Boussinesq Modeling of Alongshore Swash Zone Currents

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ABSTRACT

The report documents the theoretical and numerical investigations on wave propagation over porous beds as well as on alongshore surf and swash currents. The study has been carried out in the framework of wave-resolving Boussinesq-type models. First, we have derived a new set of Boussinesq-type equations for nonlinear waves and surf-zone currents over a permeable beach (Chen 2006). A Stokes-type analysis and rational expansions were carried out to examine the fundamental damping and dispersion properties of the new set of equations (Cruz and Chen 2006a). The vortical properties of the new and pre-existing Boussinesq equations have been carefully investigated (Gobbi et al 2006). Second, numerical implementation of porous effects into a one-dimensional Boussinesq wave model has been completed. Preliminary results were documented in Cruz and Chen (2004). We have tested the numerical model against laboratory experiments on wave transformation over heterogeneous porous beds and submerged rubble mounds. The results have been published in Cruz and Chen (2006b). Third, efforts have been devoted to the extension and testing of the Boussinesq wave model, FUNWAVE 2D, with respect to surf zone currents and swash motions. We have examined three different schemes for the treatment of a moving shoreline with an emphasis on the swash velocity. Tests of the improved FUNWAVE model against analytical, laboratory and field data of swash and surf zone currents have been carried out (Chen and Briganti 2006). Fourth, Lagrangian descriptions of the fluid motion have been employed to study the mixing of surf zone currents, which serves as an alternative to Eulerian analyses of the flow field (Briganti et al. 2006). The enhanced numerical model, tested under the SandyDuck field conditions, has produced fairly good results. Insight into the capability of predicting the alongshore drift velocity of the water mass in the swash zone was gained.

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INTRODUCTION

The swash zone on a beach forms the mobile interface of the seawater and the land. The swash motion is characterized by the turbulent bore on the front face of a broken wave, the infiltration and exfiltration on the permeable beach, and the moving shoreline boundary. Previous studies (e.g. Bodge and Dean 1987, and Kamphuis 1991) have revealed bimodal distributions of the longshore sediment transport rate across the surf zone with a peak shoreward of the breaking point and the second peak in the swash zone. In the case of a large surf similarity parameter, experimental data suggested that up to 60% of the total longshore sediment transport can occur in the swash zone.

Although considerable effort has been devoted to the study of swash motions in the literature, wave-resolving, numerical modeling of the swash zone dynamics needs further improvements. In particular, an accurate model for swash motions over complex topography is needed whenever we want to simulate both gravity waves and currents on the beach under field conditions, which is crucial for sediment transport and morphodynamic modeling.

In comparison with the understanding of the swash motion in the cross-shore direction, little is known about the magnitude and structure of alongshore currents in the swash zone generated by obliquely incident waves. The field experiment SWASHX conducted at Scripps Beach, California in 2000 has shown that there exist strong alongshore currents in the swash zone and further field measurements were carried out in the Nearshore Canyon Experiment (NCEX) in 2003. These datasets are useful for model development and verification.

A large portion of the nearshore seabed along the world's populated coasts is covered by permeable sediments, ranging from sand and pebble to large cobbles. Although a sandy beach is often treated as an impermeable bottom because of its low permeability, there are many beaches on Earth where the effects of the porous medium can not be ignored. Examples of highly permeable beaches include shingle beaches in the United Kingdom and along the northern coasts of the United States. In addition to the natural seabed, many coastal structures, such as rubble mound breakwaters, groins, jetties and revetments, all are made of porous materials. The effects of a porous bed on surface waves include the attenuation of wave energy and modification of wave kinematics, which become even more evident in the swash zone in comparison with waves in deeper water.

Considerable research effort in the coastal and ocean engineering literature has been devoted to the calculation of damping rates for water waves over porous beds (see e.g. Liu 1973, Liu and Dalrymple 1984, Gu and Wang 1991, and Lee et al. 2002). Most of the studies were restricted to linear, dispersive waves on horizontal, permeable beds. Numerical models based on the nonlinear shallow water equations and porous flow equations have been used to study the swash motion of non-dispersive, long waves on porous beaches and coastal structures by Packwood (1983), Wurjanto and Kobayashi (1993), Van Gent (1995), and Clarke et al. (2004), among others. Flaten and Rygg (1991) were the first to employ the Boussinesq approach together with Darcy's law to model weakly dispersive and weakly nonlinear water waves over an uneven, permeable seabed. The dispersion properties of the Boussinesq-type equations with a porous layer were improved by Cruz et al. (1997) who utilized Madsen et al.'s (1992) dispersion enhancement technique. Recently, Hsiao et al. (2002) introduced a new set of Boussinesq-type equations for highly nonlinear waves over a porous bed. They used Nwogu's (1993) velocity variable to optimize the dispersion properties and retained all the nonlinear terms similar to Liu (1994) and Wei et al.'s (1995) treatment. Unlike the use of linearized flow resistance inside the porous layer by Flaten and Rygg (1991) and Cruz et al. (1997), a nonlinear
formulation of damping was employed by Hsiao et al. (2002). Albeit the improvements in dispersion and nonlinearity, Hsiao et al.'s equations using the optimized velocity variable are limited to wave motions with rather weak vertical vorticity, or to nearly irrotational flows over a permeable bed.

Significant advances in Boussinesq modeling of nonlinear water waves and breaking-generated nearshore circulation over impermeable bottoms have been made over the past decade. Recent reviews on the subject were given by Madsen and Schaffer (1999) and Kirby (2003). As foreseen by Basco (1983) and proved by Sorensen et al. (1998), Chen et al. (1999) and Chen et al. (2003), Boussinesq-type models incorporating the momentum mixing owing to wave breaking are not only able to simulate nearshore wave transformation, but also capable of modeling wave-induced surf-zone currents, including rip currents and longshore currents. However, it is worth mentioning that not all of the Boussinesq-type equations in the literature are able to model wave-induced currents. To transform a Boussinesq wave model into a wave-current model, two important issues have to be resolved besides the parameterization of energy dissipation owing to wave breaking and bottom friction. First, the correct Doppler shift for wave-current interaction needs to be enforced (Yoon and Liu 1989, and Chen et al. 1998). Second, Boussinesq-type equations derived from the assumption of potential flow have to consistently recover the vertical component of the vorticity in the momentum equations to ensure the transport of vertical vorticity associated with surf-zone currents, as demonstrated by Chen (2006). Shen (2001) was the first to retain both the vertical and horizontal components of vorticity in his Boussinesq-type equations for weakly non-hydrostatic stratified flows and surface waves over an impermeable bottom.

**LONG-TERM GOALS**

The long-term goal of the study is to develop and integrate numerical models for the understanding and prediction of nearshore processes. The focus of the project is to develop the capability of modeling alongshore surf and swash zone currents under field conditions based on Boussinesq-type formulations and field observations.

**OBJECTIVES**

The specific objectives of this project are to:

- Derive a complete set of fully nonlinear Boussinesq-type equations for waves and currents over a permeable beach, including the swash zone, and simulate waves propagating over heterogeneous porous beds.
- Extend the two-dimensional, phase-resolving Boussinesq wave model to the swash zone with an emphasis on alongshore swash motions.
- Integrate the extended model with field data to gain insight into alongshore swash currents, including the correlation between the swash motions and the energetic shear waves.

**APPROACH**

The study involves theoretical formulation, model development and verification, and integration with field observations of surf and swash motions. The starting point of the theoretical formulation is the Euler equation of motion for waves and currents above the permeable seabed and the locally-averaged Navier-Stokes equations for the flow inside the porous layer. A complete set of fully
nonlinear Boussinesq-type equations for waves and currents over a permeable beach is developed. Particular attention is paid to the conservation property of potential vorticity and the poor performance of pre-existing equations when the ratio of the porous layer thickness to the water depth is large, such as the swash zone. Stokes-type analyses are carried out to examine the dispersion and damping properties of the new set of equations.

In order to simulate nonlinear wave propagation over heterogeneous porous beds, it is desirable to solve the new set of Boussinesq equations using higher-order numerical methods. One of the numerical problems associated with the simulation of swash motions is the requirement of very fine resolution in the swash zone in order to resolve the steep bore front and to prevent numerical instabilities caused by the moving shoreline. A conventional rectangular grid mesh covering a littoral area usually has difficulty to provide enough resolution in the surf and swash zone because of the computational restraint. One of the solutions to the resolution problem is to transform the Boussinesq equations derived in a Cartesian coordinate system to the generalized curvilinear coordinates, as shown in Shi et al. (2001). The transformation allows for varying the grid spacing to give fine enough resolution in the swash zone and avoid over-resolving waves in the deep water. Furthermore, the small grid spacing in the swash zone permits the use of the wetting-drying method on a fixed grid for the treatment of a moving boundary. A comparison of different schemes for the moving boundary in the Boussinesq model is carried out to determine the optimum technique with both good accuracy and efficiency for simulating cross-shore and alongshore velocities in the swash zone. Field data collected by Dr. Britt Raubenheimer’s research group at the Woods Hole Oceanographic Institute (WHOI) and Dr. Jack Puleo at the University of Delaware (UD) will be utilized to verify the Boussinesq model with respect to wave runup and swash motion on beaches.

WORK COMPLETED

First, we have derived a new set of Boussinesq-type equations for nonlinear waves and surf-zone currents over a permeable beach (Chen 2006). A Stokes-type analysis and rational expansions were carried out to examine the fundamental damping and dispersion properties of the new set of equations (Cruz and Chen 2006a). The vortical properties of the new and pre-existing Boussinesq equations have been carefully investigated (Gobbi et al, 2006). Second, numerical implementation of porous effects into a one-dimensional Boussinesq wave model has been completed. Preliminary results were documented in Cruz and Chen (2004). We have tested the numerical model against laboratory experiments on wave transformation over heterogeneous porous beds and submerged rubble mounds. The results have been published in Cruz and Chen (2006b). Third, efforts have been devoted to the extension and testing of the Boussinesq wave model, FUNWAVE 2.0, with respect to swash motions. We have examined three different schemes for the treatment of a moving shoreline with an emphasis on the swash velocity. Tests of the improved FUNWAVE model against analytical, laboratory and field data of swash and surf zone currents have been carried out (Chen and Briganti 2006). Fourth, Lagrangian descriptions of the fluid motion have been employed to study the mixing of surf zone currents, which serves as an alternative to Eulerian analyses of the flow field (Briganti et al. 2006).

RESULTS

The major results obtained in the project are: 1) the development of a complete set of Boussinesq-type equations suitable for water waves and wave-induced nearshore circulation over an inhomogeneous, permeable seabed, 2) the implementation and testing of a new numerical model, namely PFUNWAVE 1D, 3) the implementation and testing of numerical schemes for the simulation
of alongshore swash currents under the framework of FUNWAVE2D, and 4) numerical simulations of surf and swash zone currents under field conditions.

**Fully Nonlinear Boussinesq-Type Equations for Waves and Currents over Porous Beds**

Our research has resulted in a complete set of Boussinesq-type equations suitable for water waves and wave-induced nearshore circulation over an inhomogeneous, permeable seabed. The derivation starts with the conventional expansion of the fluid particle velocity as a polynomial of the vertical coordinate $z$ followed by the depth integration of the vertical components of the Euler equations for the fluid layer and the volume-averaged equations for the porous layer to obtain the pressure field. Inserting the kinematics and pressure field into the Euler and volume-averaged equations on the horizontal plane yields a set of Boussinesq-type momentum equations with vertical vorticity and $z$-dependent terms. We developed a new approach to eliminating the $z$-dependency in the Boussinesq-type equations. This technique allows for the existence and advection of the vertical vorticity in the flow field with the accuracy consistent with the level of approximation in the Boussinesq-type equations for the pure wave motion. The complete set of equations is an extension of Hsiao et al.’s (2002) equations for nonlinear waves over a porous bed without invoking their assumptions of weak vertical vorticity and weak variation of damping coefficients. In addition, the new set of equations (Chen 2006) is applicable to impermeable, solid beds, which enhances the conservation property of potential vorticity compared to pre-existing equations of Wei et al. (1995), Liu (1994) and Chen et al (2003).

Motivated by the poor performance of pre-existing Boussinesq-type equations when the thickness of the porous layer is several times larger than the water depth, we have examined the scaling of the resistance force and realized the significance of the vertical velocity to the pressure field in the porous layer. This leads to the retention of higher-order terms associated with the damping in the momentum equations. An analysis of the vortical property of the resultant equations indicates that the energy dissipation in the porous layer can serve as a source of vertical vorticity up to the leading order. In comparison with the pre-existing Boussinesq-type equations for both permeable and impermeable bottoms, the complete set of equations improves the accuracy of potential vorticity as well as the damping rate. The equations retain the conservation of potential vorticity up to $O(\mu^2)$, where $\mu$ is the measure the frequency dispersion. Such a property is desirable for modeling wave-induced nearshore circulation. Moreover, the procedure of consistently recovering the vertical vorticity and eliminating the $z$-dependency can be used to extend a variety of Boussinesq-type equations originally derived for potential flows to quasi-rotational wave-current motions in the nearshore.

A Stokes-type analysis and rational expansions were carried out to extract the fundamental properties from the complex Boussinesq equations. The linear dispersion relationship and the damping rate owing to the porous layers were compared with the exact solutions for linear waves over a homogeneous, porous, flat bottom (Cruz and Chen 2006a). We developed a new optimization technique to determine the two model parameters for the new equations (Cruz and Chen 2004). The new procedure has been extended to realistic bottom topography, allowing for variations of the model parameters to account for spatial variations of bottom permeability and porous-layer thickness.

Figure 1 illustrates the improvement of the complete set of equations in comparison to the existing model. The horizontal axis is the ratio of the water depth to the wave length in the deep water while the vertical axis represents the damping rate over a local wave length. The ratio of the porous layer thickness to the water depth is 10. It is seen in the left panel that the damping rate of the new set
of equations agrees very well with the exact solution (solid line). By contrast, the damping rate of the pre-existing equations starts deviating from the exact solution when the relative water depth is greater than 0.04, as shown in the right panel. This demonstrates that the damping property of the new set of Boussinesq-type equations is much better than the pre-existing equations when the thickness of the porous layer is much greater than the local water depth. Because of the improvements in the vortical and damping properties, the new set of Boussinesq-type equations will lead to a better numerical model for nonlinear waves and surf-zone currents over permeable beds.

![Figure 1. Comparison of the modeled damping rates (dashed lines) with the exact solution (solid lines). The new set of equations (left) is much better than the pre-existing equations (right) when the thickness of the porous layer is much greater than the local water depth. (Chen 2006)](image)

A Numerical Model for Wave Propagation over Heterogeneous Permeable Seabed

We have developed a new Boussinesq model for nonlinear waves propagating over porous beds (Cruz and Chen 2006b). The numerical model was tested against the analytical solution of linear waves on a horizontal, homogenous permeable bottom. Excellent agreement has been found. Figure 2 shows a comparison of the modeled wave profiles on an impermeable bed (solid line) and over a permeable (red dotted line) layer of coarse sand. It is seen that the porous damping results in smaller wave height and weaker higher harmonics than does the impermeable sloping bottom. However, the wave celerity, or wave length is hardly influenced by the porous layer during wave shoaling. On the horizontal bed, the waves over the solid bottom are slightly faster than the waves over the porous bed because of the accelerating effects of wave nonlinearity and wave breaking.

We have tested the new numerical model against two sets of laboratory data of waves over porous beds. Figure 3 shows a model and data comparison. The top panel depicts the wave envelope and a snapshot of the computed free surface elevation over a porous sill. The grand size is 6.7 mm and the porosity is 0.44. It is seen in the middle panel that the computed wave heights are in good agreement with the laboratory measurement at eight gage stations.

To verify the model accuracy for nonlinear waves propagating over uneven porous beds, the model was further tested against the laboratory data with detailed measurements of free surface
profiles. Figure 4 shows the computed spatial profiles, wave heights, and comparison of modeled and measured time profiles of waves over an underwater gravel mount. The solid lines in the bottom panels are the model results (Cruz and Chen 2006b), while the symbols are the measurements from Hsiao et al. (2002). It is seen that PFUNWAVE simulates well the shape and magnitude of time profiles at most of the 9 stations, although there are deviations in the last three gages. The new numerical model has the potential to model nonlinear waves over heterogeneous porous beds.

![Figure 2: Comparison of modeled wave profiles on an impermeable bed (solid line) and over a permeable (red dotted line) layer of coarse sand. H = 0.5 m, T = 8 sec, the porous layer: n = 0.40, \( d_{50} = 2 \text{ mm} \).](image)

![Figure 3: Model and data comparison of wave transformation over a porous sill on an impermeable flat bottom. Solid lines: computational results. Red dotted line: laboratory measurements. H = 0.022 m, T = 1.02 sec, porous sill: n = 0.44, \( d_{50} = 6.7 \text{ mm} \).](image)
Figure 4: Special profiles, wave heights, and comparison of modeled and measured time profiles of waves over an underwater gravel mount. Solid lines: model results (Cruz and Chen 2006b); Symbols: measurements (Hsiao et al. 2002).
Improvement of Swash Modeling

To simulate swash motions and longshore currents, it is necessary to include an appropriate treatment of the moving shoreline boundary condition in a Boussinesq wave model. Various techniques have been proposed in the literature (e.g. Madsen et al. 1997, Kennedy et al. 2000, Bellotti and Brocchini 2001, and Lynett et al. 2002). In the present study, we have focused on three relatively simple methods, namely the slotted-beach technique, the thin-film technique, and the wetting-drying method, for handling the shoreline movement with breaking-induced currents.

Instead of tracking the wet and dry cells during wave run-up/run-down on the beach, the slotted-beach technique treats the entire computational domain as an active fluid domain. The basic idea behind this technique is to replace the solid bottom, where there is very little or no water covering the land, by a porous seabed, or to assume that the solid bottom contains narrow slots. This allows the water level to fall below the beach elevation. The replacement of the solid bottom by narrow slots results in a modification of the mass equation. Three parameters influence the extent of such a modification. They are the relative width of slot with respect to a unit width of beach ($\delta$), the shape parameter ($\lambda$) for the smooth transition from unity to a narrow slot, and the depth of a slot, or the offshore still water depth ($h_0$) where a slot begins.

Madsen et al. (1997) showed that, even though a very narrow slot width is used, there is still about a 10% error in the computed maximum run-up in comparison with the analytical solution of Carrier and Greenspan (1958). This is attributed to the additional cross-sectional area introduced by the narrow slot, because the maximum run-up is very sensitive to the total volume of mass at the run-up tip. In an attempt to improve the conservation property for mass in the presence of a slot, Kennedy et al. (2000) retained an equivalent cross-sectional area of a unit width of beach by slightly elevating the physical seabed elevation, leading to the improvement in the simulation of wave run-up. The optimal values of $\delta$ and $\lambda$ are found to be 0.002 and 80, respectively, which give the best agreement with the analytical solution obtained by Carrier and Greenspan (1958). Although the slotted-beach technique implemented in the Boussinesq model FUNWAVE2D is simple for the complex problem of moving boundaries, the accuracy of the technique is limited by the need for a larger slot width and numerical filtering near the shoreline to suppress the noise generated by numerical instabilities associated with the moving shoreline for wave run-up on a steep slope and under field conditions. The accuracy of the velocity prediction near the shoreline also is not as good as the run-up prediction.

Similar to the slotted-beach technique, the thin-film scheme treats the entire computational domain as an active fluid (wet) domain. Different from the slotted-beach technique, the thin-film scheme imposes an artificial thin film of water on the surface of the dry beach without alternating the solid bottom. The governing equations are then solved in the entire wetted domain, including the artificially wetted beach area (Oey 2005, Long et al. 2006). Although this technique also gives good estimates of the shoreline position during wave runup and rundown on the beach, the accuracy of the predicted swash velocity is still limited in comparison with the benchmark test based on the analytical solution of the shallow water equation for long wave runup on a constant slope.

The third technique we have employed in the present study is the wetting-drying method of Lynett et al. (2002) who used a conceptually simple method of linear extrapolation to extend the dependent variables in the wet cells to the dry beach without alternating the seabed. In contrast to the slotted-beach method, the computational domain is explicitly divided into wet and dry cells, and the equations of motion are solved in the wet region only. The distinction between wet and dry regions is
performed on the basis of a threshold value for the local water depth. This threshold is set to be $a_0/50$, where $a_0$ is the offshore wave amplitude. If the local total depth is smaller than or equal to the threshold value, it is considered as a dry cell.

To treat the boundary condition at the interface between the wet and dry regions, a linear extrapolation method is used to assure the existence of the shoreline between two grid points and to provide the boundary conditions at the shoreline. This approach is simple and straightforward in one-dimension (1D) but becomes quite involved for complex shoreline geometry in two horizontal dimensions (2HD). In order to model alongshore surf and swash zone currents, periodic lateral boundary conditions are desirable. We therefore implemented and extended Lynett et al.’s (2002) run-up scheme to periodic cross-shore boundaries in FUNWAVE2D. In the case of 2HD, an extrapolated variable in a dry grid point is obtained from an average of the values resulting from 1D linear extrapolation of the surrounding wet points. At each of the lateral boundary, the extrapolation and average have to involve the counterparts at the opposite lateral boundary to assure the alongshore periodicity of the domain.

A large number of model tests have been carried out to examine the skills of the extended wetting-drying scheme implemented into FUNWAVE2D with an emphasis on the velocity and current prediction. The tests include 1) Carrier and Greenspan’s (1998) analytical solution of nonlinear long wave run-up on a sloping bottom, 2) Thacker’s (1981) analytical solution of wave resonance in a circular parabolic basin, 3) Briggs et al’s (1995) laboratory experiment on solitary wave run-up on a circular island, and 4) field observations of longshore currents at the U.S. Army Corps of Engineers Field Research Facility in Duck, North Carolina. We shall focus on the comparison with the analytical solutions and the dataset collected during the SandyDuck field experiments in 1997.

The analytical solution of Carrier and Greenspan (1958) has served as the classic benchmark test for many wave run-up schemes. We consider a 10s wave train with an initial wave height of 0.006m in a wave flume containing a 1:25 slope. The still water depth in the flat portion of the flume is 0.5m. Figure 5 shows the comparison of the numerical (dashed lines) and exact (solid lines) solutions of the free surface profiles and velocity profiles. It is seen in the left panel that the wetting-drying scheme implemented into FUNWAVE2D is able to accurately reproduce Carrier and Greenspan’s analytical solution, including the maximum swash excursion, and the nodes and anti-nodes of the standing waves. More importantly, the modeled swash velocities are in excellent agreement with the exact solution, as seen in the right panel.

Another benchmark test often utilized in the literature for run-up schemes is Thacker’s (1981) analytical solution of wave resonance in a circular basin with a parabolic bottom profile under the framework of nonlinear shallow water equations. Given a parabolic bottom profile in a circular basin of radius $R$ at the still water level and the maximum still-water depth $h_0$, the radial velocity $u$ is

$$u = -\frac{\varphi r A \sin(\varphi t)}{2(1 - A \cos(\varphi t))} \quad (1)$$

$$A = \frac{R^4 - r^4}{R^4 + r^4} \quad (2)$$
where $\phi =$ resonance frequency, $r_o =$ the initial radius of the shoreline.

Using the same input conditions as those in Lynett et al. (2002) with $r_o = 2000 \text{ m}$ and $R = 2500 \text{ m}$, we obtained both numerical and analytical velocity and surface profiles as shown in Figure 6, where the full lines are model result while the dotted lines represent the exact solutions. It is seen from the figure that the numerical result of FUNWAVE2D with the extended wetting-drying scheme matches very well the analytical solution in both elevation and velocity. Notice that the Boussinesq dispersive terms were switched off in order to be consistent with the nonlinear shallow water equations used in Thacker's (1981) analytical solution. It is seen in Figure 6 that the wetting-drying scheme implemented into FUNWAVE2D predicts the swash velocities very well under non-breaking, long wave conditions with simple bathymetry.
Alongshore Drift Velocity in the Swash Zone

After validating the model using the simple, non-linear shallow water solutions with an emphasis on the cross-shore swash velocity, it is important to explore the capabilities of the model in predicting the main features of the swash zone with broken waves. An interesting aspect of swash motions is the drift velocity of the longshore flow in the swash zone. The importance of this component was portrayed in Brocchini (2006) and Antuono et al. (2006), who have suggested that swash motions can influence the entire surf zone dynamics.

We computed the alongshore drift velocity of the water mass in the swash zone by integrating the instantaneous longshore velocity component over the cross-shore excursion of the swash zone and averaging it over two short-wave periods. The drift velocity serves as a measure of the swash influence at the seaward limit of the swash zone on the alongshore surf zone current. For simplicity, we considered an obliquely incident, monochromatic wave train, modulated by a longer wave and propagating on a longshore uniform beach with a constant slope of 1:30. The offshore water depth is 0.4 m, and the incoming short waves have an amplitude \( a_0 = 0.024 \text{ m} \) and a period of 1 s. This simple simulation is also useful for testing the periodic cross-shore boundary conditions in the extended Boussinesq model with respect to surface waves and longshore flows in both surf and swash zones.

Figure 7 shows a snapshot of the surface elevation and the drift velocity after 40s from the beginning of the simulation, respectively. First, it is seen that the periodic cross-shore boundary conditions work well. Second, we notice that the alongshore drift velocity averaged over two wave periods and over the cross-shore excursion of the swash zone varies in the longshore direction and grows with time in the early stages of development. Further improvement in the algorithm for tracking the seaward limit of the swash motion and the velocity in the swash zone is needed to remove the kinks in Figure 7. Obviously, the magnitude of the alongshore drift velocity in the swash zone is sensitive to the bottom friction, similar to the longshore current in the surf zone. Detailed comparisons with the analytical solution in Antuono et al. (2006) are currently undergoing.

Simulations of Field Experiments

The SandyDuck coastal field experiment was conducted in October 1997 at the U.S. Army Corps of Engineers Field Research Facility (FRF) at Duck, North Carolina. Figure 8 shows the bottom topography at the FRF close to October 2, 1997 when the field dataset for the time period 0400-0700
EST is used to test the model. The model bathymetry was constructed from a composite of high resolution mini-grid data and an interpolation between the two larger-area surveys that bracket the study time. It is seen that the bottom depression under the FRF research pier significantly alters the barred-beach, and consequently influences the surface waves and wave-driven longshore currents. It is desirable to test the extended Boussinesq model with the enhanced conservation property of potential vorticity and swash velocity prediction under the field conditions.

Input wave conditions were derived from published frequency-direction spectra which are available from the FRF. The random waves are mapped onto a finite number of components that satisfy the requirement of longshore periodicity (Chen et al. 2003). Realizations of 3.5 hour duration were constructed using linear superposition of directional components and random phases, which has a root-mean-square wave height of 1.06 m and a peak wave period of 6.6 s. The peak angle of the directional spectrum is 26°, which means that the waves came from the northeast. The bottom friction coefficient is set to be 0.0015.

Figure 8: Instrument array (dots) and bathymetry at the FRF.

Figure 9 illustrates the time sequence of the velocity and vertical vorticity fields predicted by the enhanced Boussinesq wave model under the SandyDuck field conditions. The solid lines are the contours of water depth and the shoreline is on the left. We notice that the wetting-drying scheme extended to periodic cross-shore boundary conditions handles the moving shoreline and swash motion fairly well although filtering is required to stabilize the model.

The longshore current shown in Figure 9 was obtained by averaging the computed fluid particle velocity over 5 peak wave periods to filter out the short wave motion. It is seen that the longshore current meandered and the flow pattern changed with time. Strong vortices accompanied by transient seaward currents were triggered by shear instabilities and bathymetric variations, in particular at the seabed depression under the research pier. The predicted flow field differs from previous studies using Boussinesq wave models, as the vertical vorticity is now transported by the depth-average velocity, similar to phase-averaged models based on the nonlinear shallow water equations. A comparison of the modeled and measured mean longshore currents averaged over two hours is shown in Figure 10. Fairly good agreement is found.
Figure 9: Time sequence of the velocity (arrows) and vertical vorticity fields predicted by the enhanced Boussinesq model under the SandyDuck field conditions. The solid lines are contours of water depth. The shoreline is on the left.

Figure 10: Comparison of the modeled and measured mean longshore currents under the SandyDuck field conditions using the enhanced Boussinesq wave model.
To investigate the effect of different treatments of shoreline boundary conditions on the prediction of longshore surf zone currents, a series of numerical experiments using a longshore uniform beach have been carried out. We constructed the model bathymetry using the cross-shore beach profile along the instrumentation array at 827.5 m (see the bottom panel in Figure 10) as the representative beach profile from the field experiment. The wave input uses a monochromatic wave train to represent the wave conditions in the field. Comparisons were made among the flow fields predicted by the Boussinesq model with the slotted-beach and wetting-drying techniques for a treatment of the moving shoreline. We have focused our attention on the mean longshore current velocity and vertical vorticity component as well as the relative standard deviation that is a measure of the variability of the flow field owing to shear instabilities.

Albeit a strong similarity among the predicted maximum and mean longshore currents inside the surf zone by the shoreline treatment techniques, it is evident that several differences exist. The significant alternation of the seabed by the slots can increase the wave celerity and the mean longshore current outside the surf zone. Therefore, care must be taken when choosing the three slot parameters. The considerable differences in the vertical vorticity and velocity shear near the shoreline suggest that more in-depth investigation on the alongshore drift velocity produced by the three shoreline treatment techniques is indeed needed. Such a study requires three things. First, much higher resolution of the flow in the swash zone is necessary in comparison to the typical resolution of the surf zone in a Boussinesq model with rectangular grids. Second, a consistent and accurate algorithm for defining and tracking the seaward limit of the swash zone should be used for both moving shoreline schemes in order to properly partition the swash and inner surf zones where the mean alongshore drifts are defined differently. Third, a better-devised filter is desirable in order to suppress noises near the shoreline associated with the moving shoreline treatment techniques because none of them is free from numerical instabilities under breaking wave and realistic seabed conditions. This is especially important for the simulation of the NCES dataset.

Detailed comparisons with the field data of SandyDuck and NCEX are being pursued. In addition to the theoretical and numerical results summarized above, we have utilized a particle tracking technique to examine the Lagrangian particle statistics of the flow fields produced by three sets of Boussinesq-type equations implemented in the FUNWAVE2D, including the equations developed in this project by Chen (2006). Figure 11 shows the time sequence of the particle trajectories inferred from the output of the Boussinesq wave models and SHORECIRC under the SandyDuck field conditions (Briganti et al. 2006). The solid lines represent the contours of water depth and the shoreline is on the right (note that the coordinates are different from the FRF coordinates).

A cluster of weightless particles were seeded shoreward of the sand bar crest at t=0. The particles were transported and dispersed by the surf zone currents generated by wave breaking. Owing to the cross-shore currents, a number of the particles were transported outside the surf zone at the early stages of the simulation and through the bottom depression under the research pier. Once they were outside the surf zone, the particles moved very slowly because of the weak currents. It is seen that only a portion of the cluster remains in the bar trough where they were transported by the strong longshore current. The particle tracking technique serves as an alternative to conventional Eulerian descriptions of surf zone currents. Particle statistics are being computed to examine the mixing properties of longshore currents predicted by phase-resolving and phase-averaged nearshore circulation models (Briganti et al 2006 and Kirby et al. 2006).
Figure 11: Time sequence of the particle trajectories inferred from the output of the improved Boussinesq wave model under the SandyDuck field conditions (Briganti et al. 2006). The solid lines represent the contours of water depth and the shoreline is on the right. Black: Chen et al. (2003), Red: Chen (2006), and Blue: SHORECIRC.
IMPACT/APPLICATIONS

The results obtained from this project are expected to improve the Navy’s capability of modeling swash zone processes. First, the study extends the applicability of Boussinesq models (Chen et al., 1999, and Chen et al., 2003) to the swash zone and permeable beds. This will provide sediment transport models with more realistic estimates of cross-shore and alongshore velocities in the swash zone. Therefore, improvements in predicting sediment transport in the swash zone are anticipated. Second, the complex nature of nearshore processes calls for the integration of numerical models with field measurements in our research. The extended model will become available for researchers at the Naval Research Laboratories and other institutions to complement their field study of swash zone processes. In addition, the project has also complemented the NOPP project led by Dr. Jim Kirby at the University of Delaware to develop and verify a community model for nearshore processes (Kirby et al. 2006). The phase-resolving Boussinesq model with extension to the swash zone provides the phase-averaged wave and current models developed in the NOPP project with useful information about the swash motion and the shoreline boundary conditions. The results will give new insight into alongshore swash zone currents. In addition, the Boussinesq model for irregular waves propagating over porous beds is useful for the MURI study on remote sensing of seabed properties.

PUBLICATIONS


**REFERENCES**


