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THE PHYSICAL BASIS OF THE LEAP SECOND

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ABSTRACT

International Atomic Time (TAI) is the internationally recognized timescale based on the second of the Système International d'Unités produced by the Bureau International des Poids et Mesures using data from timing laboratories around the world. TAI is an atomic timescale without steps. Coordinated Universal Time, the basis of civil time, is derived from TAI but is currently defined such that it is maintained within 0.9 s of Universal Time (UT1), the measure of time defined by the Earth's rotation angle, through the insertion of 1 s increments called leap seconds. The difference between UT1 and TAI that motivates the use of leap seconds is related to the tidal deceleration of the Earth's rotation. However, a recent paper by Deines and Williams claims that the divergence is caused by a relativistic time dilation effect. The purpose of this paper is to explain the physical basis of the leap second and to point out that leap seconds are unrelated to relativity.

Key words: celestial mechanics – Earth – reference systems – relativity – time

1. INTRODUCTION

International Atomic Time (TAI) is a timescale maintained by the Bureau International des Poids et Mesures (BIPM) using clock information contributed by metrological laboratories and observatories around the world. TAI is an atomic timescale without steps whose unit interval is the second in the Système International d'Unités (SI). Coordinated Universal Time (UTC), the fundamental scale of civil time today, is an atomic timescale that is derived from TAI. It is occasionally adjusted by the insertion of 1 s steps, called leap seconds, to keep it close to Universal Time (UT1), the measure of the Earth's rotation angle expressed in time units, which is only treated conventionally as an astronomical timescale.

A recent paper by Deines and Williams (2007) claims that the difference between UT1 and TAI is caused by a relativistic time dilation effect. They dispute the long-accepted explanation that leap seconds are by-products of the secular slowing of the Earth's rotation rate. The purpose of this paper is to explain the physical basis of the leap second and to point out that leap seconds are unrelated to relativity. The tidal deceleration of the Earth's rotation rate remains the most significant contribution to the secular change in the length of the day, which, in addition to the way in which the SI second is defined, causes "Earth rotation time" (UT1) and atomic time (TAI) to diverge.

2. TIMESCALES

As the requirements for uniform timescales in astronomy and physics have grown, the concepts and definitions for timescales have also continued to evolve.

2.1. Universal Time

UT1 describes the Earth's rotation by its relationship to the Earth rotation angle θ through the identity

$$\mathrm{UT1} = \mathrm{UTC} + (\mathrm{UT1} - \mathrm{UTC}) = \mathrm{TAI} + (\mathrm{UT1} - \mathrm{TAI})$$

and by using the conventional expression (Capitaine et al. 1986, 2003)

$$\theta(t_U) = 2\pi(0.7790572732640 + 1.00273781191135448\, t_U),$$

where t_U is defined as Julian UT1 date—2 451 545.0.

The rate of rotation of the Earth is not uniform. There are three types of variations: (1) a secular deceleration, (2) a periodic variation, and (3) random fluctuations.

From the time of Halley (1693, 1695), astronomers realized that there were problems in explaining the apparent secular acceleration in the Moon's longitude. Kant (1754) was the first to suggest that the action of the tides raised by the Moon on the Earth should slow the Earth's rotation rate and that this process might be reflected in observations of the Moon's motion. However, it took more than 100 years before the retardation of the Earth's rotation was suggested independently by Delaunay (1859, 1866) and Ferrel (1865) as an explanation of the fact that the mean motion of the Moon predicted from celestial mechanics was significantly less than the mean motion determined from the previous 2500 years of observations. They explained the secular acceleration as a result of the tidal retardation of the Earth's rotation and the consequent variation in the orbital velocity of the Moon according to the conservation of angular momentum. Stephenson & Morrison (1995) point out that over the past 2700 years, the length of day (LOD) has increased at an average rate of 1.7 ms per day per century and that geophysical analyses of tidal braking indicate that lunar tidal deceleration should contribute an increase of 2.3 ms per day per century. The discrepancy of 0.6 ms per day per century is apparently caused by the changing of the shape of the Earth in response to glacial melting. A comprehensive review is given by Stephenson (1997).

The periodic variation in the Earth's rotation rate is associated with the circulation of the atmosphere, which causes a seasonal variation in the length of the day on the order of 0.5 ms per day about the mean. The rotation of the Earth runs slow by about 30 ms in May and runs fast by a similar amount in November. In addition, the Earth's LOD is subject to frequent, apparently random changes of less than a few tenths of a millisecond per day. These fluctuations typically persist for about a decade.

Observations of the difference between astronomical time and atomic time, UT1 − TAI, are used to describe the variations in the Earth's rotation. The difference of UT1 − TAI is determined by using very long baseline interferometry (VLBI) measurements of the orientation of the Earth with respect to the celestial reference frame defined by the directions to quasars. The numerical expression for UT1 is consistent with the IAU 2000/2006 precession and nutation models, and was obtained following a procedure that ensured consistency at the microarcsecond level with the previous expression as well as continuity in UT1 at the date of change on 2003 January 1 (Capitaine et al. 1986, 2003).

2.2. *Ephemeris Time*

Because the Earth's rotation rate does not meet the need for a uniform timescale, ephemeris time (ET) was originally conceived in the 1950s as a nonrelativistic astronomical timescale realized by the motions of the celestial bodies in the solar system (Clemence 1971). ET may be characterized as the measure of time that brings the observed positions of celestial bodies into accord with their positions computed according to the Newtonian laws of dynamics; in effect, it is defined by these laws (Explanatory Supplement 1961, p. 68).

On the basis of Newcomb's formula for the geometric mean longitude of the Sun, the second of ET was defined by the 11th Conférence Générale des Poids et Mesures (CGPM) in 1960 as "the fraction 1/31,556,925.9747 of the tropical year for 1900 January 0 at 12 hr ephemeris time" (BIPM 2006, p. 149). Newcomb's formula was derived from astronomical observations performed over the interval from 1750 to 1892 (Newcomb 1895a, 1895b). Consequently, the second of ET had the same duration as a second of UT1 that would have been observed in about 1820, the approximate mean epoch of the observations analyzed by Newcomb. (Note, incidentally, that 1900 was the epoch of a tropical year of 31,556,925.9747 s of ET, while 1820 was the epoch of a LOD of 86,400 s of UT1.)

2.3. *Relativistic Timescales*

The theory of general relativity distinguishes between two kinds of time. Proper time is the reading of an ideal clock as realized, for example, by an atomic clock. Coordinate time is the time coordinate used in a particular four-dimensional coordinate system. It is the independent variable of the equations of motion of elementary particles and celestial bodies and of the equations of propagation of electromagnetic signals. The choice of coordinate time is arbitrary. It is suggested merely by convenience and the geometry of the problem under investigation. In any particular coordinate system, the relation between proper time and coordinate time is given by the invariant spacetime interval.

Coordinate time is an intermediate variable that is ultimately eliminated from a problem, as time comparisons can be made only between two clocks, each of which registers proper time. Similarly, coordinate time is used in relating the positions of the celestial bodies that are governed by the principles of relativistic dynamics. In a particular coordinate system, the coordinate time may be determined by those positions.

ET did not include relativistic effects, nor did it distinguish between proper time and coordinate time. The eventual need for such considerations led to a series of improvements in the concept of ET that resulted in the definition of timescales that do account for relativity.

Three relativistic timescales are of importance. (1) Terrestrial Time (TT) is the coordinate time associated with a coordinate system whose origin is on the surface of the Earth. Its rate corresponds to the rate of proper time given by an ideal clock on the geoid. A practical realization of TT is TT = TAI + 32.184 s. (2) Geocentric Coordinate Time (TCG) is the coordinate time associated with a coordinate system whose origin is at the center of the Earth. TCG and TT differ in rate by a defined constant (IERS Conventions 2004). (3) Barycentric Coordinate Time (TCB) is the coordinate time associated with a coordinate system whose origin is at the barycenter of the solar system. In practice, a relativistic ephemeris of planetary motion can be computed using TCB as the independent variable. TCB can then be related to TT by making the appropriate relativistic transformations (IERS Conventions 2004). These timescales were established by the International Astronomical Union at General Assemblies held between 1976 and 2000.

3. TIDAL DECELERATION OF THE EARTH'S ROTATION

One must account for the deceleration of the Earth's rotation in order to determine the time and place of ancient astronomical events. Although the deceleration is very small, its effect is cumulative. Over the past 2000 years, the error in UT1 caused by the long-term trend in the Earth's rotational deceleration is roughly 3 hr. For example, if a uniform rate of rotation were assumed, the calculated path of the total eclipse of the Sun witnessed in Babylon in 136 BCE would be displaced by $48\overset{\circ}{.}8$, corresponding to a time difference of 11,700 s. The record of this eclipse is preserved in two tablets, which are now in the British Museum, and is the most reliable account of any ancient solar eclipse (Stephenson 1997, pp. 64–67).

The trend in the increase in the LOD has persisted virtually from the time of formation of the Earth–Moon system (Jeffreys 1976, pp. 339–340). Evidence in the form of coral fossils that have both daily and annual growth rings from the middle of the Devonian period, some 370 million years ago, indicates that the number of days in the year was between 385 and 410 (Runcorn 1966). This evidence suggests that the rate of deceleration was about the same then as it is now. Stephenson et al. (1984) provide the mathematical relationships among accumulated time difference, rotational deceleration, and excess length of day in tabular form for convenient reference. Thus tidal deceleration fully accounts for the observed deceleration of the Earth's rotation, as attested by observations spanning hundreds of millions of years.

4. THE LEAP SECOND

The SI second was defined by the 13th CGPM in 1967 as "the duration of 9192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium 133 atom" (BIPM 2006, p. 153). This definition was based on the calibration of the frequency of the radiation using the second of ET as realized by astronomical observations of the Moon (Markowitz et al. 1957). For all practical purposes, the durations of the SI second and the ET second are the same. However, as stated in Section 2.2, the second of ET was equivalent to the second of UT1 in about 1820. Thus, due to the circumstances of its historical evolution, the SI second is also equivalent to the second of UT1 in about 1820. Consequently, the LOD was 86,400 SI seconds in about 1820.

But as the rotation rate of the Earth is constantly slowing, with a corresponding increase in the LOD of approximately 1.7 ms per day per century, the day is now on average roughly 86,400.0025 SI seconds in duration, or 2.5 ms longer than it was in 1820. This difference accumulates to about 1 s in a year. It is this difference that is compensated by the occasional insertion of a leap second into UTC. Therefore, the leap second is motivated by the fact that the SI second is now shorter than the second defined by observations of the Earth's rotation angle (Nelson et al. 2001).

UTC is defined to have the same rate as TAI but is kept within 0.9 s of UT1 by inserting leap seconds as needed. The decision to introduce a leap second is made by the International Earth Rotation Service (IERS). Thus, UTC is offset from TAI by an integral number of seconds. Presently (2008), TAI is ahead of UTC by 33 s.

Since 1999, there has been only one leap second (in 2005) because of fluctuations in the rotation of the Earth caused by internal processes. Due to variations in the LOD that persist for about a decade, the LOD has recently been closer to 86,400 s and leap seconds have not been necessary. As the trend in the increase in the LOD has been inexorable, it is anticipated that under the current definition of UTC, multiple leap seconds will be needed in the near future, with perhaps more than one leap second in a single year to compensate for those that have been omitted in the recent past. In fact, a leap second is scheduled to be implemented on 2008 December 31.

5. SIGNIFICANCE OF RELATIVITY, UT1, AND ET

Deines & Williams (2007) base their claim that time dilation is responsible for leap seconds on a mathematical expression that they developed for the difference between a proper timescale on the Earth's surface and a coordinate timescale at the solar system barycenter. They attempt to derive this expression from the relativistic spacetime metric for an accelerated, rotating coordinate system derived by Nelson (1990). However, they mistakenly associate UT1, a measure of the Earth's angle of rotation, with proper time, and confuse ET, a Newtonian timescale that did not distinguish among reference frames, with a relativistic coordinate time defined with respect to the solar system barycenter.

The argument of Deines & Williams (2007) fails to recognize that UT1 is the rotation angle of the Earth. According to modern astronomical interpretations, UT1 is treated as a measure of time only by convention. In practice, no clock provides UT1, and it is realized by astronomical observations of the Earth's orientation made in a celestial reference frame taking into account the appropriate defining relationships. In view of its definition, UT1 cannot be regarded as a measure of proper time.

Their argument also fails to recognize that ET lacks a relativistic definition and is based on observations made from the surface of the Earth. The closest relativistic parallel to ET would be TT, not TCB. Thus, the assertion made by Deines and Williams (2007) is a misapplication of the concepts of proper time and coordinate time. There is no foundation for the conclusion that time dilation is the cause of the divergence between an atomic timescale and a timescale based on the Earth's rotation.

6. CONCLUSION

The two key points we wish to emphasize are the following:

1. Tidal deceleration fully accounts for the observed deceleration of the Earth's rotation, as demonstrated by observations spanning hundreds of millions of years.
2. The leap second is motivated by the fact that the SI second is now shorter than the second defined by observations of the Earth rotation angle.

These points are based on the fact that the SI second is equivalent to an older measure of the second of UT1, which was too small to start with and further, as the duration of the UT1 second increases, the discrepancy widens. Contrary to the assertions of Deines & Williams (2007), the divergence of UT1 and TAI is unrelated to relativity.

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