AN INTEGER LINEAR PROGRAM TO COMBINE CONTAINER HANDLING AND YARD CRANE DEPLOYMENT

by

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The number of containers handled by container terminals has increased significantly over the last fifty years and has stimulated researchers to improve storage yard operations. Container handling and crane deployment are two major yard operations that can impact the performance of a whole container terminal. This thesis establishes an Integer Linear Program (ILP) to combine container handling and yard crane deployment for Rubber Tired Gantry Cranes (RTG). Using real world data, we test the ILP for two different yard sizes. We find the resulting ILPs difficult to solve directly. In order to decrease the computation time, we apply a cascade method that solves the problem as a sequence of restricted subproblems. Each subproblem is restricted to a sequence of containers and the output of each subproblem provides an input to the next subproblem. This method provides better solutions than the solution that we get by solving the problem directly. The cascade method also decreases the computation time significantly. The results demonstrate the ability to combine container handling and yard crane deployment in a single model and they verify that the cascade method works well with the ILP.
AN INTEGER LINEAR PROGRAM TO COMBINE CONTAINER HANDLING
AND YARD CRANE DEPLOYMENT

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ABSTRACT

The number of containers handled by container terminals has increased significantly over the last fifty years and has stimulated researchers to improve storage yard operations. Container handling and crane deployment are two major yard operations that can impact the performance of a whole container terminal. This thesis establishes an Integer Linear Program (ILP) to combine container handling and yard crane deployment for Rubber Tired Gantry Cranes (RTG). Using real world data, we test the ILP for two different yard sizes. We find the resulting ILPs difficult to solve directly. In order to decrease the computation time, we apply a cascade method that solves the problem as a sequence of restricted subproblems. Each subproblem is restricted to a sequence of containers and the output of each subproblem provides an input to the next subproblem. This method provides better solutions than the solution that we get by solving the problem directly. The cascade method also decreases the computation time significantly. The results demonstrate the ability to combine container handling and yard crane deployment in a single model and they verify that the cascade method works well with the ILP.
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EXECUTIVE SUMMARY

The rapid growth of containerization significantly increased the number of containers and the competition among seaport container terminals. As a result of competition among container terminals, managers and operators began to search for new methods to improve terminal performance.

Berth time of a vessel is usually accepted as the primary measure of performance for container terminals, and this time depends on a sequence of operations that are conducted in the container yard where containers are stored temporarily. Loading and unloading containers and assigning yard cranes to specific areas in the yard are two major yard operations that can impact the performance of a whole container terminal, unless they are well planned and coordinated. Although there have been various studies to improve these operations separately, this thesis is the first to establish a combined model for both operations.

In this thesis, we minimize the time of placing a container which is unloaded from a vessel (discharge) in the container yard and the time of loading a container to a vessel in such a way that the same type of discharge containers are stored together and the total workload is divided fairly, as much as possible, among Rubber Tired Gantry Cranes (RTG). We let the workload of an RTG exceed a fair limit when it is worth paying a deviation penalty. We define the planning horizon in terms of the sequence of containers.

We formulate the problem as an Integer Linear Program (ILP). The objective function seeks to minimize the operation time which includes transfer times of RTGs between areas, average time that an RTG spends with discharging or loading a container, time to carry containers from the vessel to a storage area or vice versa, and the reshuffling time that a container causes if it is stored with different type of containers.
Using real world data, we test the ILP for two different yard sizes. We take 60 containers as a base case for our study. The ILP contains about 1,400,000 equations, 507,000 continuous and 487,000 binary variables and takes more than twenty-four hours to provide a near optimal solution. In order to decrease the computation time and to satisfy memory requirement, we apply a cascade method that solves the problem as a sequence of restricted subproblems. Each subproblem is restricted to a sequence of containers and the output of each subproblem provides an input to the next subproblem. This method provides better solutions than the solution that we get by solving the problem directly. The cascade method also decreases the computation time significantly. The results demonstrate the ability to combine container handling and yard crane deployment in a single model and they verify that cascade method works well with the ILP.
I. INTRODUCTION

A. BACKGROUND

The number of containers handled by container terminals has increased significantly, as international sea freight transportation has grown rapidly over the last fifty years. As a result of competition among container terminals, managers and operators began to search for new methods to improve terminal performance. The berth time of a vessel is usually accepted as the primary measure of performance for container terminals and this time depends on a sequence of operations that are conducted in the container yard where containers are stored temporarily. Loading and unloading containers and assigning yard cranes to specific areas in the yard are two major yard operations that can impact the performance of a whole container terminal, unless they are well planned and coordinated. Although there have been various studies to improve container handling and yard crane deployment (separately), this thesis is the first to establish a combined model for both operations.

This thesis establishes an Integer Linear Program (ILP) to combine container handling and yard crane deployment strategies. We choose to model the deployment of Rubber Tired Gantry Cranes (RTG), among different types of yard cranes, because RTGs are used extensively in container terminals around the world. Our ILP finds the best place in the yard under given constraints for containers that are going to be unloaded from a vessel; it also picks an appropriate container from several possible locations to be loaded. We find the area covered by each RTG that minimizes the time to load and unload containers by minimizing the interblock movements of the RTGs, as well as balancing the workload among them.
1. Container Terminals

*Container terminals* are the facilities where cargo containers are transshipped between different transport vehicles for onward transportation. A container terminal can be partitioned into three areas: *quayside* (or berth) where containers are loaded or discharged onto/from a vessel, *container yard*, and the *landside* (gatehouse) which is the landside entrance of a terminal where the container inspection and paperwork is done, and container-carrying trains and trucks come into or go out of the terminal (Figure 1).

![Container Terminal Layout and Flow of Operations](image)

Figure 1. Container Terminal Layout and Flow of Operations

In container yards, containers are stacked in areas called *blocks*. A block usually consists of approximately twenty containers in length, six to eight containers in width and four to six containers in height. The length of a block is called *bay*; the width and height are named *row* and *tier*, respectively. Therefore, the location of a container in a block is defined by its bay, row and tier number.
2. Terminal Equipment

A *container* or a *cargo container* is a large, typically metal, box used for transporting goods (Figure 2). There are several ways to classify containers, namely by dimension, by weight, and by content. Of these classifications, dimension is the most common and TEU (twenty-foot equivalent unit) is the standardized unit of measure. Forty-foot and forty-five-foot long containers are counted as two TEUs. Dimensions of a short container (so called standard container) are 20x8x8.5 or 20x8x9.5 (in feet), whereas dimensions of a forty foot long (Two TEU) container are 40x8x8.5 or 40x8x9.5.

![Figure 2. 40-foot container (Left) and 20-foot container (Right) (From: Huynh and Walton [2005])](image)

Containers are transported by trucks or *Automated Guided Vehicles* (AGV) (Figure 3) at terminals. Trucks that are used only in terminal operations are named *Terminal Tractors* (Figure 4) or *Internal Trucks* (IT), whereas the trucks that transport the containers into terminals or out of terminals are called *External Trucks* (ET). In some container terminals, Automated Guided Vehicles carry containers between quay cranes and the container yard with their positions controlled via wires or transponders.
There are different types of cranes used to load and unload containers. Cranes that load and unload containers from/to vessels are called Quay Cranes (QC) (Figure 5). These cranes play a major role in the performance of a container terminal. Other types of cranes are usually used to stack containers in container yards. The three main types of these cranes are Automated Stacking Crane (ASC) (Figure 6), Rail Mounted Gantry Crane (RMG) (Figure 7), and Rubber Tired Gantry Crane (RTG) (Figure 8). Of these three types of yard cranes, ASC is the only one that can operate without an operator. ASCs move on rails and they don’t have the ability to move between blocks. Because RTGs
operate on rubber tires, they are much more versatile than ASCs and RMGs. While the movements of ASCs and RMGs are restricted to a block or to the blocks on the same lane, respectively, RTGs can move between blocks in a yard, even between the ones that are located on different lanes. The only issue concerning the movements of RTGs is that more time is required to change lanes than to change blocks within the same lane. When an RTG is transferred to a block on a different lane, it moves out of the block to the open space at the end of the current block, makes a 90-degree turn, moves in parallel to the block width, lines to the correct lane of an adjacent block, makes a 90-degree turn again, then enters the block. These 90-degree turns take extra time that delays not only the operation of the RTG, but also the traffic that flows on the road when an RTG is turning.

Figure 5. Quay Crane (From: Kalmar Industries [2007])
Figure 6. Automated Stacking Crane (From: Kalmar Industries [2007])

Figure 7. Rail Mounted Gantry Crane (From: Doosanheavy Industries and Construction [2007])
There are also other types of vehicles and equipment in container terminals such as straddle carriers, forklifts, reachstackers and trains. However, these are not considered in this thesis.

Containers that are unloaded from a vessel and are sent to a yard to be stacked are called discharges or import containers, and the ones that are sent from a container yard to a quay crane to be loaded onto vessel are called loads or export containers.

3. Terminal Operations

Basic operations on the quayside are discharging and loading ships. When containers are discharged from the vessel by quay cranes, ITs and AGVs carry those containers to the container yard, and then carry the load containers from the yard to vessels.

There are several operations conducted in container yards. Yard cranes place discharge containers in blocks, pickup the load containers, and load them onto ITs or AGVs when a vessel is berthed at a port. These cranes also load import containers onto ETs and unload export containers from ETs when there is no waiting discharge or load move from/to the vessel. They also relocate some
of the containers to facilitate future loads when they are idle. Because ship turnaround time is one of the major measures of performance of terminals, quayside operations have priority over yard and landside operations.

On the landside at the gatehouses, paperwork is done for the entrance and exit of ETs. There might be container-handling operations at the train station if there is a railway access to a container terminal.

B. ORGANIZATION OF THE STUDY

This study includes five chapters. Chapter II provides an overview of some previous studies that have been conducted to improve container terminal operations. Chapter III presents the problem that is the source of this study. In the same chapter, the characteristics of the model that combines the container handling and crane scheduling operations are explained. Chapter IV describes the test data provided by Navis Llc and and presents results. Chapter V includes conclusions and suggestions for further studies.
II. LITERATURE REVIEW

A. INTRODUCTION

Due to the numerous advantages that containerization brought to international trade, it has become one of the most essential parts of international sea freight transportation over the last fifty years. The rapid growth of containerization significantly increased the number of containers and the competition among seaport container terminals [Steenken et al. 2004].

Berth time of a vessel is usually accepted as a measure of performance for container terminals and this time depends on a sequence of operations that are conducted in the yard. Container handling and yard crane scheduling are two major yard operations that can easily become a bottleneck for a container terminal unless they are well planned and coordinated. Although there have been various studies to improve container handling and crane scheduling separately, this thesis is the first to establish a model that combines both operations.

B. PREVIOUS STUDIES

As Steenken et al. [2004] report, containers are designed to make international transportation of goods easier by using a unit-load concept. Cordeau et al. [2005] state that containerization requires less product packaging, it also reduces damages and yields higher productivity during various handling processes.

Zhang et al. [2006] report that containerization has grown at an annual rate of nine percent recently. They also state that it is expected that by 2010, ninety percent of all international sea freight will be containerized. Due to this high rate of growth and the high cost of terminal structure and equipment investments, terminal operators have been searching for more efficient ways of handling containers without enlarging the footprint of terminals.
Although all terminal operations are strongly interrelated, due to the large number of decisions and the multi-objective nature of the problem, the uncertainty and the complexity of the decisions result in a focus on one or two specific operations.

Some recent research areas include berth allocation (e.g., Cordeau et al. [2005]), quay crane scheduling (e.g., Park and Kim [2003], Kim and Park [2004]), ship planning (e.g., Steenken et al. [2001]), automated guided vehicles scheduling (e.g., Kim and Bae [2004], Rashidi and Tsang [2005]), container handling (e.g., Kozan and Preston [1999], Alvarez [2006]), crane routing (e.g., Kim and Kim [1999a]), straddle carrier routing (Kim and Kim [1999b]), crane deployment (e.g., Chung et al. [2002]), automated stacking cranes scheduling (e.g., Zyngiridis [2005]), and classification of container terminal operations (e.g., Steenken et al. [2004]).

Because getting real-time solutions is essential in solving container terminal problems, computational complexity of the problem is a real concern for researchers. Hence, heuristic optimization techniques have been proposed by several researchers (e.g., Linn and Zhang [2002]).

Daganzo [1990] studies a queuing problem that arises at multipurpose port terminals. He presents queuing models to predict the stochastic characteristics of the traffic overflow from a multipurpose terminal to the rest of a port.

Castilho and Daganzo [1993] focus on container import operations at container terminals. They present methods for measuring the required amount of handling effort based on two different strategies, one of which tries to keep all stacks the same size and the other that segregates containers according to arrival time.

Kozan and Preston [1999] use genetic algorithms to reduce the container handling/transfer times and ships’ turnaround times by speeding up handling operations. They report that a scheduled storage policy where containers are
stored closer to the berth is better than a random storage policy. They also state that the storage area fullness does not have a significant effect on either the random or the scheduled storage policy.

Kim and Kim [1999] formulate the optimal routing of a single yard crane as a mixed integer program (MIP), to minimize the container handling time of a transfer crane that includes the setup time at each yard-bay and travel time between yard-bays.

Zhang et al. [2002] formulate the deployment of RTGs among blocks as a MIP, and solve it by Lagrangean relaxation to determine the routes of crane movements in a way such that the total delayed work is minimized. It is assumed that the locations of the discharge containers are known prior to the operation.

Linn et al. [2003] formulate the yard crane deployment problem as a MIP to investigate dynamic crane deployment in the container yard on a shift-to-shift basis and solve it with a least cost heuristic method. Again, it is assumed that the locations of the discharge containers are known prior to the operation.

Chung et al. [2002] also study the problem of scheduling the yard crane movements. They formulate the problem as a MIP and solve it by using Lagrangean decomposition. They also report a method named piecewise-linear approximation that is efficient for large-size problems and again, it is assumed that the locations of the discharge containers are known prior to the operation.

Zyngiridis [2005] focuses on the problem of automated stacking crane scheduling and develops Integer Linear Programs to prescribe routes for one and two equally sized ASCs. He finds that one ASC working alone over four hours requires up to 70% more time than two ASCs working together to accomplish the same required container movements.

This thesis differs from the previous studies by combining the container handling and yard crane deployment strategies to improve the efficiency of yard management in container terminals. This thesis does not aim to provide real time
solutions in a few seconds. Our goal is to provide a near-optimal solution in a reasonable time that can be a benchmark for heuristic solutions.
III. MODEL

A. INTRODUCTION

As mentioned in Chapter II, two of the major measures of performance for container terminals are vessel berth time and quay crane rate. However, all terminal operations are interrelated and a delay in one of those operations may cause a delay in quay operations and decrease the performance of a container terminal. Container handling and crane scheduling are two of these yard operations that can easily cause a performance bottleneck for a container terminal unless they are well planned and coordinated.

Although all terminal operations are interrelated, the uncertainty and complexity of the large number of decisions and the multi-objective nature of the problems prevent us from formulating all terminal operations in a single model. Therefore, we focus on two major operations in a container terminal. One of those operations is distributing discharge containers in a container yard and the other one is deployment of RTGs to the areas. When a container is discharged from a vessel by a quay crane, it is loaded onto an internal truck or on an AGV to be carried to the yard where it is going to be stored by the yard cranes. It usually takes a few minutes to carry a container from the berth to the yard, depending on the distance between them and the speed of the carrying vehicle. There is an interval called push rate between discharge container arrivals to the yard. Push rate can be adjusted to prevent congestion in the yard. However, it is supposed to be as small as possible to minimize the discharging time of the vessel.

In this thesis, we minimize the time of discharge and load operations in such a way that the same type of discharge containers are stored together and the total workload is divided fairly, as much as possible, among RTGs. We let the workload of an RTG exceed the limit when it is worth paying a deviation penalty. We define the planning horizon in terms of the sequence of containers. Our aim is to keep this horizon as long as possible but in real operations, conditions in the
yard change and RTG assignments to areas must be reevaluated periodically. Additionally runtime of the model and the memory of the computers we use limit the number of containers that can be handled.

Regarding the above-mentioned characteristics of the problems, we have the following assumptions:

- The sequence of the movement type (either discharge or load) is known.
- Only RTGs are used for stacking containers in the yard and their initial positions are known.
- All RTGs are the same type and size. Therefore, they cannot cross over each other.
- When an RTG needs to be transferred from one block to another, the delay it causes in the yard traffic is included in RTG transfer time.

Container handling includes placing discharge containers in the container yard and picking up the load containers from appropriate areas (among the alternatives). We want the same type of containers to be stored in the same area, so we penalize the discharge moves that do not store similar containers together. We also want to ensure that a load container is obtained from a specific area where that type of container is already stored, so we provide the locations of those containers in the data. We also ensure that all containers are handled.

Crane deployment covers the deployment of RTGs in specific blocks where they operate, until all containers are handled. We want each RTG to be matched with one or more areas. Thus, each RTG is matched with the area where it is initially located. When an RTG needs to change its area, a cost is defined in crane minutes and it is assumed that this cost includes the cost of the delay of the yard traffic that the RTG causes. When an RTG handles a container, it cannot be assigned to handle the next few containers in the sequence because it can delay yard operations while waiting for the RTG to be available. To prevent RTG congestions in areas, we only let one RTG be matched with an area. However, an RTG can be matched with multiple areas. We calculate the
workload of an RTG by adding up the time that an RTG spends while it is changing areas (transfer time) and the average time it takes to discharge or to load a container. Therefore, the workload is represented by crane minutes.

B. FORMULATION

We formulate the problem as an Integer Linear Program (ILP). The objective function seeks to minimize the operation time which includes transfer times of RTGs between areas, average time, that an RTG spends with discharging or loading a container, time to carry containers from vessel to the storage area or vice versa, and the reshuffling time that a container causes if it is stored with different type of containers.

1. Indices

\[ t, t' \in T \quad \text{containers in the time sequence of their movement} \]
\[ t \in \{1, 2, \ldots, T\} \]
\[ a, a' \quad \text{area where containers are stored or picked up for loading} \]
\[ a \in \{1, 2, \ldots, A\} \]
\[ d \in D \quad \text{containers to be discharged} \]
\[ d \subset T \]
\[ l \in L \quad \text{containers to be loaded} \]
\[ l \subset T \]
\[ g, g' \quad \text{RTG} \]
\[ g \in \{1, 2, \ldots, G\} \]

2. Sets

\[ t' \in N_t \quad \text{Set of containers that an RTG cannot handle if it handles a container at time } t. \]
\[ (a, g) \in InitialG \quad \text{Set of area crane pairs where RTG } g \text{ is initially located.} \]
\[ a \in Loc_i \quad \text{Set of areas where load container } l \text{ can be obtained.} \]
3. Scalars

TOTALG : Total number of RTGs in use
AREAS : Upper limit for the number of areas that an RTG can cover
MOVES : Upper limit for the number of movements of an RTG between areas up to container $t$

4. Parameters

Travel : Time to carry a container from a vessel to an area or vice versa.
PenDC : Penalty of discharging container $t$ into area $a$.
Transfer : Time to transfer an RTG from area $a'$ to area $a$.
Initial : 1 if RTG $g$ starts in area $a$.
ADT : Average discharge time for area $a$.
ALT : Average loading time for area $a$.

Delta : Shows how much the workload of an RTG can exceed the average workload.
CmlWL : Cumulative workload of an RTG up to container $t$.
PEN : Penalty for exceeding the workload limit.

5. Positive Variables

WL : Workload of an RTG.
Dev : Deviation from the sum of Average WL and Delta.

6. Binary Variables

DC : 1 if container $t$ is moved by RTG $g$ to area $a$.
LD : 1 if container $t$ is moved by RTG $g$ from area $a$.
Y : 1 if RTG $g$ covers area $a$. 
\[ Z_{a,g,t} \begin{cases} 1 & \text{if RTG } g \text{ has control over area } a \text{ at time } t. \\ M_{a',a,g,t} \begin{cases} 1 & \text{if RTG } g \text{ moves from area } a' \text{ to area } a \text{ at time } t. 
\end{cases} \end{cases} \]

7. Objective Function

\begin{align*}
(1) \quad \text{Min} & \quad \sum_{a,g,t} \left( \text{Travel}_{a} + \text{PenDC}_{a,t} \right) \cdot \text{DC}_{a,g,t} \\
& + \sum_{a,g,t} \text{Travel}_{a} \cdot \text{LD}_{a,g,t} \\
& + \sum_{a',a,g,t} \text{Transfer}_{a',a} \cdot M_{a',a,g,t} \\
& + \sum_{g,t} \text{PEN} \cdot \text{Dev}_{g,t} 
\end{align*}

8. Constraints

\begin{align*}
(2) \quad \sum_{a,g} \text{DC}_{a,g,d} &= 1 \quad \forall d \\
(3) \quad \sum_{a=\text{Loc}_i} \sum_{g} \text{LD}_{a,g,l} &= 1 \quad \forall l \\
(4) \quad \sum_{a} \sum_{t \in N_t} \text{DC}_{a,g,t} + \text{LD}_{a,g,t} &\leq 1 - \sum_{a} \text{DC}_{a,g,t} + \text{LD}_{a,g,t} \quad \forall g,t \\
(5) \quad \text{DC}_{a,g,t} &\leq Z_{a,g,t} \quad \forall a,g,t > 1 \text{ and } t \in D \\
(6) \quad \text{LD}_{a,g,t} &\leq Z_{a,g,t} \quad \forall a,g,t > 1 \text{ and } t \in L \\
(7) \quad Y_{a,g} &= 1 \quad \forall (a,g) \in \text{InitialG} \\
(8) \quad \sum_{g} Y_{a,g} &\leq 1 \quad \forall a \\
(9) \quad \sum_{a} Y_{a,g} &\leq \text{AREAS} \quad \forall g \\
(10) \quad Z_{a,g,t} &\leq Y_{a,g} \quad \forall a,g,t > 1 \\
(11) \quad Z_{a',g,t} &= 1 \quad \forall (a',g) \in \text{InitialG} \\
(12) \quad \sum_{a} Z_{a,g,t} &= 1 \quad \forall g,t \\
(13) \quad Z_{a,g,t+1} &\leq Z_{a,g,t} + \sum_{a'} M_{a',a,g,t} \quad \forall a,g,t < T \\
(14) \quad M_{a',a,g,t} &\leq Z_{a,g,t} \quad \forall g,a',a (a' \neq a), t 
\end{align*}
9. The Objective Function and Constraint Description

The objective function seeks to minimize the time and the penalty of placing all containers, as well as minimizing the time for transferring RTGs between areas. Constraint sets (2) and (3) ensure each discharge and load container are handled. Constraint set (4) prevents a container to be matched with an unavailable RTG. Constraint sets (5) and (6) require that when a container is discharged to area \( a \) or it is loaded from area \( a \) by RTG \( g \) at time \( t \), for RTG \( g \) to have control over that area at time \( t \). Constraint set (7) states that each RTG has to be assigned to the area where it is initially located. Constraint set (8) limits an area to be matched only with one RTG. Constraint set (9) limits the number of areas that an RTG can cover. Constraint set (10) ensures an RTG can have control over an area if it is assigned to that area. Constraint set (11) ensures each RTG is in its initial location at the start. Constraint set (12) ensures each RTG can be only in one area at time \( t \). The following four constraint sets regulate the transfers of RTGs between areas: Constraint set (13) states if an RTG has control over area \( a \) at time \( t+1 \), it has to have control over the same area at time \( t \) or it has to be transferred to that area from some area \( a' \) at time \( t \). Constraint set (14) states that if an RTG is transferred from area \( a' \) to area \( a \) at time \( t \), then it

\[
\begin{align*}
(15) & \quad M_{a',a,g,t} \leq Z_{a,g,t+1} \quad \forall g,a',a(a' \neq a), t > 1 \\
(16) & \quad M_{a',a,g,t} \geq Z_{a',g,t} + Z_{a,g,t+1} - 1 \quad \forall g,a',a(a' \neq a), t > 1 \\
(17) & \quad \sum_{a} M_{a',a,g,t} \leq 1 \quad \forall (a',g) \in \text{InitialG} \\
(18) & \quad \sum_{a,a',t} M_{a',a,g,t} \leq \text{MOVES}_t \quad \forall g,t \\
(19) & \quad WL_{g,t} = \sum_{a,a'} \text{Transfer}_{a,a'} \cdot M_{a',a,g,t} + \sum_{a} \left( \text{ADT}_a \cdot \text{DC}_{a,g,t} \right) + \left( \text{ALT}_a \cdot \text{LD}_{a,g,t} \right) \\
& \quad \forall g,t \\
(20) & \quad \sum_{t' \leq t} WL_{g,t'} \leq \frac{\sum_{g,t'} WL_{g,t'}}{G} + \text{Delta}_{t} + \text{Dev}_{g,t} \quad \forall g,t 
\end{align*}
\]
has to have control over area \( a' \) at time \( t \). Constraint set (15) states if an RTG is transferred from area \( a' \) to area \( a \) at time \( t \), then it has to have control over area \( a \) at time \( t+1 \). Constraint set (16) states if an RTG is transferred from area \( a' \) to area \( a \) at time \( t \), then it has to have control over area \( a' \) at time \( t \) and it has to have control over area \( a \) at time \( t+1 \) as well. Constraint set (17) states that if RTG \( g \) moves from area \( a' \) at the start, it has to be located in that area initially. Constraint set (18) limits the interblock movements of each RTG. Constraint set (19) defines the workload of each RTG. Finally, constraint set (20) limits the workload of an RTG at time \( t \) to the average workload and a specific amount called \( \text{Delta} \) at time \( t \) plus the deviation.
IV. COMPUTATIONAL STUDY

A. TEST DATA

Navis LLc provided a data set consisting of over 6,000 containers, half of which are discharge containers and the other half are load containers [Ayik and Golbasi 2007]. Navis LLc reports that the data set shows a daily workload of a typical container terminal.

We are given the container names and the locations of the load containers. A load container may be obtained from several different areas. We know the distances between quay cranes and areas. We assume that ITs carry containers at a constant speed. Time to carry a container from a quay crane to an area or from an area to a quay crane is found by dividing the distances by the IT-speed. There are costs associated with RTG movements; not only for inter-area movements but also for the in-area movements. There are also costs associated with the areas where a container is discharged. Cost of storing a container with a different type of container is higher than storing it with containers of its type.

B. ANALYSIS

We break the data set into parts so that we can get a solution for each small data set in a reasonable time without exceeding the memory limits of our computers. In this study, we take one hour as a reasonable time to get a solution. We use GAMS [GAMS 2007] Version 2.0.33.5 and Cplex Version 10.0.1 to solve instances using a desktop computer with 3.73 GHz CPU and 3.00 GB of RAM.

We consider a portion of the container terminal and two quay cranes discharging and loading a vessel that berths at a port. We consider two quay cranes just to simplify how we obtain a list of containers. The list of containers could easily be a representative of more than two quay cranes. There are six
RTGs operating in the yard that consists of twelve lanes, each of which has three blocks. The lanes are parallel to the berth (Figure 9). Initial positions of the RTGs are randomly chosen. It is assumed that only the sequence of the discharge and load moves is known prior to the arrival of the vessel. The locations where discharge containers are going to be stored are not given. So, we randomly assign areas of preference for discharge containers.

![Yard Layout](image)

Figure 9. Yard Layout

We take 60 containers as a base case for our study. 30 of those 60 containers are discharge and 30 are load containers. First, we run the model for 60 containers. The ILP contains about 1,400,000 equations, 507,000 continuous and 487,000 binary variables. After about 28 hours, the model runs out of memory. The lower bound it provides is about 336 and the best integer value is 520. This means a 35.38% relative gap (ratio of the difference between the integer value and the best possible). This result and the length of the computation time lead us to use an optimization based heuristic method to get a better solution in less than one hour. We use a *Cascade Method* (Baker and
Rosenthal [1998] and Baker et al. [2002]). With this method, we solve the sixty-container problem part by part. Namely, we run the model for a small part of data at a relative gap of 5% for each part and then we run the model again for a relatively larger number of containers after fixing some of the variables to the values that we obtain at the first run. This process goes on until a solution is obtained for 60 containers. Note that each part of the problem is solved by an exact algorithm (CPLEX Version 10.0.1).

We set up different cascade scenarios and run the model for each of them. Table 1 shows the scenarios that we tested.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1st Step</th>
<th>2nd Step</th>
<th>3rd Step</th>
<th>4th Step</th>
<th>5th Step</th>
<th>MIP Solution</th>
<th>Run Time (min.)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0-20</td>
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Table 1. MIP Solutions and Computation Times with Thirty-six Areas

In the first scenario, at the first step, we run the model for 20 containers. At the second step, we fix the variables associated with the first 10 containers to the values that we obtain at the first step and run the model for 40 containers. At the third step, we fix the variables associated with the first 20 containers to the values that we obtain at the second step. We follow this process until we reach a solution for 60 containers. At the fifth step, we run the model for 60 containers after fixing the variables associated with the first 40 variables. We get an integer
solution with the value of 526 with this scenario in a total of 30 minutes. The fourth scenario provides the best result regarding the integer solution and the computation time. It provides an integer solution with the value of 479 in 37 minutes. The relative gap is 29.85% (better than the 35.38% that we got before). The computation time is also much shorter.

The assignments of RTGs also change with each scenario. Figure 10 shows the assignments of RTGs to the areas.

Figure 10. RTG Assignments With 36 Areas

In Figure 10, rectangles represent areas. There are two numbers in each rectangle. The number on the right is the area number. The circled ones on the
left show the RTG that is assigned to that area. Shaded rectangles represent the areas where RTGs are initially located. Some of the areas are not covered because there are several areas where a discharge container can be stored. Also, a load container can usually be picked up from multiple areas.

We also set up the similar scenarios for a lesser number of areas. We decrease the number of areas from 36 to 18, and define new locations for load containers and initial locations for RTGs. We keep the rest of the data the same and run the model again. First, we run the model for the base case scenario, which has no fixed values. The ILP contains about 360,000 equations, 137,000 continuous and 127,000 binary variables. After about 28 hours, the model runs out of memory. The lower bound it provides is 225 and the best integer value is 283. This equals a 20.15% relative gap. Table 2 shows the results of different scenarios when we use the cascade method.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>1st Step</th>
<th>2nd Step</th>
<th>3rd Step</th>
<th>4th Step</th>
<th>5th Step</th>
<th>MIP Solution</th>
<th>Run Time (min.)</th>
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</tbody>
</table>

Table 2. MIP Solutions and Computation Times with Eighteen Areas

This time the first scenario provides the best integer value. The relative gap associated with that integer value is 14.77%. The computation time is only 3
minutes. This is a reasonable time when compared to the computation time of the base case, which is more than 28 hours.

Figure 11 shows the assignments of RTGs to the areas, for the scenarios of 18 areas.

Figure 11 shows that the model tends to assign the RTGs to adjacent areas. This is due to the low transfer cost between those areas. However, in some situations, an RTG may be assigned to an area that is far away from its current location (e.g., in the 2nd scenario RTG5 is assigned to area 01A). This happens when there is a container to be handled in that area and the closest RTG is not available. Then, among the available RTGs, the RTG that is located in an area associated with the lowest transfer cost is assigned to that area.
V. CONCLUSION

This thesis is the first to combine the container handling and yard crane deployment strategies to improve the efficiency of yard management in container terminals.

We establish a model that finds the best area for a container that is unloaded from a vessel, selects a container to load among several possible areas and assigns yard cranes to the areas where containers are stored. We formulate the problem as an ILP and test it with real world data. Due to the complexity of the problem, we break the data into small parts. Even for the moderate-size data, it takes more than twenty-four hours for the ILP to provide a near optimal solution. In order to decrease the computation time and to satisfy memory requirement, we apply a cascade method that solves the problem as a sequence of restricted subproblems. Each subproblem is restricted to a sequence of containers and the output of each subproblem provides an input to the next subproblem. This method provides better solutions than the solution that we get by solving the problem directly. The cascade method also decreases the computation time significantly. The results demonstrate the ability to combine container handling and yard crane deployment in a single model and they verify that cascade method works well with the ILP. As a general conclusion, this model might be used to provide a benchmark for the quality of the solutions provided by heuristic algorithms.

In this study, we combine two of the three major operations in a container terminal. Further studies may add the landside operations and provide more general solutions for container terminal operations. Additionally, heuristic algorithms may be used to provide real-time solutions.
LIST OF REFERENCES


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1. Defense Technical Information Center  
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