STINFO FINAL REPORT

This work has been submitted to AIAA for publication in the AIAA Atmospheric Flight Mechanics Conference proceedings. If this work is published, it is considered a work of the U.S. Government and is not subject to copyright in the United States.
NOTICE

Using Government drawings, specifications, or other data included in this document for any purpose other than Government procurement does not in any way obligate the U.S. Government. The fact that the Government formulated or supplied the drawings, specifications, or other data does not license the holder or any other person or corporation; or convey any rights or permission to manufacture, use, or sell any patented invention that may relate to them.

This report was cleared for public release by the Air Force Research Laboratory Wright Site (AFRL/WS) Public Affairs Office (PAO) and is releasable to the National Technical Information Service (NTIS). It will be available to the general public, including foreign nationals.


THIS TECHNICAL REPORT IS APPROVED FOR PUBLICATION.

/s/        /s/
_______________________________________  ______________________________________
David Doman          Deborah S. Grismer
Senior Aerospace Engineer        Chief
Control Design and Analysis Branch        Control Design and Analysis Branch
Air Force Research Laboratory        Air Force Research Laboratory
Air Vehicles Directorate        Air Vehicles Directorate

/s/
_______________________________________
Brian W. Van Vliet
Chief
Control Sciences Division
Air Force Research Laboratory
Air Vehicles Directorate

This report is published in the interest of scientific and technical information exchange and its publication does not constitute the Government’s approval or disapproval of its ideas or findings.
**ABSTRACT**

A method for estimating the effects of nose cone ablation on the aerodynamic forces and moments of a reentry vehicle is presented. The method provides a means of generating a rough, but representative model that makes use of interpolated test or prediction data using instantaneous nose cone mass as an interpolation variable. An interpolation function is proposed that can be used to approximate the location of points on the outer-moldline as well as the aerodynamic characteristics between node points. While the changes in overall vehicle mass properties due to ablation are typically small, one can make use of the interpolated outer-mold-line information to estimate mass properties, such as moments of inertia or center of gravity. The interpolated aerodynamic and mass properties provide a means for generating data for dynamic simulations for use in vehicle proof-of-concept studies.
Ablation Modeling for Dynamic Simulation of Reentry Vehicles

David Doman*

Air Force Research Laboratory/VACA, Wright-Patterson AFB, OH 45433-7531

1. Introduction

Dynamic simulation of a reentry vehicle is often required to perform trajectory optimization and analysis and to analyze the robustness of guidance and control system. Reentry vehicle trajectories vary widely depending on mission requirements and vehicle design. Vehicles that experience relatively low dynamic pressure (<300 psf) with reusable thermal protection systems such as the shuttle experience little if any ablation. At the other extreme, ballistic missiles can experience high dynamic pressures (> 20000 psf) and the associated high reentry heating will typically cause the nose-cone material to ablate. A number of vehicle characteristics change as a result of ablation. The mass properties of the vehicle change due to the loss of nose cone material and the aerodynamic forces and moments acting on the vehicle change as a result of the change in OML. These changes can have an adverse effect on aerodynamic efficiency and guidance and control system performance. A number of experimental techniques exist that allow one to measure the changes to the OML\textsuperscript{1,2} from test data. An empirical method is described that can be used to translate limited test data into a rough, but representative model that can be used to estimate the effects of ablation on a vehicle’s ability to follow a prescribed trajectory and on guidance and control performance and robustness.

II. Generation of Outer Mold Lines from Test Data

In order to generate an aerodynamic model of an ablated vehicle, an approximation for the outer mold line must be constructed. Measurements of reentry vehicle nose cones exist from tests where the nose cone of the flight test vehicle was equipped with a radioactive recession sensor which allowed the nose cone shape to be reconstructed from flight test data. The shape of the nose-cone are sometimes provided in the form of an upper and lower surface curve in the x-z plane in body-axis coordinates. Hypersonic aerodynamics codes such as SHABP, require a 3-D grid that describes the outer-mold line. Since only 2-D profiles are obtained from recession measurements, OML grid points outside of the x-z plane must be estimated. The unablated nose cones of many reentry vehicles are axisymmetric and the remainder of this section presents a method that can be applied to such bodies.

The estimation of the out-of-plane grid points can be performed by interpolating between two surfaces of revolution about the x-body axis. Figure 1 shows how the upper and lower curves defining a body in the x-z plane can be used to generate an interpolated body of revolution in the y-z plane.

The requirements are that the interpolated OML must to be coincident with the upper surface when $\theta = \pi/2$ and with the lower surface when $\theta = -\pi/2$, where $\theta$ is the angle (in the y-z plane) measured from the y-axis to a point on one of the surfaces. Furthermore, the interpolated surface must be continuous at $\theta = \pm \pi/2$. These boundary conditions are motivated by the fact that the OML of the unablated vehicle could be described by a surface of revolution. For lifting reentry vehicles, test data shows that leeward surfaces ablate less than the windward surfaces. Thus, it is assumed that maximum ablation occurs on the windward side and that minimum ablation occurs on the leeward side and that all other points lie somewhere in-between. We now construct an interpolation function that satisfies these boundary conditions.

Let $\lambda(\theta) \in [0,1]$ be an interpolating function such that:

$$r_i(\theta) = \lambda(\theta)r_u + \left|1 - \lambda(\theta)\right|r_l$$

*Senior Aerospace Engineer, AFRL/VACA, 2210 Eight Street, Bldg 146,Wright-Patterson AFB, OH. 45433-7531, Associate Fellow AIAA, david.doman@wpafb.af.mil

American Institute of Aeronautics and Astronautics
Figure 1. Cubic interpolation between two bodies of revolution
where \( r_i \) is the radius of the interpolated surface, \( r_u \) is the radius of upper surface (leeward side) and \( r_l \) is radius of the lower surface (windward side). Note that a simple linear interpolating function will not satisfy the continuity conditions because a linear function does not contain enough free parameters. A cubic polynomial interpolating function will satisfy our four required conditions, i.e.

\[
\begin{align*}
  r_i(\pi/2) &= r_u \\
  r_i(-\pi/2) &= r_l \\
  \frac{d\lambda}{d\theta}(\pm\pi/2) &= 0
\end{align*}
\]  

(2)

The general form for the third order interpolating polynomial is given by:

\[
\lambda(\theta) = a\theta^3 + b\theta^2 + c\theta + d
\]

(3)

The coefficients of the polynomial equation must satisfy the following equation:

\[
\begin{pmatrix}
  (\pi/2)^3 & (\pi/2)^2 & (\pi/2) & 1 \\
  (-\pi/2)^3 & (-\pi/2)^2 & (-\pi/2) & 1 \\
  3(\pi/2)^2 & 2(\pi/2) & 1 & 0 \\
  3(-\pi/2)^2 & 2(-\pi/2) & 1 & 0
\end{pmatrix}
\begin{pmatrix}
  a \\
  b \\
  c \\
  d
\end{pmatrix}
= \begin{pmatrix}
  1 \\
  0 \\
  0 \\
  0
\end{pmatrix}
\]

(4)

Solving for the polynomial coefficients one obtains:

\[
\begin{pmatrix}
  a \\
  b \\
  c \\
  d
\end{pmatrix} = \begin{pmatrix}
  -0.064503008886399 \\
  0 \\
  0.477464829275686 \\
  0.5
\end{pmatrix}
\]

(5)

Substituting back into Equation 1 one can estimate the radius of the outer mold line in the \( y-z \) plane at a given \( x \)-station for known values of \( r_u \) and \( r_l \) via

\[
\begin{align*}
  y_i &= r_i \cos(\theta) \\
  z_i &= r_i \sin(\theta)
\end{align*}
\]

(6)

### III. Computing Instantaneous Mass of Nose Cone

First we will assume that the nose cone density \( \rho_n \) is constant throughout. It is also assumed that test data is available that provides the upper and lower curves for one or more ablated cases. The volume and thus the mass of the ablated and unablated nose cones can be computed using the 3-D OMLs generated using the techniques described in the previous section. The following formula for predicting ablation mass loss rates is proposed in:\(^3\)

\[
\frac{dm}{dt} = -\frac{C_H}{H^2} \rho V^3 S
\]

(7)

where \( m \) is the mass of the nose cone, \( C_H \) is a dimensionless heat transfer coefficient less than unity, \( H \) is the effective heat of ablation, \( \rho \) is the air density, \( V \) is the velocity of the body with respect to the air mass, and \( S \) is the current cross sectional frontal area of the body. Note that \( C_H \) and \( H \) are vehicle and material dependent parameters and we hereby denote the ratio \( C_H/H \) as \( K_H \). Integrating Equation 7 over the nominal profile, we obtain:

\[
m_f - m_i = -\frac{1}{2} K_H S \int_0^t \rho(t)V(t)^3 dt
\]

(8)

where \( m_f \) and \( m_i \) are the initial and final values of the nosecone mass. Denoting \( \Delta m = m_f - m_i \) and solving for \( K_H \) we have:

\[
K_H = 2 \frac{\Delta m}{S \int_0^t \rho(t)V(t)^3 dt}
\]

(9)

Estimates of \( \rho \) can be obtained from an atmospheric model, \( V \) can be measured or estimated from test instrumentation, and changes in mass can be determined from recession measurements. Thus, Equation 9 can be used to empirically estimate \( K_H \) for the flight profile for which the recession measurements were obtained. The integral is not analytically tractable, thus the the integral must be evaluated by quadrature.

---

American Institute of Aeronautics and Astronautics
IV. Interpolating Outer Mold Lines

While the centroid and elements of the inertia tensor of the nose cone can change significantly as a result of ablation, those effects usually have little influence upon the mass properties of the vehicle as a whole. For the purposes of dynamic simulation, the changes in vehicle mass properties due to ablation can often be ignored. Computation of the OML based on instantaneous mass however, is important because the aerodynamic properties are a strong function of the OML. Since the aerodynamic properties are primarily influenced by OML, and OML and mass are related, a method is proposed that will allow the aerodynamic characteristics of an ablated profile to be estimated using using instantaneous mass as an interpolation variable.

Generally, a small number of OMLs are known from test or prediction data. We would like to estimate the shape and aerodynamic characteristics of the vehicle based on measurements or predictions of a finite number of OMLs and estimates instantaneous mass. During a simulation run, the time rate of change of vehicle mass due to ablation can be computed $dm/dt$ via Equation 7. Thus the mass of the vehicle can be computed as a function of time in a dynamic simulation. Given the masses associated with known OMLs, it is desired to estimate the instantaneous OML location and the associated aerodynamic characteristics for any specified mass. In order to obtain a point on an OML for a given mass, one must interpolate between two points on adjacent OMLs. It is postulated that ablation occurs normal to the innermost surface when interpolating between an inner and outer surface as shown in Figure ???. Thus, one can match control points on the inner surface to corresponding points on the outer surface. It is also assumed that points on an
interpolated OML move along straight lines that connect the control points on the inner and outer surfaces. Points on the interpolated OML are computed as follows:

\[
x_{OML} = \lambda_m(m)x_i + (1 - \lambda_m(m))x_o
\]
\[
z_{OML} = \lambda_m(m)z_i + (1 - \lambda_m(m))z_o
\]  

(10)

where \( \lambda_m(m) \in [0, 1] \) is an interpolation function that relates changes in distance to changes in the instantaneous mass of the vehicle. Since mass or volume is a cubic function of distance, we postulate the following form for the interpolation function:

\[
\lambda_m(m) = K_1m^\frac{3}{2} + K_2
\]  

Clearly \( \lambda_m(m_o) = 0 \) and \( \lambda_m(m_i) = 1 \), thus

\[
K_1 = \frac{1}{m_i^\frac{3}{2} - m_o^\frac{3}{2}}
\]
\[
K_2 = \frac{-m_o^\frac{3}{2}}{m_i^\frac{3}{2} - m_o^\frac{3}{2}}
\]  

(12)

The distance that a point moves along a line connecting the inner and outer control points is assumed to be related to the cube root of the volume or mass. The result is exact for interpolation between spheres. The errors are small for transition between objects that are nearly geometrically similar.

The interpolation of aerodynamic tables using instantaneous mass proceeds along similar lines since the aerodynamic properties are primarily influenced by OML. Typically aerodynamic force and moment coefficients are tabulated as a function of angle-of-attack, sideslip angle, Mach number and control surface deflections. For a vehicle that experiences ablation, the aerodynamic data will also be a function of the instantaneous outer mold line of the vehicle. It is proposed that additional aerodynamic tables be constructed for a representative set of ablated OMLs. Between the node points established for the OMLs it is proposed that the tables be interpolated using the interpolation function given in Equation 11 because of the strong influence of OML on aerodynamic coefficients. For an arbitrary aerodynamic coefficient the following interpolation scheme is proposed:

\[
C_e(M, \alpha, \beta, \delta, m) = \lambda(m)C_e(M, \alpha, \beta, \delta, m_i) + [1 - \lambda(m)]C_e(M, \alpha, \beta, \delta, m_o)
\]  

(13)

Thus, to interpolate the aerodynamic tables, one must estimate the instantaneous mass of vehicle using Equation 7, and use the interpolation function given by Equation 11 in conjunction with Equation 13.

V. Summary

A method for estimating the effects of nose cone ablation on the aerodynamic forces and moments of a reentry vehicle has been presented. The method provides a means of generating a rough, but representative model that makes use of interpolated test or prediction data using instantaneous nose cone mass as an interpolation variable. Since aerodynamic properties are primarily influenced by outer-mold-line, an interpolation function was proposed that can be used to approximate the location of points on the outer-mold-line as well as the aerodynamic characteristics between node points. While the changes in overall vehicle mass properties due to ablation are typically small, one can make use of the interpolated outer-mold-line information to estimate mass properties, such as moments of inertia or center of gravity. The interpolated aerodynamic and mass properties provide a means for generating data for dynamic simulations for use in vehicle proof-of-concept studies.

References