be predicted from the behaviors of the individual subsystems and their interconnections. The need for decentralized control design of large-scale systems is a direct consequence of the physical size and complexity of the dynamical model. In particular, computational complexity may be too large for model analysis while severe constraints on communication links between system sensors, actuators, and processors may render centralized control architectures impractical. Moreover, even when communication constraints do not exist, decentralized processing may be more economical.

\(^1\)Here we have in mind large flexible space structures, aerospace systems, electric power systems, network systems, economic systems, and ecological systems, to cite but a few examples.
The complexity of modern controlled large-scale dynamical systems is further exacerbated by the use of hierarchical embedded control subsystems within the feedback control system; that is, abstract decision-making units performing logical checks that identity system mode operation and specify the continuous-variable subcontroller to be activated. Such systems typically possess a multielectron hybrid decentralized control architecture characterized by continuous-time dynamics at the lower levels of the hierarchy and discrete-time dynamics at the higher levels of the hierarchy. The lower-level units directly interact with the dynamical system to be controlled while the higher-level units receive information from the lower-level units as inputs and provide (possibly discrete) output commands which serve to coordinate and reconcile the (sometimes competing) actions of the lower-level units. The hierarchical controller organization reduces processor cost and controller complexity by breaking up the processing task into relatively small pieces and decomposing the fast and slow control functions. Typically, the higher-level units perform logical checks that determine system mode operation, while the lower-level units execute continuous-variable commands for a given system mode of operation.

Since implementation constraints, cost, and reliability considerations often require decentralized controller architectures for controlling large-scale systems, decentralized control has received considerable attention in the literature. A straightforward decentralized control design technique is that of sequential optimization, wherein a sequential centralized subcontroller design procedure is applied to an augmented closed-loop plant composed of the actual plant and the remaining subcontrollers. Clearly, a key difficulty with decentralized control predicated on sequential optimization is that of dimensionality. An alternative approach to sequential optimization for decentralized control is based on subsystem decomposition with centralized design procedures applied to the individual subsystems of the large-scale system. Decomposition techniques exploit subsystem interconnection data and in many cases, such as in the presence of very high system dimensionality, is absolutely essential for designing decentralized controllers.

In this research [6], we develop a novel energy-based hybrid decentralized control framework for lossless and dissipative large-scale dynamical systems based on subsystem decomposition. The notion of energy here refers to abstract energy notions for which a physical system energy interpretation is not necessary. These dynamical systems cover a very broad spectrum of applications including aerospace systems, fluid systems, electromechanical systems, electrical systems, combustion systems, structural vibration systems, biological systems, physiological systems, power systems, telecommunications systems, and economic systems, to cite but a few examples. The concept of an energy-based hybrid decentralized controller can be viewed as a feedback control technique that exploits the coupling between a physical large-scale dynamical system and an energy-based decentralized controller to efficiently remove energy from the physical large-scale system. Specifically, if a dissipative or lossless large-scale system is at high energy level, and a lossless feedback decentralized controller at a low energy level is attached to it, then subsystem energy will generally tend to flow from each subsystem into the corresponding subcontroller, decreasing the subsystem energy and increasing the subcontroller energy [1]. Of course, emulated energy, and not physical energy, is accumulated by each subcontroller. Conversely, if each attached subcontroller is at a high energy level and the corresponding subsystem is at a low energy level, then energy can flow from each subcontroller to each corresponding subsystem, since each subcontroller can generate real, physical energy to effect the required energy flow. Hence, if and when the subcontroller states coincide with a high emulated energy level, then we can react these states to remove the emulated energy so that the emulated energy is not returned to the plant. In this case, the overall closed-loop system consisting of the plant and the controller possesses discontinuous flows since it combines logical switchings with continuous dynamics, leading to impulsive differential equations [1].

2.2. Control Vector Lyapunov Functions for Large-Scale Hybrid Dynamical Systems

In this research [5], we provide generalizations to the recent extensions of vector Lyapunov theory for continuous-time systems [2] to address stability and control design of impulsive dynamical systems via vector Lyapunov functions. Vector Lyapunov theory has been developed to weaken the hypothesis of standard Lyapunov theory in order to enlarge the class of Lyapunov functions that can be used for analyzing system stability. Lyapunov methods have also been used by control system designers to obtain stabilizing feedback controllers for nonlinear systems. In particular, for smooth feedback, Lyapunov-based methods were inspired by Jurdjevic and Quinn who give sufficient conditions for smooth stabilization based on the ability of constructing a Lyapunov function for the closed-loop system. More recently, Artstein introduced the notion of a control Lyapunov function whose existence guarantees a feedback control law which globally stabilizes a nonlinear dynamical system. Even though for certain classes of nonlinear dynamical systems a universal construction of a feedback stabilizer can be obtained using control Lyapunov functions, there does not exist a unified procedure for finding a Lyapunov function candidate that will stabilize the closed-loop system for general nonlinear systems.

In an attempt to simplify the construction of Lyapunov functions for the analysis and control design of nonlinear dynamical systems, several researchers have resorted to vector
Lyapunov functions as an alternative to scalar Lyapunov functions. Vector Lyapunov functions were first introduced by Bellman and Matrosov, and are ideal for analyzing large-scale systems [1, 2, 5]. The use of vector Lyapunov functions in dynamical system theory offers a very flexible framework since each component of the vector Lyapunov function can satisfy less rigid requirements as compared to a single scalar Lyapunov function. Weakening the hypothesis on the Lyapunov function enlarges the class of Lyapunov functions that can be used for analyzing system stability. In particular, each component of a vector Lyapunov function need not be positive definite with a negative or even negative-semidefinite derivative. Alternatively, the time derivative of the vector Lyapunov function need only satisfy an element-by-element inequality involving a vector field of a certain comparison system. Since in this case the stability properties of the comparison system imply the stability properties of the dynamical system, the use of vector Lyapunov theory can significantly reduce the complexity (i.e., dimensionality) of the dynamical system being analyzed.

The results of this research [5] build on those of [2] and include a generalized comparison principle involving hybrid comparison dynamics that are dependent on the comparison system states as well as the nonlinear impulsive dynamical system states. Next, we develop stability theorems based on hybrid comparison inequalities as well as partial stability results for impulsive systems using vector Lyapunov functions. Furthermore, we extend the newly developed notion of control vector Lyapunov functions presented in [2] to impulsive dynamical systems and show that in the case of a scalar comparison system the definition of a control vector Lyapunov function collapses into a combination of the classical definition of a control Lyapunov function for continuous-time dynamical systems and the definition of a control Lyapunov function for discrete-time dynamical systems. In addition, using control vector Lyapunov functions, we present a universal hybrid decentralized feedback stabilizer for a decentralized affine in the control nonlinear impulsive dynamical system with guaranteed gain and sector margins. These results are then used to develop hybrid decentralized controllers for large-scale impulsive dynamical systems with robustness guarantees against full modeling and input uncertainty.

2.3. Consensus and Semistability in Network Dynamical Systems with Arbitrary Time-Delays

As discussed in Section 2.1, modern complex dynamical systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks. By properly formulating these systems in terms of subsystem interaction involving energy/mass transfer, the dynamical models of many of these systems can be derived from mass, energy, and information balance considerations that involve dynamic states whose values are nonnegative. Hence, it follows from physical considerations that the state trajectory of such systems remains in the nonnegative orthant of the state space for nonnegative initial conditions. Such systems are commonly referred to as nonnegative dynamical systems in the literature. A subclass of nonnegative dynamical systems are compartmental systems. Compartmental systems involve dynamical models that are characterized by conservation laws (e.g., mass and energy) capturing the exchange of material between coupled macroscopic subsystems known as compartments. Each compartment is assumed to be kinetically homogeneous, that is, any material entering the compartment is instantaneously mixed with the material of the compartment. The range of applications of nonnegative systems and compartmental systems includes biological and physiological systems, chemical reaction systems, queuing systems, large-scale systems, stochastic systems (whose state variables represent probabilities), ecological systems, economic systems, demographic systems, telecommunications systems, transportation systems, power systems, thermodynamic systems, and structural vibration systems, to cite but a few examples.

A key physical limitation of compartmental systems is that transfers between compartments are not instantaneous and realistic models for capturing the dynamics of such systems should account for material, energy, or information in transit between compartments. Hence, to accurately describe the evolution of the aforementioned systems, it is necessary to include in any mathematical model of the system dynamics some information of the past system states. In this case, the state of the system at a given time involves a piece of trajectories in the space of continuous functions defined on an interval in the nonnegative orthant of the state space. This of course leads to (infinite-dimensional) delay dynamical systems.

Nonnegative and compartmental models are also widespread in agreement problems in networks with directed graphs and switching topologies. Specifically, distributed decision-making for coordination of networks of dynamic agents involving information flow can be naturally captured by compartmental models. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles, distributed sensor networks, swarms of air and space vehicle formations, and congestion control in communication networks. In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. In particular, it is important to develop consensus protocols for networks of dynamic agents with directed information flow, switching network topologies, and possible system time-delays. In this research [13], we use compartmental dynamical system models to characterize dynamic algorithms for linear and nonlinear networks of dynamic agents in the presence of inter-agent communication delays that possess a continuum of semistable equilibria, that is, protocol algorithms that guaran-
toe convergence to Lyapunov stable equilibria. In addition, we show that the steady-state
distribution of the dynamic network is uniform, leading to system state equi-partitioning
or consensus. These results extend the results in the literature on consensus protocols for linear
balanced networks to linear and nonlinear unbalanced networks with switching topologies
and time-delays.

2.4. Finite-Time Semistable Consensus Protocols for Dynamical
Networks

In recent research [19] a unified stability analysis framework for systems having a con-
tinuum of equilibria was developed. Since every neighborhood of a nonisolated equilibria
contains another equilibrium, a nonisolated equilibrium cannot be asymptotically stable.
Hence, asymptotic stability is not the appropriate notion of stability for systems having a
continuum of equilibria. Two notions that are of particular relevance to such systems are
convergence and semistability. Convergence is the property whereby every system solution
converges to a limit point that may depend on the system initial condition. Semistability
is the additional requirement that all solutions converge to limit points that are Lyapunov
stable. Semistability thus implies Lyapunov stability, and is implied by asymptotic stability.

The dependence of the limiting state on the initial state is seen in numerous dynamical
systems including compartmental systems which arise in chemical kinetics, and biomedical,
environmental, economic, power, and thermodynamic systems [10]. For these systems, every
trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a
(possibly different) Lyapunov stable equilibrium, and hence, these systems are semistable.
In addition to semistability, it is desirable that a dynamical system that exhibits semistability
also possesses the property that trajectories that converge to a Lyapunov stable system state
must do so in finite time rather than merely asymptotically.

In this research [19], we merge the theories of semistability and finite-time stability to
develop a rigorous framework for finite-time semistability. In particular, finite-time semis-
Tability for a continuum of equilibria of continuous autonomous systems is established.
Continuity of the settling-time function as well as Lyapunov and converse Lyapunov theorems
for semistability are also developed. In addition, necessary and sufficient conditions for
finite-time semistability of homogeneous systems are addressed by exploiting the fact that a
homogeneous system is finite-time semistable if and only if it is semistable and has a negative
degree of homogeneity.

Next, we use these results to develop a general framework for designing semistable pro-
tocols in dynamical networks for achieving coordination tasks in finite time. Distributed
decision-making for coordination of networks of dynamic agents involving information flow
can be naturally captured by graph-theoretic notions. These dynamical network systems
cover a very broad spectrum of applications including cooperative control of unmanned air
vehicles (UAVs), autonomous underwater vehicles (AUVs), distributed sensor networks,
air and ground transportation systems, swarms of air and space vehicle formations, and
congestion control in communication networks, to cite but a few examples. Hence, it is
not surprising that a considerable research effort has been devoted to control of networks
and control over networks in recent years. However, finite-time coordination has not been
addressed in the literature.

In many applications involving multiagent systems, groups of agents are required to agree
on certain quantities of interest. In particular, it is important to develop information con-
sensus protocols for networks of dynamic agents wherein a unique feature of the closed-loop
dynamics under any control algorithm that achieves consensus in a dynamical network is the
existence of a continuum of equilibria representing a state of consensus. Under such dynam-
ics, the limiting consensus state achieved is not determined completely by the dynamics, but
depends on the initial system state. Hence, using the results on finite-time semistability, we
develop a unified framework for addressing the consensus problem in networks of agents in
finite time. Specifically, we develop nonlinear finite-time controllers using undirected and
directed graphs to accommodate for a full range of possible graph information topologies
without limitations of bidirectional communication.

2.5. Continuous and Hybrid Distributed Control for Multiagent
Coordination: Consensus, Flocking, and Cyclic Pursuit

Modern complex dynamical systems are highly interconnected and mutually interde-
dent, both physically and through a multitude of information and communication networks.
Distributed decision-making for coordination of networks of dynamic agents involving in-
formation flow can be naturally captured by graph-theoretic notions. These dynamical
network systems cover a very broad spectrum of applications including cooperative control
of unmanned air vehicles (UAVs), autonomous underwater vehicles (AUVs), distributed
sensor networks, air and ground transportation systems, swarms of air and space vehicle
formations, and congestion control in communication networks, to cite but a few examples.

A key application area within aerospace systems is cooperative control of vehicle forma-
tions using distributed and decentralized controller architectures. Distributed control refers
to a control architecture wherein the control is distributed via multiple computational units
that are interconnected through information and communication networks, whereas decen-

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eralized control refers to a control architecture wherein local decisions are based only on local information. Vehicle formations are typically dynamically decoupled, that is, the motion of a given agent or vehicle does not directly affect the motion of the other agents or vehicles. The multiagent system is coupled via the task which the agents or vehicles are required to perform.

In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. In particular, it is important to develop information consensus protocols for networks of dynamic agents wherein a unique feature of the closed-loop dynamics under any control algorithm that achieves consensus is the existence of a continuum of equilibria representing a state of equipartitioning or consensus. Under such dynamics, the limiting consensus state achieved is not determined completely by the dynamics, but depends on the initial system state as well. As discussed in Section 2.4, for such systems possessing a continuum of equilibria, semistability, and not asymptotic stability, is the relevant notion of stability. Semistability is the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium.

Alternatively, in other applications of multiagent systems, groups of agents are required to achieve and maintain a prescribed geometric shape. This formation problem includes flocking and cyclic pursuit, wherein parallel and circular formations of vehicles are sought. For formation control of multiple vehicles, cohesion, separation, and alignment constraints are typically required for individual agent steering which describe how a given vehicle maneuver based on the positions and velocities of nearby agents. Specifically, cohesion refers to a steering rule wherein a given vehicle attempts to move toward the average position of local vehicles, separation refers to collision avoidance with nearby vehicles, while alignment refers to velocity matching with nearby vehicles.

Using graph-theoretic notions in this research [17,58] we develop a unified framework for addressing consensus, flocking, and cyclic pursuit problems for multiagent dynamical systems. Specifically, we present continuous and hybrid distributed and decentralized controller architectures for multiagent coordination. In contrast to virtually all of the existing results in the literature on control of networks, the majority of the proposed controllers are dynamic compensators. The proposed controller architectures are predicated on the recently developed notion of system thermodynamics [10] resulting in thermodynamically consistent continuous and hybrid controller architectures involving the exchange of information between agents that guarantee that the closed-loop dynamical network is consistent with basic thermodynamic principles. Another unique feature of our framework is that several of the proposed controller architectures are hybrid, and hence, the overall closed-loop dynamics under these controller algorithms achieving consensus, flocking, or cyclic pursuit possesses discontinuous flows since they combine logical switchings with continuous dynamics, leading to impulsive differential equations [1]. The proposed controllers use undirected and directed graphs to accommodate for a full range of possible graph information topologies without limitations of bidirectional communication.

2.6. Robust Control Algorithms for Network Consensus Protocols with Uncertain Communication Graph Topologies

Due to advances in embedded computational resources over the last several years, a considerable research effort has been devoted to the control of networks and control over networks. Network systems involve distributed decision-making for coordination of networks of dynamic agents involving information flow enabling enhanced operational effectiveness via cooperative control in autonomous systems. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV’s) and autonomous underwater vehicles (AUV’s) for combat, surveillance, and reconnaissance; distributed reconfigurable sensor networks for managing power levels of wireless networks; air and ground transportation systems for air traffic control and payload transport; and traffic management; swarms of air and space vehicle formations for command and control between heterogeneous air and space vehicles; and congestion control in communication networks for routing the flow of information through a network.

To enable the applications for these multiagent systems, cooperative control tasks such as formation control, rendezvous, flocking, cyclic pursuit, cohesion, separation, alignment, and consensus need to be developed. To realize these tasks, individual agents need to share information of the system objectives as well as the dynamical network. In particular, in many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. Information consensus over dynamic information-exchange topologies guarantees agreement between agents for a given coordination task. Distributed consensus algorithms involve neighbor-to-neighbor interaction between agents wherein agents update their information state based on the information states of the neighboring agents. A unique feature of the closed-loop dynamics under any control algorithm that achieves consensus in a dynamical network is the existence of a continuum of equilibria representing a state of consensus. Under such dynamics, the limiting consensus state achieved is not determined completely by the dynamics, but depends on the initial state as well. As discussed in Section 2.4, in systems possessing a continuum of equilibria, semistability, and not asymptotic
stability is the relevant notion of stability.

Even though many consensus protocol algorithms have been developed over the last several years in the literature, robustness properties of these algorithms have been ignored. Robustness here refers to sensitivity of the control algorithm achieving semistability and consensus in the face of communication uncertainty and communication dropouts. In this research [36], we build on the results of [17, 19] to develop robust control algorithms for network consensus protocols with uncertain communication topologies of a specified structure. In particular, we construct homogeneous control protocol functions that scale in a consistent fashion with respect to a scaling operation on an underlying space with the additional property that the protocol functions can be written as a sum of functions, each homogeneous with respect to a fixed scaling operation, that retain system semistability and consensus.

2.7. Finite-Time Stabilization of Large-Scale Nonlinear Dynamical Systems

The notions of asymptotic and exponential stability in dynamical systems theory imply convergence of the system trajectories to an equilibrium state over the infinite horizon. In many applications, however, it is desirable that a dynamical system possesses the property that trajectories that converge to a Lyapunov stable equilibrium state do so in finite time rather than merely asymptotically. Most of the existing control techniques in the literature ensure that the closed-loop system dynamics of a controlled system are Lipschitz continuous, which implies uniqueness of system solutions in forward and backward times. Hence, convergence to an equilibrium state is achieved over an infinite time interval. In order to achieve convergence in finite time, the closed-loop system dynamics need to be non-Lipschitzian giving rise to non-uniqueness of solutions in backward time. Uniqueness of solutions in forward time, however, can be preserved in the case of finite-time convergence.

In this research [20], we develop a general framework for finite-time stability analysis of nonlinear dynamical systems using vector Lyapunov functions. Specifically, we construct a vector comparison system that is finite-time stable and, using the vector comparison principle [2], relate this finite-time stability property to the stability properties of the nonlinear dynamical system. Furthermore, we design universal finite-time stabilizing decentralized controllers for large-scale dynamical systems based on the newly proposed notion of a control vector Lyapunov function [2]. In addition, we present necessary and sufficient conditions for continuity of such controllers. Moreover, we specialize these results to the case of a scalar Lyapunov function to obtain universal finite-time stabilizers for nonlinear systems that are affine in the control.

2.8. Complexity, Robustness, Self-Organization, Swarms, and System Thermodynamics

Due to technological advances in sensing, actuation, communication, and computation over the last several years, a considerable research effort has been devoted to the control of networks and control over networks. Network systems involve distributed decision-making for coordination of dynamic agents involving information flow enabling enhanced operational effectiveness via cooperative control in autonomous systems. These dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV's) and autonomous underwater vehicles (AUV's) for combat, surveillance, and reconnaissance; distributed reconfigurable sensor networks for managing power levels of wireless networks; air and ground transportation systems for air traffic control and payload transport and traffic management; swarms of air and space vehicle formations for command and control between heterogeneous air and space vehicles; and congestion control in communication networks for routing the flow of information through a network.

To enable the autonomous operation for these multiagent systems, the development of functional algorithms for agent coordination and control is needed. In particular, control algorithms need to address agent interactions, cooperative and non-cooperative control, task assignments, and resource allocations. To realize these tasks, appropriate sensory and cognitive capabilities such as adaptation, learning, decision-making, and agreement (or consensus) on the agent and multiagent levels are required. The common approach for addressing the autonomous operation of multiagent systems is using distributed control algorithms involving neighbor-to-neighbor interaction between agents wherein agents update their information state based on the information states of the neighboring agents. Since most multiagent network systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks, these systems are characterized by high-dimensional, large-scale interconnected dynamical systems. To develop distributed methods for control and coordination of autonomous multiagent systems, many researchers have looked to autonomous swarms systems appearing in nature for inspiration.

Biology has shown that many species of animals such as insect swarms, ungulate flocks, fish schools, ant colonies, and bacterial colonies self-organize in nature. These biological aggregations give rise to remarkably complex global behaviors from simple local interactions between large numbers of relatively unintelligent agents without the need for centralized control. The spontaneous development (i.e., self-organization) of these autonomous biological systems and their spatio-temporal evolution to more complex states often appears without any external system interaction. In other words, structure morphing into coherent groups
is internal to the system and results from local interactions among subsystem components that are independent of the physical nature of the individual components. These local interactions often comprise a simple set of rules that lead to remarkably complex global behaviors. Complexity here refers to the quantity of a system wherein interacting subsystems self-organize to form hierarchical evolving structures exhibiting emergent system properties. Hence, a complex dynamical system is a system that is greater than the sum of its subsystems or parts. In addition, the spatially distributed sensing and actuation control architecture prevalent in such systems is inherently robust to individual subsystem (or agent) failures and unplanned behavior at the individual subsystem (or agent) level.

The connection between the local subsystem interactions and the globally complex system behavior is often elusive. Complex dynamical systems involving self-organizing components forming spatio-temporally evolving structures that exhibit a hierarchy of emergent system properties are not limited to biological aggregation systems. Such systems include, for example, nervous systems, immune systems, ecological systems, quantum particle systems, chemical reaction systems, economic systems, cellular systems, and galaxies, to cite but a few examples. These systems are known as dissipative systems [24] and consume energy and matter while maintaining their stable structure by dissipating entropy to the environment. For example, as in biology, in the physical universe billions of stars and galaxies interact to form self-organizing dissipative nonequilibrium structures. The fundamental common phenomenon among these systems are that they evolve in accordance to the laws of (nonequilibrium) thermodynamics which are among the most firmly established laws of nature. System thermodynamics, in the sense of [10], involves open interconnected dynamical systems that exchange matter and energy with their environment in accordance with the first law (conservation of energy) and the second law (nonconservation of entropy) of thermodynamics. Self-organization can spontaneously occur in such systems by invoking the two fundamental axioms of the science of heat. Namely, i) if the energies in the connected subsystems of an interconnected system are equal, then energy exchange between these subsystems is not possible, and ii) energy flows from more energetic subsystems to less energetic subsystems. These axioms establish the existence of a system entropy function as well as equi-partition of energy [10] in system thermodynamics and information consensus [17, 19] in cooperative networks; an emergent behavior in thermodynamic systems as well as swarm systems. Hence, in complex interconnected dynamical systems, self-organization is not a property of the system’s parts but rather emerges as a result of the nonlinear subsystem interactions.

In light of the above discussion, engineering swarm systems necessitates the development of relatively simple autonomous agents that are inherently distributed, self-organized, and truly scalable. Scalability follows from the fact that such systems do not involve centralized control and communication architectures. In addition, engineered swarming systems should be inherently robust to individual agent failures, unplanned task assignment changes, and environmental changes. Mathematical models for large-scale swarms can involve Lagrangian and Eulerian models. In a Lagrangian model, each agent is modeled as a particle governed by a difference or differential equation, whereas an Eulerian model describes the local energy or information flux for a distribution of swarms with an advection-diffusion (conservation) equation. The two formulations can be connected by a Fokker-Planck approximation relating jump distance distributions of individual agents to terms in the advection-diffusion equation.

In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. In particular, it is important to develop information consensus protocols for networks of dynamic agents wherein a unique feature of the closed-loop dynamics under any control algorithm that achieves consensus is the existence of a continuum of equilibria representing a state of equipartitioning or consensus. Under such dynamics, the limiting consensus state achieved is not determined completely by the dynamics, but depends on the initial system state as well. For such systems possessing a continuum of equilibria, semistability [9], and not asymptotic stability, is the relevant notion of stability. Semistability is the property whereby every trajectory that starts in a neighborhood of a Lyapunov stable equilibrium converges to a (possibly different) Lyapunov stable equilibrium. From a practical viewpoint, it is not sufficient to only guarantee that a swarm converges to a state of consensus since steady state convergence is not sufficient to guarantee that small perturbations from the limiting state will lead to only small transient excursions from a state of consensus. It is also necessary to guarantee that the equilibrium states representing consensus are Lyapunov stable, and consequently, semistable.

In this research [24], we develop distributed boundary control algorithms for addressing the consensus problem for an Eulerian swarm model. The proposed distributed boundary controller architectures are predicated on the recently developed notion of system thermodynamics [10] resulting in controller architectures involving the exchange of information between uniformly distributed swarms over an m-dimensional (not necessarily Euclidian) space that guarantee that the closed-loop system is consistent with basic thermodynamic principles. For our thermodynamically consistent model we further establish the existence of a unique continuously differentiable entropy functional for all equilibrium and nonequilibrium states of our system. Information consensus and semistability are shown using the well-known Sobolev embedding theorems and the notion of generalized (or weak) solutions. Finally, since the closed-loop system is guaranteed to satisfy basic thermodynamic principles, robustness to individual agent failures and unplanned individual agent behavior is automatically guaranteed.
2.9. Semistability, Differential Inclusions, and Consensus Protocols for Dynamical Networks with Switching Topology

Modern complex dynamical systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks. Distributed decision-making for coordination of networks of dynamic agents involving information flow can be naturally captured by graph-theoretic notions. As noted in Section 2.8, these dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles (UAV's), autonomous underwater vehicles (AUV's), distributed sensor networks, air and ground transportation systems, swarms of air and space vehicle formations, and congestion control in communication networks, to cite but a few examples. Hence, it is not surprising that a considerable research effort has been devoted to control of networks and control over networks in recent years.

Since communication links among multiagent systems are often unreliable due to multipath effects and exogenous disturbances, the information exchange topologies in network systems are often dynamic. In particular, link failures or creations in network multiagent systems result in switchings of the communication topology. This is the case, for example, if information between agents is exchanged by means of line-of-sight sensors that experience periodic communication dropouts due to agent motion. Variation in network topology introduces control input discontinuities, which in turn give rise to discontinuous dynamical systems. In addition, the communication topology may be time-varying. In this case, the vector field defining the dynamical system is a discontinuous function of the state and time, and hence, system stability can be analyzed using nonsmooth Lyapunov theory involving concepts such as weak and strong stability notions, differential inclusions, and generalized gradients of locally Lipschitz functions and proximal subdifferentials of lower semicontinuous functions.

In many applications involving multiagent systems, groups of agents are required to agree on certain quantities of interest. In particular, it is important to develop information consensus protocols for networks of dynamic agents wherein a unique feature of the closed-loop dynamics under any control algorithm that achieves consensus is the existence of a continuum of equilibria representing a state of equipartitioning or consensus. Under such dynamics, the limiting consensus state achieved is not determined completely by the dynamics, but depends on the initial system state as well. For such systems possessing a continuum of equilibria, semistability [9], and not asymptotic stability, is the relevant notion of stability.

To address agreement problems in switching networks with time-dependent and state-dependent topologies, in this research [36,74] we extend the theory of semistability to discontinuous time-invariant and time-varying dynamical systems. In particular, we develop necessary and sufficient conditions to guarantee weak and strong invariance of Filippov solutions under the assumption that the discontinuous system vector field is uniformly bounded. Moreover, we present Lyapunov-based tests for strong semistability, weak semistability, as well as uniform semistability for autonomous and nonautonomous differential inclusions.

2.10. $\mathcal{H}_2$ Optimal Semistable Control for Linear Dynamical Systems

Dynamical network systems cover a very broad spectrum of applications including cooperative control of unmanned air vehicles, autonomous underwater vehicles, distributed sensor networks, air and ground transportation systems, swarms of air and space vehicle formations, and congestion control in communication networks, to cite but a few examples. A unique feature of the closed-loop dynamics under any control algorithm in dynamical networks is the existence of a continuum of equilibria representing a desired state of convergence. Under such dynamics, the desired limiting state is not determined completely by the system dynamics, but depends on the initial system state as well [17,19].

The dependence of the limiting state on the initial state is not limited to dynamical network systems; it is also seen in the dynamics of compartmental systems which arise in chemical kinetics, and biomedical, environmental, economic, power, and thermodynamic systems. In all such systems possessing a continuum of equilibria, semistability, and not asymptotic stability, is the relevant notion of stability.

In this research [28,35], we use linear matrix inequalities (LMIs) to develop $\mathcal{H}_2$ optimal semistable controllers for linear dynamical systems. Linear matrix inequalities provide a powerful design framework for linear control problems. Since LMIs lead to convex or quasiconvex optimization problems, they can be solved very efficiently using interior-point algorithms. Unlike the standard $\mathcal{H}_2$ optimal control problem, a complicating feature of the $\mathcal{H}_2$ optimal semistable stabilization problem is that the closed-loop Lyapunov equation guaranteeing semistability can admit multiple solutions. An interesting feature of the proposed approach, however, is that a least squares solution over all possible semistabilizing solutions corresponds to the $\mathcal{H}_2$ optimal solution. It is shown that this least squares solution can be characterized by a linear matrix inequality minimization problem.
2.11. Finite-Time Stabilization for Nonlinear Impulsive Dynamical Systems

The mathematical descriptions of many hybrid dynamical systems can be characterized by impulsive differential equations [1]. Impulsive dynamical systems can be viewed as a subclass of hybrid systems and consist of three elements—namely, a continuous-time differential equation, which governs the motion of the dynamical system between impulsive or resetting events; a difference equation, which governs the way the system states are instantaneously changed when a resetting event occurs; and a criterion for determining when the states of the system are to be reset. Since impulsive systems can involve impulses at variable times, they are in general time-varying systems, wherein the resetting events are both a function of time and the system's state. In the case where the resetting events are defined by a prescribed sequence of times which are independent of the system state, the equations are known as time-dependent differential equations [1]. Alternatively, in the case where the resetting events are defined by a manifold in the state space that is independent of time, the equations are autonomous and are known as state-dependent differential equations [1].

Finite-time stability implies Lyapunov stability and convergence of system trajectories to an equilibrium state in finite-time, and hence, it is a stronger notion than asymptotic stability. For continuous-time dynamical systems, finite-time stability implies non-Lipschitzian dynamics [9] giving rise to non-uniqueness of solutions in reverse time. Uniqueness of solutions in forward time, however, can be preserved in the case of finite-time convergence. Finite-time convergence to a Lyapunov stable equilibrium for continuous-time systems, that is, finite-time stability, was rigorously studied in [9] using Hölder continuous Lyapunov functions.

Finite-time stability of impulsive dynamical systems, however, has not been studied in the literature. For impulsive dynamical systems, it may be possible to reset the system states to an equilibrium state, in which case finite-time convergence of the system trajectories can be achieved without the requirement for non-Lipschitzian dynamics. In addition, due to system resettings, impulsive dynamical systems may exhibit non-uniqueness of solutions in reverse time even when the continuous-time dynamics are Lipschitz continuous.

In this research [15], we develop sufficient conditions for finite-time stability of nonlinear impulsive dynamical systems. Furthermore, we present stability results using vector Lyapunov functions wherein finite-time stability of the impulsive system is shown via finite-time stability of a hybrid comparison system. We use these results to further construct hybrid finite-time stabilizing controllers for impulsive dynamical systems. In addition, we construct decentralized finite-time stabilizers for large-scale impulsive dynamical systems.

2.12. Neural Network Hybrid Adaptive Control for Nonlinear Uncertain Impulsive Dynamical Systems

Modern complex engineering systems involve multiple modes of operation placing stringent demands on controller design and implementation of increasing complexity. Such systems typically possess a multichannel hierarchical hybrid control architecture characterized by continuous-time dynamics at the lower levels of the hierarchy and discrete-time dynamics at the higher levels of the hierarchy [1]. The lower-level units directly interact with the dynamical system to be controlled while the higher-level units receive information from the lower-level units as inputs and provide (possibly discrete) output commands which serve to coordinate and reconcile the (sometimes competing) actions of the lower-level units. The hierarchical controller organization reduces processor cost and controller complexity by breaking up the processing task into relatively small pieces and decomposing the fast and slow control functions. Typically, the higher-level units perform logical checks that determine system mode operation, while the lower-level units execute continuous-variable commands for a given system mode of operation. The mathematical description of many of these systems can be characterized by impulsive differential equations [1].

The purpose of feedback control is to achieve desirable system performance in the face of system uncertainty. To this end, adaptive control along with robust control theory have been developed to address the problem of system uncertainty in control-system design. In contrast to fixed-gain robust controllers, which maintain specified constants within the feedback control law to sustain robust performance, adaptive controllers directly or indirectly adjust feedback gains to maintain closed-loop stability and improve performance in the face of system uncertainties. Specifically, indirect adaptive controllers utilize parameter update laws to identify unknown system parameters and adjust feedback gains to account for system variation, while direct adaptive controllers directly adjust the controller gains in response to plant variations. The inherent nonlinearities and system uncertainties in hierarchical hybrid control systems and the increasingly stringent performance requirements required for controlling such modern complex embedded systems necessitates the development of hybrid adaptive nonlinear control methodologies.

Neural network-based adaptive control algorithms have been extensively developed in the literature, wherein Lyapunov-like functions are used to ensure that the neural network controllers can guarantee ultimate boundedness of the closed-loop system states rather than closed-loop asymptotic stability. Ultimate boundedness ensures that the plant states converge to a neighborhood of the origin [9]. The reason why stability in the sense of Lyapunov is not guaranteed stems from the fact that the uncertainties in the system dynamics cannot be
perfectly captured by neural networks using the universal function approximation property and the residual approximation error is characterized via a norm bound over a given compact set. Ultimate boundedness guarantees, however, are often conservative since standard Lyapunov-like theorems that are typically used to show ultimate boundedness of the closed-loop hybrid system states provide only sufficient conditions, while neural network controllers may possibly achieve plant state convergence to an equilibrium point.

In this research [16], we develop a neural hybrid adaptive control framework for a class of nonlinear uncertain impulsive dynamical systems which ensures state convergence to a Lyapunov stable equilibrium as well as boundedness of the neural network weighting gains. Specifically, the proposed framework is Lyapunov-based and guarantees partial asymptotic stability of the closed-loop hybrid system; that is, Lyapunov stability of the overall closed-loop states and convergence of the plant state. The neuroadaptive controllers are constructed without requiring explicit knowledge of the hybrid system dynamics other than the fact that the plant dynamics are continuously differentiable and that the approximation error of the unknown system nonlinearities lies in a small gain-type norm bounded conic sector over a compact set. Hence, the overall neuroadaptive control framework captures the residual approximation error inherent in linear parameterizations of system uncertainty via basis functions. Furthermore, the proposed neuroadaptive control architecture is modular in the sense that if a nominal linear design model is available, then the neuroadaptive controller can be augmented to the nominal design to account for system nonlinearities and system uncertainty.

Finally, we emphasize that we do not impose any linear growth condition on the system resetting (discrete) dynamics. In the literature on classical (non-neural) adaptive control theory for discrete-time systems, it is typically assumed that the nonlinear system dynamics have the linear growth rate which is necessary in proving Lyapunov stability rather than practical stability (ultimate boundedness). Our novel characterization of the system uncertainties (i.e., the small gain-type bound on the norm of the modeling error) allows us to prove asymptotic stability without requiring a linear growth condition on the system dynamics.

2.13. Controller Synthesis with Guaranteed Closed-loop Phase Constraints

The ability to address gain and phase uncertainties is essential for maximizing achievable performance in controlling uncertain dynamical systems. The small gain theorem guarantees robust stability by requiring that the loop gain (including desired weighting functions for loop shaping) be less than unity at all frequencies. The small gain theorem, however, does not make use of phase information in guaranteeing stability. To some extent, phase information is accounted for by means of positivity theory. In this theory, a positive real plant and a strictly positive real uncertainty are both assumed to have phase less than 90° so that the loop transfer function has less than 180° of phase shift, hence guaranteeing robust stability in spite of gain uncertainty. Other notable results addressing phase information include concepts such as principal phases, multivariable phase margin, phase spread, phase envelope, phase matching, phase-sensitive structured singular value, and plant uncertainty templates. With the exception of positivity theory all of the aforementioned methods are restricted to frequency domain characterizations and are not amenable to state space formulations necessary for developing controller synthesis methods with guaranteed phase constraints.

In this research [22], we present an analysis and synthesis approach for guaranteeing that the phase of a single-input, single-output closed-loop transfer function is contained in the interval $[-\alpha, \alpha]$ for a given $\alpha > 0$ at all frequencies. Specifically, we first derive a sufficient condition involving a frequency domain inequality for guaranteeing a given phase constraint. Next, we use the Kalman-Yakubovich-Popov (KYP) theorem to derive an equivalent time domain condition. In the case where $\alpha = \frac{\pi}{2}$, we show that frequency and time domain sufficient conditions specialize to the positivity theorem. Furthermore, using linear matrix inequalities (LMIs), we develop a controller synthesis approach for guaranteeing a phase constraint on the closed-loop transfer function. Finally, we extend this synthesis approach to address mixed gain and phase constraints on the closed-loop transfer function.

2.14. Adaptive Control for Nonlinear Uncertain Systems with Actuator Amplitude and Rate Saturation Constraints

In light of the increasingly complex and highly uncertain nature of dynamical systems requiring controls, it is not surprising that reliable system models for many high performance engineering applications are unavailable. In the face of such high levels of system uncertainty, robust controllers may unnecessarily sacrifice system performance, whereas adaptive controllers are clearly appropriate since they can tolerate far greater system uncertainty levels to improve system performance. However, an implicit assumption inherent in most adaptive control frameworks is that the adaptive control law is implemented without any regard to actuator amplitude and rate saturation constraints. Of course, any electromechanical control actuation device is subject to amplitude and/or rate constraints leading to saturation nonlinearities enforcing limitations on control amplitudes and control rates. As a consequence, actuator nonlinearities arise frequently in practice and can severely degrade closed-loop system performance, and in some cases drive the system to instability. These
effects are even more pronounced for adaptive controllers which continue to adapt when the feedback loop has been severed due to the presence of actuator saturation causing unstable controller modes to drift, which in turn leads to severe windup effects.

Many practical applications involve nonlinear dynamical systems with simultaneous control amplitude and rate saturation. The presence of control rate saturation may further exacerbate the problem of control amplitude saturation. For example, in advanced tactical fighter aircraft with high maneuverability requirements, pilot induced oscillations can cause actuator amplitude and rate saturation in the control surfaces, leading to catastrophic failures.

In this research [26], we develop a direct adaptive control framework for adaptive tracking of multivariable nonlinear uncertain systems with amplitude and rate saturation constraints. In particular, we extend the Lyapunov-based direct adaptive control framework developed in [11] to guarantee asymptotic stability of the closed-loop tracking system; that is, asymptotic stability with respect to the closed-loop system states associated with the tracking error dynamics in the face of actuator amplitude and rate saturation constraints. Specifically, a reference (governor or supervisor) dynamical system is constructed to address tracking and regulation by deriving adaptive update laws that guarantee that the error system dynamics are asymptotically stable, and the adaptive controller gains are Lyapunov stable. In the case where the actuator amplitude and rate are limited, the adaptive control signal to the reference system is modified to effectively robustify the error dynamics to the saturation constraints, thus guaranteeing asymptotic stability of the error states.

2.15. A New Neuroadaptive Control Architecture for Nonlinear Uncertain Dynamical Systems

One of the primary reasons for the large interest in neural networks is their capability to approximate a large class of continuous nonlinear maps from the collective action of very simple, autonomous processing units interconnected in simple ways. Neural networks have also attracted attention due to their inherently parallel and highly redundant processing architecture that makes it possible to develop parallel weight update laws. This parallelism makes it possible to effectively update a neural network on line. These properties make neural networks a viable paradigm for adaptive system identification and control of complex highly uncertain systems, and as a consequence the use of neural networks for identification and control has become an active area of research.

The goal of adaptive and neuroadaptive control is to achieve system performance without excessive reliance on system models. Both controller approaches directly or indirectly adjust feedback controller gains and improve system performance in the face of system uncertainty. Specifically, indirect adaptive and neuroadaptive controllers utilize parameter update laws to identify unknown system parameters and adjust feedback gains to account for system variation, while direct adaptive and neuroadaptive controllers adjust the controller gains in response to system variations.

The fundamental difference between adaptive control and neuroadaptive control can be traced back to the modeling and treatment of the system uncertainties. In particular, adaptive control is based on constant, linearly parameterized system uncertainty models of a known structure but unknown variation. This uncertainty characterization allows for the system nonlinearities to be parameterized by a finite linear combination of basis functions within a class of function approximators such as rational functions, spline functions, radial basis functions, sigmoidal functions, and wavelets. However, this linear parameterization of basis functions cannot, in general, exactly capture the unknown system parameters. In such a case, the uncertainty is expressed in terms of a neural network involving a parameterized nonlinearity. Hence, in contrast to adaptive control, neuroadaptive control is based on the universal function approximation property, wherein any continuous nonlinear system uncertainty can be approximated arbitrarily closely on a compact set using a neural network with appropriate weights. This difference in the modeling and treatment of the system uncertainties results in the ability of adaptive controllers to guarantee asymptotic closed-loop system stability versus ultimate boundedness as is the case with neuroadaptive controllers [11].

To improve robustness and the speed of adaptation of adaptive and neuroadaptive controllers several controller architectures have been proposed in the literature. These include the - and -modification architectures used to keep the system parameter estimates from growing without bound in the face of system uncertainty [40]. In this research [40], a new neuroadaptive control architecture for nonlinear uncertain dynamical systems is developed. Specifically, the proposed framework involves a new and novel controller architecture involving additional terms, or -modification terms, in the update laws that are constructed using a moving window of the integrated system uncertainty. The -modification terms can be used to identify the ideal neural network system weights which can be used in the adaptive law. In addition, these terms effectively suppress system uncertainty.

Even though the proposed approach is reminiscent to the composite adaptive control framework, the -modification framework does not involve filtered versions of the control input and system state in the update laws. Rather, the update laws involve auxiliary terms predicated on an estimate of the unknown neural network weights which in turn are characterized by an auxiliary equation involving the integrated error dynamics over a moving
time interval. In this research [40], we consider vector uncertainty structures with both linear and nonlinear parameterizations. In addition, state and output feedback controllers are developed. Finally, to illustrate the efficacy of the proposed approach we apply our results to an aircraft model with wing rock dynamics as well as a spacecraft model involving an unknown moment of inertia matrix and compare our results with standard neuroadaptive control methods.

2.16. $H_2$ Suboptimal Estimation and Control for Nonnegative Dynamical Systems

Nonnegative dynamical systems involve dynamic states whose values are nonnegative. A subclass of nonnegative dynamical systems are compartmental systems. Compartmental systems involve dynamical models that are characterized by conservation laws (e.g., mass, energy, fluid, etc.) capturing the exchange of material between coupled macroscopic subsystems known as compartments. These models are widespread in biological, physiological, and ecological sciences as well as engineering systems such as queuing, large-scale, telecommunications, transportation, power, and network systems, to cite but a few examples. Since nonnegative and compartmental systems have specialized structures, special control law strategies need to be developed that guarantee that the trajectories of the closed-loop plant system states remain in the nonnegative orthant of the state space for nonnegative initial conditions. In addition, for certain applications of nonnegative systems, such as active control for clinical pharmacology, we require the control (source) inputs to be nonnegative.

Even though nonnegative systems are often encountered in numerous application areas, nonnegative orthant stabilizability and holdability has received little attention in the literature. In this research [27], we use linear matrix inequalities (LMIs) to develop $H_2$ (sub)optimal estimators and controllers for nonnegative dynamical systems. Linear matrix inequalities provide a powerful design framework for linear control problems. Since LMIs lead to convex or quasiconvex optimization problems, they can be solved very efficiently using interior-point algorithms. An interesting feature of nonnegative orthant stabilizability is that it can be formulated as a solution to an LMI problem. However, $H_2$ optimal nonnegative orthant stabilizability cannot, in general, be formulated as an LMI problem. In this research [27], we formulate a series of generalized eigenvalue problems subject to a set of LMI constraints for designing $H_2$ suboptimal estimators, static controllers, and dynamic controllers for nonnegative dynamical systems.

2.17. Adaptive Disturbance Rejection Control for Compartmental Systems

As discussed in Section 2.16, nonnegative and compartmental systems are essential in capturing the behavior of a wide range of dynamical systems involving dynamic states whose values are nonnegative. These systems are derived from mass and energy balance considerations and are comprised of homogeneous interconnected microscopic subsystems or compartments which exchange variable quantities of material via intercompartmental flow laws. Since biological and physiological systems have numerous input, state, and output properties related to conservation, dissipation, and transport of mass and energy, nonnegative and compartmental systems are remarkably effective in describing the phenomenological behavior of these dynamical systems. The range of applications of nonnegative and compartmental systems is not limited to biological and medical systems. Their usage includes demographic, epidemic, ecological, economic, telecommunications, transportation, power, and large-scale systems.

In a recent series by the Principal Investigator (see the references in [29]) a direct adaptive control framework for linear and nonlinear nonnegative and compartmental systems was developed. This framework is Lyapunov-based and guarantees partial asymptotic set-point regulation, that is, asymptotic set point stability with respect to the closed-loop system states associated with the plant. In addition, the adaptive controllers guarantee that the physical system states remain in the nonnegative orthant of the state space. In this research [23], we develop a direct adaptive control framework for adaptive stabilization and disturbance rejection for compartmental dynamical systems with exogenous system disturbances. The main challenge here is to construct nonlinear adaptive disturbance rejection controllers without requiring knowledge of the system dynamics or the system disturbances while guaranteeing that the physical system states remain in the nonnegative orthant of the state space.

While such an adaptive control framework can have wide applicability in areas such as economics, telecommunications, and power systems, its use in the specific field of anesthetic pharmacology is particularly noteworthy. Specifically, during stress (such as hemorrhage) in an acute care environment, such as the operating room, perfusion pressure falls and hypertonic saline solutions are typically intravenously administered to regulate hemodynamic effects and avoid hemorrhagic shock. This exogenous disturbance drives the system pharmacokinetics and pharmacodynamics and can be captured as a system disturbance. In addition, exogenous system disturbances can be used to capture unmodeled physiological and pharmacological system dynamics. Although the proposed framework develops adaptive controllers
for general compartmental systems with exogenous disturbances, the specific focus of the research is on pharmacokinetic models with hemorrhage and hemodilution effects.

2.18. Neuroadaptive Output Feedback Control for Automated Anesthesia with Noisy EEG Measurements

The dosing of most drugs is a process of empirical administration of a low dose with observation of the biological effect and subsequent adjustment of the dose in the hopes of achieving the desired effect. This is true of anesthetic drugs, just as it is of chronically administered medications (for example, anti-hypertensive agents). In the acute environment of the operating room and intensive care unit (ICU), this can result in inefficient, and possibly even dangerous, titration of drug to the desired effect. There has been a long interest in use of the electroencephalograph (EEG) as an objective, quantitative measure of consciousness that could be used as a performance variable for closed-loop control of anesthesia [29]. Processed electroencephalogram algorithms have been extensively investigated as monitors of the level of consciousness in patients requiring surgical anesthesia [29]. However, the EEG is a complex of multiple time series and in earlier work it was difficult to identify one single aspect of the EEG signal that correlated with the clinical signs of anesthesia.

Subsequent to this early research there has been substantial progress in the development of processed EEG monitors that analyze the raw data to extract a single measure of the depth of anesthesia. The best known of these monitors is the bispectral or BIS monitor, which calculates a single composite EEG measure that is well correlated with the depth of anesthesia [29]. The BIS signal ranges from 0 (no cerebral electrical activity) to 100 (the normal awake state). Available evidence indicates that a BIS signal less than 55 is associated with lack of consciousness. While BIS monitoring has proven useful in the operating room environment, there have been inconsistencies reported and attempts to extend BIS monitoring for the evaluation of sedation outside of the operating room have been unsuccessful [29]. One of the key reasons for this is due to the fact that the signal-averaging algorithm within the BIS monitor ignores signal noise, and when there is excessive noise, the BIS monitor does not generate a signal.

It is widely appreciated that BIS monitoring, or for that matter, any EEG monitoring, can be fraught with error because of the potential for outside interference to produce an unfavorable signal-to-noise ratio yielding spurious results. Nonphysiologic artifactual signals may be generated from sources external to the patient that include lights, electric cautery devices, ventilators, pacemakers, patient warming systems, and electrical noise related to the many different kinds of monitors normally found in an operating room or ICU. Physio-

logic movements such as eye movements, myogenic activity, perspiration, and ventilation can produce artifactual increases in the BIS score. In particular, it is apparent that electromyographic (EMG) activity can spuriously increase the BIS score [29]. The co-administration of neuromuscular blockade eliminates artifacts from muscle movement, which can be superimposed on the BIS score; and this undoubtedly contributes to the widespread use and value of the BIS device during surgery. However, to extend this technology outside of the operating room, or for that matter, to nonparalyzed patients in the operating room, further refinements are needed. In addition, if the BIS signal is to be used to quantify levels of consciousness for feedback control in general anesthesia, then the observation noise needs to be accounted for in the control system design process.

The challenge to the use of the BIS signal for closed-loop control of anesthesia is that the relationships between drug dose and tissue concentration (pharmacokinetics) and between tissue concentration and physiological effect (pharmacodynamics) is highly variable between individuals. In addition, observation noise in the BIS signal results in feedback measurement signals with high signal-to-noise ratios that need to be accounted for in the control algorithm. Adaptive feedback controllers seem particularly promising given this interpatient variability as well as BIS signal variation due to noise. In previous work, we have used nonnegative and compartmental dynamical systems theory to develop adaptive and neuroadaptive controllers for controlling the depth of anesthesia [49]. One of our initial efforts was the development of a direct adaptive control framework for uncertain nonlinear nonnegative and compartmental systems with nonnegative control inputs [49]. This framework is Lyapunov-based and guarantees partial asymptotic set-point regulation, that is, asymptotic setpoint stability with respect to part of the closed-loop system states associated with the physiological state variables. In addition, the adaptive controllers, which are constructed without requiring knowledge of the pharmacokinetic and pharmacodynamic parameters, provide a nonnegative control input for stabilization with respect to a given setpoint in the nonnegative orthant. Subsequently, we also developed a neuroadaptive output feedback control framework for uncertain nonlinear nonnegative and compartmental systems with nonnegative control inputs [7]. This framework is also Lyapunov-based and guarantees ultimate boundedness of the error signals corresponding to the physical system states in the face of interpatient pharmacokinetic and pharmacodynamic variability.

In a recent paper [49] we presented numerical and clinical results that compares and contrasts our adaptive control algorithm with our neural network adaptive control algorithm for controlling the depth of anesthesia in the operating theater during surgery. Specifically eleven clinical trials were performed with our adaptive control algorithm and seven clinical trials were performed with our neural network algorithm at the Northeast Georgia Medical
Center in Gainesville, Georgia. The proposed automated anesthesia controllers demonstrated excellent regulation of unconsciousness and allowed for a safe and effective administration of the anesthetic agent propofol. However, the adaptive and neuroadaptive controllers presented in [49] did not account for measurement noise in the EEG signal. Clinical testing has clearly demonstrated the need for developing adaptive and neuroadaptive controllers that can address system measurement noise [40].

In this research [37], we extend the neuroadaptive controller framework developed in [7] to address measurement noise in the BIS signal. Specifically, we develop an output feedback neural network adaptive controller that operates over a tapped delay line (TDL) of available input and filtered output measurements. The neuroadaptive laws for the neural network weights are constructed using a linear observer for the nominal normal form error system dynamics. The proposed framework is Lyapunov-based and guarantees ultimate boundedness of the error signals. In addition, the nonnegative neuroadaptive controller guarantees that the physiological system states remain in the nonnegative orthant of the state space. Finally, we present numerical and clinical results for controlling the depth of anesthesia in the operating theater during surgery. The proposed automated anesthesia neuroadaptive controller demonstrates excellent regulation of unconsciousness and allows for a safe and effective administration of the anesthetic agent propofol in the face of noisy EEG measurements.

2.19. Direct Adaptive Control for a Multi-Compartamental Model of a Pressure-Limited Respirator and Lung Mechanics System

Mechanical ventilation of a patient with respiratory failure is one of the most common life-saving procedures performed in the intensive care unit. However, mechanical ventilation is physically uncomfortable due to the toxic interface between the ventilator and patient and mechanical ventilation evokes substantial anxiety on the part of the patient. This will often be manifested by the patient “fighting the ventilator.” In this situation, there is dysynchrony between the ventilatory effort of the patient and the ventilator. The patient will attempt to exhale at the time the ventilator is trying to expand the lungs or the patient will try to inhale when the ventilator is decreasing airway pressure to allow an exhalation. When patient-ventilator dysynchrony occurs, at the very least, there is excessive work of breathing with subsequent ventilatory muscle fatigue and in the worst case, elevated airway pressures that can actually rupture lung tissue. In this situation, it is a very common clinical practice to sedate patients to minimize “fighting the ventilator.” Sedative-hypnotic agents act on the central nervous system to ameliorate the anxiety and discomfort associated with mechanical ventilation and facilitate patient-ventilator synchrony. In this research, we developed an adaptive feedback controller for alleviating the dysynchrony.

In a recent paper [56], we extended the existing models for ventilation systems, to obtain a general mathematical model for the dynamic behavior of a multi-compartment respiratory system in response to an arbitrary applied inspiratory pressure. Specifically, we used compartmental dynamical system theory to model and analyze the dynamics of a pressure-limited respirator and lung mechanics system, and showed that the periodic orbit generated by this system is globally asymptotically stable. Furthermore, we showed that the individual compartmental volumes, and hence the total lung volume, converge to steady-state end-inspiratory and end-expiratory values. In this research [73], first, we develop a model reference direct adaptive controller framework where the plant and reference model dynamics are switching and time-varying. Next, we apply the proposed adaptive framework to the multi-compartmental model of a pressure-limited respirator and lung mechanics system. Specifically, we develop an adaptive feedback controller that stabilizes a given limit cycle corresponding to a respiratory pattern identified by the clinician as a plausible breathing pattern. Finally, we provide simulations that quantify dysynchrony in a controlled mechanical ventilator model.

3. Research Personnel Supported

Faculty
Waseem M. Haddad, Principal Investigator

Graduate Students
Qing Hui, Ph. D.

Several other students (K Volanskiy, J. J. Im, and H. Li) were involved in research projects that were closely related to this program. Although none of these students were financially supported by this program, their research did directly contribute to the overall research effort. Furthermore, one Ph. D. dissertation was completed under partial support of this program, namely

Q.Hui, Nonlinear Dynamical Systems and Control for Large-Scale, Hybrid and Network Systems, August 2008.

Dr. Hui is presently an Assistant Professor of Mechanical Engineering at Texas Tech.
4. Interactions and Transitions

4.1. Participation and Presentations

The following conferences were attended over the past three years.

American Control Conference, Minneapolis, MN, June 2006.
IEEE Conference on Decision and Control, San Diego, CA, December 2006.
IEEE Conference on Decision and Control, Cancun, Mexico, December 2008.

Furthermore, conference articles [44-74] were presented.

4.2. Transitions

Our work on adaptive and neuroadaptive control of drug delivery partially supported
under this program has transitioned to clinical studies at the Northeast Georgia Medical
Center in Gainesville, Georgia, under the direction of Dr. James M. Bailey (770-534-1312),
director of cardiac anesthesia and consultant in critical care medicine. To date, we have
performed over forty clinical trials.

In critical care medicine it is current clinical practice to administer potent drugs that
profoundly influence levels of consciousness, respiratory, and cardiovascular function by manu-

nal control based on the clinician’s experience and intuition. Open-loop control by clinical

personnel can be tedious, imprecise, time-consuming, and sometimes of poor quality, de-

pending on the skills and judgment of the clinician. Military physicians may face the most
demanding of critical care situations when dealing with the casualties of hostile action and

in these situations, precise control of the dosing of drugs with potent cardiovascular and

central nervous system effects is critical.

It has been an aphorism among anesthesiologists since World War II that “thiopental (a

common drug for the induction of anesthesia) killed more Americans at Pearl Harbor than

the enemy,” referring to the consequences of cardiovascular collapse induced by thiopental

in trauma patients. Furthermore, military medicine faces unique challenges compared to the
civilian sector. The necessity of triage has, sadly, been a not rare event in times of war due to
unexpected numbers of casualties overwhelming available resources and furthermore, health
care providers may be among the casualties. Because of the possibility of demands on health
care providers that may exceed local resources, we believe that it is crucial to investigate

the use of advanced control technology to extend the capabilities of the health care system
to handle large numbers of casualties. Closed-loop control based on appropriate dynamical
system models can improve the quality of drug administration in surgery and the intensive
care unit, lessening the dependence of patient outcome on the skills of the clinician.

This work was recently communicated to Colonel Leopoldo C. Cancio (210-916-3301) of
the US Army Institute of Surgical Research in Fort Sam Houston, San Antonio, in order to
provide improvements for combat casualty care in current and future battlefields. Transition
discussions are ongoing.
5. Research Publications

5.1. Journal Articles and Books


### 5.2. Conference Articles


