A novel diagnostic technique has been developed, utilizing the Thrust Stand Mass Balance, to directly measure a time accurate mass flow from a solid-fuel thruster for systems where the mass flow rate is of the same order as the experimental error. The mass flow measurement technique has been verified using an idealized numerical simulation. Two calibration experiments have been performed to assess the dynamic response of the mass balance. First, a set of calibration weights were placed on the mass balance and removed in order to properly characterize the mass balance motion. Second, a known mass flow rate of water was deposited onto the test stand. As a proof of concept experiment, a 3.81 cm diameter PMMA/GOx hybrid thruster core was burned and the propellant mass flow was measured. Variations in the GOx flow rate resulted in corresponding variations in the total propellant mass flow as expected, showing the utility of the Thrust Stand Mass Balance as a mass flow measurement device.
Thrust Stand Mass Balance Measurements of Hybrid Motor Mass Flow

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A novel diagnostic technique has been developed, utilizing the Thrust Stand Mass Balance, to directly measure a time accurate mass flow from a solid-fuel thruster. The mass flow measurement technique has been verified using an idealized numerical simulation. Two calibration experiments have been performed to assess the dynamic response of the mass balance. First, a set of calibration weights were placed on the mass balance and removed in order to properly characterize the mass balance motion. Second, a known mass flow rate of water was deposited onto the test stand. As a proof of concept experiment, a 3.81cm diameter PMMA/GOx hybrid thruster core was burned and the propellant mass flow was measured. Variations in the GOx flow rate resulted in corresponding variations in the total propellant mass flow as expected, showing the utility of the Thrust Stand Mass Balance as a mass flow measurement device.

Nomenclature

$\delta_n$ = logarithmic decrement [n.d]
$\zeta$ = damping ratio [n.d.]
$\omega_n$ = natural frequency [rad/s]
$\omega_d$ = damped frequency [rad/s]
$C$ = torsional damping coefficient [N m s/rad]
$F_C$ = static calibration force [N]
$g_o$ = acceleration due to gravity on earth [m/s$^2$]
$I$ = mass moment of inertia [N m s$^2$/rad][kg m$^2$/rad]
$I_{sp}$ = specific impulse [s]
$I_{tot}$ = total impulse [N s]
$K$ = torsional spring constant [N m/rad]
$M_o$ = steady state forcing moment [N m]
$m_p$ = mass flow of propellant [kg/s]
$\Delta m_p$ = total propellant mass loss [kg]
$m_C$ = mass of steady state calibration weight [kg]
$R_C$ = radial distance from pivot to calibration load [m]
$R_L$ = radial distance from pivot to LVDT [m]
$R_R$ = radial distance from pivot to test load [m]
$I$ = thrust [N]
$X$ = measured LVDT deflection [m]

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I. Introduction

Widespread use of hybrid rockets as safe, low cost, and flexible alternatives to liquid and solid thrusters [1,2] has prompted the study of a novel diagnostics tool for their investigation. This study used a scaled hybrid motor as a proof of concept for the time resolved, direct measurement of propellant mass flow. Although technologies have been developed to accurately measure the thrust [3,4,5] of small-scale propulsion devices, significant barriers have prohibited an accurate means of determining the mass flow of solid propellants, which is an important quantity for determining the specific impulse of the motor. In the case of most gaseous and liquid fuel thrusters, the mass flow of the propellant and oxidizer during a burn can be measured by a variety of well-characterized, time resolved mass flow techniques. The problem is more difficult when applied to solid propellant motors where standardized tools do not exist.

To determine the propellant mass flow from a solid system, current techniques rely on known oxidizer flow rates or chamber pressures to extrapolate the propellant mass flow over time or take an averaged mass flow rate based on the total mass loss and time of burn [2,6]. To eliminate some error [7] and allow for direct measurement of the propellant mass flow as a function of time, a Thrust Stand Mass Balance (TSMB) has been developed for systems where the experimental uncertainty is of the same order as the measurement to be made. In this study, the hybrid thruster core is aligned such that the thrust generated is perpendicular to the motion of the TSMB. In other words, the thrust generally does not contribute to the deflection of the balance. As the solid propellant burns, its change in mass is measured. The displacement of the test stand varies directly with the change in mass of the hybrid thruster system. By analyzing the displacement of the test stand as that of a damped spring-mass system, the forcing function can be derived. From investigating the rate of change of the forcing function, the mass flow rate can be measured.

II. Theoretical Framework

The TSMB behaves like an under-damped, oscillating, mass-spring system, whose behavior can be described by the following second order differential equation:

\[ I \ddot{\theta}(t) + C \dot{\theta}(t) + K \theta(t) = M(t) = F(t) \]

(1)

For small angles of deflection, \( \theta \), the motion of the stand can be linearly approximated through the small angle identity as \( \sin(\theta) = \theta = \frac{X}{r} \). Keeping in mind the left side of equation (1) is related to the measurement of the stand’s motion and the right side is related to its forcing, the stand’s motion can be written as:

\[ I \frac{\ddot{X}(t)}{R_L} + C \frac{\dot{X}(t)}{R_L} + K \frac{X(t)}{R_L} = F(t)R_R \]

(2)

Typical motion of the free moving TSMB can be seen in Figure 1 where \( T \) represents the natural period of the mass balance and \( X_i \) is the magnitude of the deflection.

![Figure 1. Typical damped oscillation of the TSMB.](image)

In order to derive the forcing function from a measured deflection, the coefficients in equation (2) must be experimentally measured from the TSMB. By applying a steady-state calibration force to the stand, the resulting deflection, as time goes to infinity, becomes:

\[ K \frac{X}{R_L} = F_C R_R \]

(3)
The remaining two coefficients can be found by analyzing the measured period and the decay of the deflection amplitude as a function of time. First, a logarithmic decrement is defined as

\[ \delta_n = \ln \left( \frac{X_i}{X_{i+n}} \right) \]  

(4)

where \( i \) is a positive integer, and \( X_i \) and \( X_{i+n} \) are peak amplitudes, separated by \( n \) periods. The damping ratio is then given by

\[ \zeta = \frac{\delta_n}{\sqrt{(n \cdot 2\pi)^2 + \delta_n^2}} \]  

(5)

The measured frequency of the stand can be found from the measured period and related to the natural frequency by

\[ \frac{2\pi}{T} = \omega_d = \omega_0 \sqrt{1 - \zeta^2} \]  

(6)

and the mass moment of inertia and damping coefficients are

\[ I = \frac{K}{\omega_0^2} \]  

(7)

\[ C = 2\zeta \sqrt{KI} \]  

(8)

Therefore, the coefficients \( I, C, \) and \( K \) may be determined from any single test trace resulting from a known, static, steady state, calibration load. It follows that if the deflection, \( X(t) \), resulting from a dynamic load, is measured on a stand with no significant change in \( I, C, \) or \( K \), the time dependent function \( F(t) \), which describes that dynamic load, may be determined through equation (2). By assuming that the stand is only forced by the change in mass of the thruster system, the rate of change of \( \Delta m_p \) is simply the time derivative of the forcing function divided by the gravitation constant

\[ \dot{m}(t) = \frac{\dot{F}(t)}{g_0} \]  

(9)

Before proceeding with the experiments in the lab, the diagnostic technique was verified using an idealized numerical model free of experimental noise and uncertainty. The system in equation (1) can be solved numerically for a steady state forcing function as [8]:

\[ \theta(t) = \frac{M_0}{K} e^{\alpha t} \left[ \theta_0 \cos(\beta t) + \frac{\theta_0 K - \alpha K \theta_0 + \alpha M_0}{K \beta} \sin(\beta t) \right] \]  

(10)

\[ \dot{\theta}(t) = e^{\alpha t} \left[ \theta_0 \cos(\beta t) + \frac{I \alpha \dot{\theta}_0 - K \theta_0 + M_0}{I \beta} \sin(\beta t) \right] \]  

(11)

\[ \ddot{\theta}(t) = e^{\alpha t} \left[ \frac{M_0 - C \dot{\theta}_0 - K \theta_0}{I} \cos(\beta t) + \frac{\alpha (M_0 - C \dot{\theta}_0 - K \theta_0) - K \dot{\theta}_0}{I \beta} \sin(\beta t) \right] \]  

(12)

where \( \alpha = -C/2I \) and \( \beta = \sqrt{K/I - \alpha^2} \).

The model simulated an arbitrary static (short duration) mass changing in value from 0g to 1.5g, 2.0g, 0.5g, 0g, 2.5g, 1.25g, and finally back to 0g at the end of the trace. The results from the simulated stand motion can be seen in Figure 2 sampled at both 60Hz and 1000Hz. The diagnostic technique correctly derived the constants \( K, C, \) and \( I \) within less than 1% of the simulated values. An overshoot in the derived forcing function can be seen in the 60Hz simulation. Since the technique properly derives the simulated constants, the source of the overshoot must lie in the numerical technique used to acquire the derivatives of the displacement. The simple rise over run scheme employed caused the overshoot when applied to the discretized sine wave associated with the sampled stand motion. By increasing the simulation sample rate to 1000Hz, the ringing is greatly reduced.

III. Experimental Setup and Methods

The TSMB has been designed based on the nNTS [5] and NIBS [8] systems, which measure thrust, or impulse, using a torsion pendulum that pivots around a vertical axis of rotation. A force imparted to those stands results in a horizontal displacement of the stand, which can be precisely measured using a linear variable differential transformer (LVDT). Robust techniques have been developed using these systems to perform thrust and impulse measurements for forces as low as 80 nano-Newton (steady state) or 7 nano-Newton-seconds (impulse).
TSMB used in this study differs from these setups in that the arms of the stand are set on a horizontal axis of rotation and have been greatly increased in dimension and sturdiness for the hybrid thruster application.

![Derived forcing functions from simulated stand traces. A) 60Hz B) 1000Hz](image)

Figure 2. Derived forcing functions from simulated stand traces. A) 60Hz B) 1000Hz

A major sizing factor for this test setup was a balance between the rigidity of the torsional flexures and the maximum deflection of the LVDT measurement device. In order to give greater moment of inertia as well as strength, the arms of the stand were scaled up from previous iterations. The goal for this setup was to measure a total mass loss of approximately 2.5 grams (i.e. a force of 25.5mN) at a radial distance of approximately 45cm.

A simple magnetic system was used to damp the stand. The movement of a conductive non-ferrous metal plate attached to the stand, through a permanent magnetic field created by two stationary rare earth magnets, induces eddy currents within the metal plate. These currents set up opposing magnetic fields that resisted the motion of the plate, and thus the stand. The magnitude of this damping could be adjusted by changing the moment arm where the plates were attached. Gaseous oxidizer for the hybrid thruster was delivered to the stand through a small flexible tube aligned with the axis of rotation to minimize its effect on the stand’s motion. Oxidizer flow rate was measured using an Omega 5000SCCM flow meter. A schematic of the experiment can be seen in Figure 3.

![Schematic of the experimental setup](image)

Figure 3. Schematic of the experimental setup
Static calibration was accomplished using four calibration masses of approximately 0.5g, 1.0g, 1.5g, and 2.0g. Each mass was measured by a NIST calibrated digital scale with a resolution of 0.1mg. Static calibration traces took 180 seconds each, consisting of 45 seconds unloaded, 120 seconds with the calibration mass on the stand, and 15 seconds with the calibration masses removed (i.e. unloaded). The beginning and end unloaded states were compared to ensure that thermal or electronic drift was negligible during the calibration. The resulting displacement was measured as a voltage difference by the LVDT and sampled by a data acquisition unit at 60 Hz. Static calibration traces were taken for each of the 4 calibration masses before and after each dynamic load test to verify that the stand properties did not change during the experiment. The static calibrations yielded the constants in equation (2).

Dynamic calibration was conducted using a known volumetric flow rate of water or ethanol from a graduated syringe. The stream from the syringe was directed onto the mass balance, with care taken to ensure the entire stream impacted the receiving cup. These traces were conducted over 210 seconds. The first 45 seconds were used to measure the unloaded value of the stand, after which the syringe was depressed by hand, expelling its contents in roughly 15 seconds. The final value of the stand deflection represented the loaded value of the stand. The mass of the syringe was measured before and after the trace (full and empty) by a NIST calibrated digital scale. For the water stream traces, a high-speed camera recorded the position of the plunger in the barrel of the syringe at a rate of 120 frames/second. The frame at which the plunger passed the hash marks was marked, from which the volumetric flow rate of the water was taken. This calibration allowed the TSMB mass flow measurement technique to be verified.

The arbitrary dynamic test conducted was the proof of concept burns of a hybrid thruster core. The propellant core consisted of a polymethyl methacrylate (PMMA) fuel cylinder with gaseous oxygen as the oxidizer. The PMMA propellant core cylinder was 5.08cm long and 3.81cm in diameter. A center bore was drilled out of the cylinders to a diameter of 0.635cm. The test casing was manufactured from a copper tube with an inner diameter of 3.81cm and a length of 6.35cm. An orifice plate with a 1.27cm hole was welded to one end while a removable cover with the oxidizer inlet line was fitted to the other. A Ni-chrom wire was used to ignite the test core. The test case and a fuel core can be seen schematically in Figure 4.

The hybrid test traces were also 210 seconds long. The first 25 seconds measured the unloaded deflection of the stand, followed by 45 seconds with only the oxidizer flowing to verify that it was not affecting the mass balance. The core was then ignited and allowed to burn for approximately 20 seconds, after which the oxidizer was turned off and the stand was allowed to settle. The oxidizer flow rate was varied from 2000 to about 4000 SCCM. The mass of the test assembly (casing, igniter, ignition wires, and core) was measured on the digital mass balance before and after each burn. For some of the tests the flow rate was started at 2000SCCM and increased to 4000SCCM half way through the burn to investigate whether the corresponding change in propellant mass flow was seen by the TSMB.

The dynamic calibration and proof of concept data were analyzed in a similar fashion. The voltage as a function of time was translated into linear displacement in meters as a function of time, X(t). The first and second derivatives of the function were taken using standard numerical differentiation methods. These time-dependent functions were combined with the pertinent values for I, C, K, R_L, and R_R, as developed in equation (2). This approach yields the forcing function, F(t), and ultimately allows for a derivation of  Özel(m(t)). Values of ∆m_p were determined by two methods: (i) computing a numeric time integral of  Özel(m(t)) over the course of an entire burn (or water test) or (ii) measuring the change in the stands deflection between its unloaded and loaded states. These ∆m_p values were compared to those found using the digital scale.

IV. Results and Discussion

Although the constants for I, C, and K varied from test to test and these test specific constants were used in the analysis of the test runs, typical values were determined for the TSMB. One complication of the TSMB as used in
this study was that the mass moment of inertia changed as the stand added or removed mass. However, the change in mass moment was small compared to the overall moment of inertia of the TSMB of 0.183±.004 Nms²/rad. The value for the TSMB spring constant was found to be 4.81±.07 Nm/rad. This is consistent with the published value from the spring manufacturer of 5.2±.5 Nm/rad for two springs. The damping coefficient was 0.068±.002 Nms/rad.

The results of the dynamic tests exhibit a ringing in the derived forcing function. For this study, a data acquisition system with high resolution (24 bit) for the LVDT voltage measurements was preferred. Unfortunately, the high-resolution data acquisition system had a sample rate limited to 60 Hz. The ringing in the data was already found to be caused by the relatively slow sampling rate, which is unavoidable with the current experimental setup since the data acquisition system cannot sample faster without losing critical sensitivity in the deflection measurement.

The mass flow from the dynamic calibration is shown in Figure 5. The data points (squares) represent the volumetric flow rate found by the high-speed video camera. This is considered the standard mass flow and is found by measuring the number of frames required for the syringe plunger to move from one gradation on the barrel to the next multiplied by the density of water. The derived flow rate measured on the TSMB shows good agreement with the volumetric flow rate recorded on the high-speed camera despite the error associated with the additional ringing oscillations from the slow sampling rate. Keeping in mind the plunger of the syringe was depressed by hand (i.e. not uniformly), the good agreement between the two values suggests that several of the oscillations are physical variations in the mass flow rate.

Figure 5. Comparison of derived mass flow rate and imparted volumetric rate from a graduated syringe.

Figure 6 shows the propellant mass loss and propellant mass flow rate of the experimental hybrid fuel core as a function of time. The GOx rate for this run was 4010 SCCM. The end of the burn shows the same ringing seen in the numerical simulations. This ringing is most prevalent at the end of the trace because the rate of change of the TSMB deflection is the greatest. The mass flow rate takes a few seconds to establish itself, consistent with the flame spreading across the interior of the hybrid motor bore. From about 80 seconds onward the mass flow has an upward trend, consistent with the increase in bore diameter, which would occur as the fuel burned.

Figure 6. Hybrid motor #10 data. (A) Derived mass loss and (B) and mass flow rate.
Figure 7 shows another run of the experimental hybrid core, this time varying the GOx rate half way through the run from 1976 SCCM to 3975 SCCM. At 90 seconds, there is a jump in the consumption of the fuel consistent with the increase in the oxidizer flow rate. Also, the propellant mass flow at 3975 SCCM is consistent with the mass flow rate seen in Figure 6 at 4010 SCCM. Figure 7 also shows the averaged mass loss and mass flow rate assuming a constant rate of change over thruster burn time. This would be the result of measuring the total system mass before and after the burn and dividing by the burn time. The beginning of the burn is considered to be the beginning of mass loss as found by the TSMB and the end of the burn is considered the end of the oxidizer flow. This averaging method successfully measures the total mass loss, but it is inadequate for approximating the time accurate solutions in the combustion process.

The total mass loss measured by the NIST calibrated digital scale was used to check to the accuracy of the experimental TSMB values. The total propellant mass loss for various motor cores can be seen in Table 1. There is good agreement between the digital scale values and those from the two forms of TSMB data analysis. The values found by comparing the loaded versus the unloaded state of the stand are typically within 2% of the values found by integrating the mass flow over the course of the run.

<table>
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<tr>
<th>Motor Core #</th>
<th>Digital Scale (kg)</th>
<th>Loaded vs. Unloaded (kg)</th>
<th>% of Scale</th>
<th>Mdot Integral (kg)</th>
<th>% of Scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>04</td>
<td>1.4800E-03</td>
<td>1.4730E-03</td>
<td>0.47%</td>
<td>1.4729E-03</td>
<td>0.48%</td>
</tr>
<tr>
<td>05</td>
<td>1.0459E-03</td>
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<tr>
<td>07</td>
<td>1.4001E-03</td>
<td>1.3890E-03</td>
<td>0.79%</td>
<td>1.3883E-03</td>
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<tr>
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<tr>
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<td>1.29%</td>
<td>1.6156E-03</td>
<td>1.31%</td>
</tr>
</tbody>
</table>

Table 1. Comparison of measured and derived mass losses for various runs.

A feasibility test for this study was performed on a test stand very similar in size and characteristic to one described by Pancotti, et al [9]. This hybrid thruster burn was measured using a faster data acquisition unit with less accuracy than the one previously described. A test burn of a hollow PMMA rod with a low GOx flow rate can be seen in Figure 8 sampled at 1000 Hz. The propellant mass flow rate here is an order of magnitude lower then shown in either Fig. 6 or 7. This test run illustrates the possibility of eliminating the experimental ringing found in the earlier 60 Hz tests. Future results will be performed using a high accuracy, high sample rate system.
V. Conclusion

A novel diagnostic technique has been developed, utilizing the Thrust Stand Mass Balance, to directly measure the propellant mass flow of a hybrid rocket core as a function of time. By analyzing the TSMB as a damped spring-mass system, the position of the stand as a function of time can be used to derive the forcing function that induced the TSMB motion. From this forcing function, accurate mass flow rates have been derived for the hybrid thruster cores. The technique has been verified using two calibration methods. The first used known weights added and removed from the stand. The second used a dynamic system with a known mass flow rates of water added the mass balance. Time accurate, propellant mass flow rates from a hybrid thruster in the range of $10^{-6}$ to $10^{-3}$ kg/sec have been demonstrated. With modifications to the TSMB, the measurement of higher mass flow rates would be conceivable. Through this study, the TSMB has proven its applicability to measuring propellant mass flow rates from solid or hybrid thrusters.

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