The research objectives of this project were focused on developing methods and procedures suitable for use with dynamic response measurements from flexible structural components and assemblies that may incorporate elements undergoing significant multidimensional nonlinear deformations. By using a powerful model-free approach to obtain computationally efficient reduced-order models, a general framework was developed for the probabilistic representation and propagation of measured uncertainties in the stochastic nonlinear test articles, their related nonparametric nonlinear model, and the corresponding probabilistic time-history response of the physical system.
Final Report

ANALYTICAL AND EXPERIMENTAL STUDIES OF THE QUANTIFICATION AND PROPAGATION OF UNCERTAINTIES IN NONLINEAR SYSTEM MODELING AND SIMULATION

AFOSR GRANT NUMBER FA9550-04-1-0147

Principal Investigator: Sami F. Masri

Department of Civil Engineering
University of Southern California
Los Angeles, California 90089-2531

Telephone: (213) 740-0602
FAX: (213) 740-3984
E-mail: masri@usc.edu

Period of Performance: 1 January 2004 – 31 December 2006

Dr. Victor Giurgiutiu
Program Manager, Structural Mechanics
Directorate of Aerospace and Materials Sciences
Air Force Office of Scientific Research
4015 Wilson Blvd, Room 713
Arlington, VA 22203-1954

Phone: (703) 696-7259; Blackberry 703 254 8247; Mobile 803 730 4478
FAX: (703) 696-8451
E-mail: victor.giurgiutiu@afosr.af.mil

22 March 2007
ANALYTICAL AND EXPERIMENTAL STUDIES OF THE QUANTIFICATION AND PROPAGATION OF UNCERTAINTIES IN NONLINEAR SYSTEM MODELING AND SIMULATION

Abstract

The project research objectives were focused on developing methods and procedures suitable for use with dynamic response measurements from flexible structural components and assemblages that may incorporate elements undergoing significant multi-dimensional nonlinear deformations. By using a powerful model-free approach to obtain computationally efficient reduced-order models, a general framework was developed for the probabilistic representation and propagation of measured uncertainties in the stochastic nonlinear test articles, their related nonparametric nonlinear model, and the corresponding probabilistic time-history response of the physical system. The research included carefully conducted experimental studies of generic types of nonlinearities likely to be encountered in aerospace structures. High-fidelity models (both parametric as well as nonparametric) were created that have the potential to provide predictive descriptions of uncertain nonlinear system behavior under arbitrary dynamic environments.

Accomplishments

Research activities of this project proceeded along two fronts: (1) an experimental phase involving the design and fabrication of an adjustable test apparatus for conducting studies on a generic multi-dimensional "joint" element which incorporates important nonlinear characteristics such as nonlinear elastic properties, hysteretic characteristics, and deadspace nonlinearities involving friction, and (2) an analytical phase focused on the development of a theoretical framework for processing experimental structural response measurements from uncertain systems and to develop and evaluate the utility of some promising analytical tools for the quantification and propagation of uncertainties in nonlinear dynamic systems.

1.0 ANALYTICAL STUDIES:

Identification and Prediction of Stochastic Dynamic Systems in a Polynomial Chaos Basis

An approach for obtaining predictions about the dynamic response of an uncertain nonlinear two-degree-of-freedom system under deterministic excitations has been developed. This approach consists of two independent procedures. In the first step, the Restoring Force Method is applied to identify the dynamic parameters of an uncertain nonlinear single-degree-of-freedom system with randomness in all its parameters. After identifying the properties of all nonlinear systems, the second phase is applied and the stochastic differential equations governing the motion of the uncertain non-linear two-degree-of-freedom system (Eq. 1) is solved using Polynomial Chaos Expansion:
\[
\begin{align*}
    m_1 \ddot{x}_1(t,\theta) + r_1(x_1(t,\theta), \dot{x}_1(t,\theta), \theta) - r_2(x_2(t,\theta), \dot{x}_1(t,\theta), \theta) - \dot{x}_1(t,\theta), \dot{x}_1(t,\theta), \theta) = F_1(t) \\
    m_2 \ddot{x}_2(t,\theta) + r_2(x_1(t,\theta), \dot{x}_1(t,\theta), \theta) - \dot{x}_1(t,\theta), \dot{x}_1(t,\theta), \theta) + r_3(x_2(t,\theta), \dot{x}_1(t,\theta), \theta) = F_2(t)
\end{align*}
\]

where the state variables and restoring forces will be functions of a random vector \( \theta \).

**Figure 1:** Nonlinear two-degree-of-freedom system.

A numerical study in which just the non-linear coupling parameters are assumed to be stochastic was carried out as an example. The uncertain viscous damping term was assumed to have a uniform distribution, the uncertain linear stiffness a Gaussian distribution, and the uncertain hardening coefficient a Gamma distribution. For all three parameters a 25% coefficient of variation was assumed. By using the nonparametric identification technique under discussion, the uncertainty in the identified parameters was determined as well as their corresponding histograms, and compared with original probability distributions (Figure 2).

Having identified the nonlinear system, a Polynomial Chaos Expansion (PCE) is performed to represent the random characteristics of the nonparametric estimated parameters. Due to the stochastic nature of the dynamic response of the system, it can be expanded again in a Polynomial Chaos basis leading to predictions of random response time histories reflecting the uncertainty in the real dynamic parameters and the error propagation in the identification process. The results obtained after solving the stochastic differential equation of motion (Eq. 1) subjected to deterministic swept-sine excitations are summarized in Figure 3. The solutions for \( x_1(t,\theta) \) and \( x_2(t,\theta) \) are given in terms of their means \( \mu_{x_1}, \mu_{x_2} \) and standard deviations \( \sigma_{x_1}, \sigma_{x_2} \). Additionally, the variances obtained by using PCE are compared with the variances calculated after solution of deterministic differential equations for 15000 Monte-Carlo events corresponding to the "real" random system. It is clear that an accurate estimation of responses in the nonlinear 2DOF system was achieved by performing the nonparametric identification technique.

Further details regarding this study are available in the work of Ghanem et al., 2005, and in Masri et al., 2005, and Masri et al., 2006.
2.0 EXPERIMENTAL STUDIES:

Test setup of a 2DOF Nonlinear Nonconservative “Joint” System

A test apparatus was designed to simulate the behavior of a nonlinear dissipative 2DOF “joint” element, in order to utilize the application of data-based model-free representations of such systems. The test setup consists of two computer-controlled electro-mechanical servo drives that generate external excitations in two independent directions. The motion of the drives is transferred through a shaft, with universal joints at each end to provide decoupled motions, to the nonlinear “joint” element. The applied forces to the system are measured through two sets of strain gauges mounted on the shaft in axial and torsional configurations. The relative motions, in the axial and rotational directions, of the “joint” are measured with four sets of optical encoders. Two linear and angular accelerometers are used to measure the absolute acceleration of the “joint”. The data acquisition system included a DAQ-board, three counter-boards, controller, and a chassis, in order to have synchronized measurements. A pictorial diagram indicating the inter-connection of the main system components, including the mechanical assembly, excitation sources, instrumentation network, and sensors, is provided in Fig. 4.

Figure 2: Statistical characteristics of estimated nonlinear SDOF. Part (a) shows the mean of the estimated Restoring Force. Parts (b), (c) and (d) show distribution of identified parameters $\hat{C}$, $\hat{K}$ and $\hat{\xi}$ compared with the corresponding parent distribution used for generating the “real” uncertain parameters.
Figure 3: Stochastic response of 2DOF nonlinear system. Left column shows the stochastic solution for \(x_1(t)\) and relative displacement \(x_2(t) - x_1(t)\). The right row shows the comparison between the standard deviations obtained by using the identified parameters and 15000 Monte-Carlo realizations of the "real" parameters.

Formulation
The nonlinear nonconservative components, in this study, are presented as massless nonlinear "joint" elements, which are located between two lumped masses of the system. Figure 5 shows the free body diagram of a massless nonlinear "joint" element that is located between DOF-\(i\) and -\(j\), with constant mass matrices of \(M_i\) and \(M_j\) that characterizes the inertia forces; \(x_i\), \(\dot{x}_i\), \(x_j\), and \(\dot{x}_j\) which are the state of the DOFs; \(r(x,x,p)\) which represents the restoring force vector of nonlinear nonconservative forces of the "joint" element; and \(F_{total_i}\), \(F_{total_j}\), \(T_{total_i}\), \(T_{total_j}\), and which are the sum of the resulting forces (external and internal) applied to the DOFs- \(i\) and -\(j\) of the system.
The nonlinear restoring force of the "joint" element shown in Fig. 5 can be obtained by subtracting the corresponding inertia forces of each DOFs from the sum of the resulting forces applied to the "joint" element of the system.

Figure 4: System architecture and wiring diagram of test setup.

Figure 5: Free body diagram of a generic massless nonlinear "joint" element.
External force applied to the \( x \) axis only.

External force applied to the \( \theta \) axis only.

External force applied to the both \( x \) and \( \theta \) axes.

\textbf{Figure 6:} Experimental data sets used in the identification of the nonlinear coupled 2DOF "joint".
Training Data Sets from Experimental Test Setup

For an accurate identification of nonlinear dynamical systems, it is essential that the identification data sets provide a complete representation of the nonlinear characteristics of the system. Therefore, the data sets selected for the identification of the simulation model, were data sets of the system responses when subjected to broadband random excitations, in order to capture all the modes of the system within the excitation frequency range. However, for capturing the coupling effects of the nonlinear component through the identification process, it was necessary that the data sets incorporate the coupling effects of the DOFs, because the identified model depends on the input and output data. Therefore, three data sets were required to capture the correct behavior of the system. These data sets, which are shown in Fig. 6, represent the system response when there was excitation on each of the axes individually, and when the excitation forces were applied to both axes simultaneously. Moreover, the data were normalized across all the sets prior to the identification process in order to have a zero mean with amplitude of ±1.

Further details regarding this study are available in the work of Caffrey et al., 2004 and Tasbihgoo et al., 2006.

Acknowledgment/Disclaimer

This work was sponsored in part by the Air Force Office of Scientific Research, USAF, under grant number FA9550-04-1-0147. The views and conclusions contained herein are those of the author(s) and should not be interpreted as necessarily representing the official policies or endorsements, either expressed or implied, of the Air Force Office of Scientific Research or the U.S. Government.

Publications:

The following papers, which are supported in part by this research effort, have been published or accepted for publication, and are listed in chronological order:


Interactions/Transitions:

Results of the research are directly applicable to modeling, monitoring and control of uncertain nonlinear systems which are widely encountered in the aerospace field.

Extensive collaborations and interactions with several researchers at different academic institutions were performed, and several published technical papers resulted from these interactions.

New Discoveries, Inventions, or Patent Disclosures: None

Honors/Awards: None
References


