### Title and Subtitle
A Unified View of Global Instabilities of Compressible Flow Over Open Cavities

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### Abstract
We report progress in our ongoing effort to compute and understand the three-dimensional instabilities (resonance) of open cavity flows from incompressible to supersonic speeds. In particular, our work is aimed at regimes where significant interactions occur between the shear layer spanning the cavity and the recirculating flow within the cavity, as encountered in many experiments and numerical simulations reported in the literature. Complementary methodologies for extracting information about global instabilities (including their receptivity and optimal control) of two- and three-dimensional cavity flows have been developed. We present here some sample calculations that show that for a low Mach number cavity with a length-to-depth ratio of two, the two-dimensional steady flow is unstable to three-dimensional (spanwise homogeneous) disturbances that consist of spanwise modulation of the recirculating vortex interior to the cavity. The oscillations are unstable over a narrow band of spanwise wavelengths comparable to the cavity depth. They are oscillatory in time, but with a very slow frequency that is about ten times slower than the incipient two-dimensional Rossiter instability. Instability seems to be related to cellular patterns observed in surface streamline patterns on cavity bottoms in some previous experiments.

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A UNIFIED VIEW OF GLOBAL INSTABILITIES OF COMPRESSIBLE FLOW OVER OPEN CAVITIES

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Abstract
We report progress in our ongoing effort to compute and understand the three-dimensional instabilities (resonance) of open cavity flows from incompressible to supersonic speeds. In particular, our work is aimed at regimes where significant interactions occur between the shear layer spanning the cavity and the recirculating flow within the cavity, as encountered in many experiments and numerical simulations reported in the literature. Complementary methodologies for extracting information about global instabilities (including their receptivity and optimal control) of two- and three-dimensional cavity flows have been developed. We present here some sample calculations that show that for a low Mach number cavity with a length-to-depth ratio of two, the two-dimensional steady flow is unstable to three-dimensional (spanwise homogeneous) disturbances that consist of spanwise modulation of the recirculating vortex interior to the cavity. The oscillations are unstable over a narrow band of spanwise wavelengths comparable to the cavity depth. They are oscillatory in time, but with a very slow frequency that is about ten times slower than the incipient two-dimensional Rossiter instability. Instability seems to be related to cellular patterns observed in surface streamline patterns on cavity bottoms in some previous experiments.

Background
Recent interest in controlling oscillations in flows over an aircraft weapons bay has led to detailed numerical and experimental investigations of flows over shallow cavities (see the reviews [1,2]). Meanwhile, theoretical models of cavity resonance, dating back to the early work of Rossiter [3], have treated the shear-layer emanating from the upstream corner of the cavity in isolation (using parallel flow instability) coupled to acoustic feedback via linear receptivity and scattering problems at the leading and trailing edges, respectively. Improved linear models of the coupled system [4] have been able to eliminate many of the empirical constants in Rossiter’s equation, but many real cavity flows show significant interaction between the shear layer and the recirculating flow within the cavity. In the most extreme form, this leads to a completely different form of cavity resonance—the so-called wake mode [5,6,7] that is more akin to vortex shedding from bluff bodies (where acoustic feedback plays no role). Between the two extremes (Rossiter and wake mode) are a large number of experimental and numerical

1 Graduate Research Assistant.
observations that involve significant interactions between the shear layer dynamics and the flow within the [e.g. 8,9,10].

The present research is aimed at calculating and understanding the instabilities and resonance of open cavities over the entire parameter space, including three-dimensional instabilities and those regimes where significant shear layer and cavity interactions occur. The development of three-dimensional instabilities has not previously been studied in detail. In order to do this, we must compute the eigenspectrum of the cavity steady flow field which is inhomogeneous in at least two (streamwise and depth) spatial directions. In the next section, we briefly review the computational framework that we have developed for these calculations. Following this we present results for three-dimensional unstable modes observed in low Mach number cavities with $L/D=2$. Finally, we discuss our recent implementation of adjoint Navier-Stokes algorithms for receptivity and control.

**Computational Framework**

Base flows and nonlinear DNS are computed with a high-order-accurate finite-difference method that has been described previously [6]. The grid is a block-structured format capable of rectilinear cavity configurations including three dimensional homogenous (periodic) and inhomogeneous (solid end walls). The code is parallelized using a domain-decomposition method. The code has optional equation sets to solve either (i) nonlinear Navier-Stokes, (ii) Navier-Stokes equations linearized about a specified (steady) based flow, and (iii) adjoint Navier-Stokes equations about a specified (steady) base flow.

Instability calculations are based on either on direct solution of the two-dimensional eigenvalue problem via the iterative (Krylov subspace) ARPACK [11] routine, or on the by solution of the linearized equations with extraction of least-damped or unstable modes from the long-time residuals [12]. In what follows, we refer to the BiGlobal instability Ansatz [see 12 for details] where flow quantities are decomposed according to $q(x,y,z,t) = \tilde{q}(x,y) + \epsilon \tilde{q}(x,y,z,t)$ with $\tilde{q} = (\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\rho}, \tilde{p})^T$ and $\tilde{q} = (\tilde{u}, \tilde{v}, \tilde{w}, \tilde{\rho}, \tilde{p})^T$ representing the steady two-dimensional basic flow and the unsteady three-dimensional infinitesimal perturbations, respectively. At this stage we consider perturbations that are homogeneous (periodic) in the spanwise direction. We thus set $\tilde{q}(x,y,z,t) = \tilde{q}(x,y)e^{i(\Omega t - \alpha z)} + c.c.$ where $\tilde{q}(x,y)$ is the eigenfunction and $\Omega$ is the complex frequency whose real part ($\omega$) represents the frequency of oscillations and whose $-/+\Omega$ imaginary part ($\alpha$) part represents its growth/damping rate, respectively.

**Three-dimensional cellular instability**

We concentrate on an operating condition with Mach number, $M=0.35$, $L/D=2$, and $L/\theta = 52$, where $L$, $D$, $\theta$ are the length, depth, and laminar boundary layer thickness at the cavity leading edge, respectively. For strictly two-dimensional flow, this set of parameters yields steady flow, but is very close to the neutral stability curve for self-sustained oscillations of the shear-layer type (Rossiter). Two-dimensional stability
calculations (i.e. with $\beta = 0$) show that the least damped disturbance is a Rossiter tone with $St=0.46$, which is well within the experimental scatter for mode 1 oscillations. Given the steady two-dimensional base flow, we compute the three-dimensional eigenfunctions ($\beta \neq 0$) and frequency and growth rates of oscillations. These are presented in Table 1 for selected values of wavelength, $\lambda = 2 \pi / \beta$.

<table>
<thead>
<tr>
<th>Spanwise wavelength $\lambda / D$</th>
<th>Frequency $St = \omega L / 2 \pi U$</th>
<th>Growth rate $\sigma L / U$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.025</td>
<td>0.010</td>
</tr>
<tr>
<td>1.5</td>
<td>0.020</td>
<td>0.001</td>
</tr>
<tr>
<td>1.25</td>
<td>0.047</td>
<td>-0.007</td>
</tr>
<tr>
<td>1.1</td>
<td>0.048</td>
<td>-0.014</td>
</tr>
<tr>
<td>1</td>
<td>0.047</td>
<td>-0.015</td>
</tr>
<tr>
<td>0.9</td>
<td>0.046</td>
<td>-0.009</td>
</tr>
<tr>
<td>0.75</td>
<td>0.045</td>
<td>0.003</td>
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<tr>
<td>0.5</td>
<td>0.047</td>
<td>0.054</td>
</tr>
<tr>
<td>0.25</td>
<td>0.038</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 1. Frequency and growth rate of three-dimensional disturbances (unstable modes shaded Green) for $M=0.35$, $L/D=2$, and $L/\theta = 52$.

A narrow range of spanwise wavelengths with $0.9 < \lambda / D < 1.25$ are unstable. They are oscillatory modes but the frequency of oscillation is about an order of magnitude smaller than the Rossiter frequency. Animations of the eigenfunctions reveal that the oscillation frequency is approximately equal to the rotation rate of the large two-dimensional vortex that exists in the mean flow inside the latter 2/3 of the cavity (Figure 1). Provided this vortex scales with cavity length and free-stream velocity, the frequency of the three-dimensional oscillation will remain an order of magnitude lower than the Rossiter frequency at other values of Mach number and $L/D$. Contours of spanwise velocity (Figure 1) for a typical eigenfunction show that the three-dimensional oscillations are most intense in the large vortex that sits inside the latter 2/3 of the cavity. The mode is stationary in the spanwise direction and consists of two cells mirrored about a plane of symmetry. Given that the instabilities occur in a narrow band of wavelengths, one might expect to find an integral number of cells with $\lambda / D \approx 1$ provided the cavity is sufficiently wide or nearly a integral multiple of cavity depths.
Figure 1. At left are streamlines in steady basic cavity flow with \( M=0.35, L/D=2, \) and \( L/\theta=52. \) At right contours of the spanwise velocity fluctuations (arbitrary linear scale) at one point in an oscillation cycle for an unstable mode with spanwise wavelength \( \lambda / D = 1. \)

These results are are now compared with some experimental observations of three-dimensional behavior observed in cavity flows. In experiments, oscillations are influenced by end effects not present in the homogeneous calculations. Indeed, the observations in [16] refer to three-dimensional effects from end-wall spillage when the width, \( W, \) is less than \( L. \) For relatively wide cavities, the cellular pattern exhibited by the unstable eigenfunctions is reminiscent of the cells observed in early experiments of Maull and East [15]. They used oil flow visualization of surface streamlines on the cavity bottom to show the existence, under certain conditions, of a nearly steady spanwise cellular pattern with the recirculating flow within the cavity. They observed that the most regular patterns existed when the cavity span was an integral number of preferred cell-widths. This behavior is consistent with a finite band of unstable spanwise wavenumbers as we found above. However, the results are not in agreement in terms of the observed wavelength or steadiness of cells. Indeed, for a cavity with \( L/D=2, \) Maull and East found cells with \( \lambda / D = 3.6 \) (note that [15] defined the cell width as \( s=\lambda/2), \) whereas the 3D instability we find has \( \lambda / D \approx 1. \) Moreover, the unstable modes we obtain here are oscillatory in time, albeit at a very slow timescale, though oil film visualizations can be difficult to interpret in unsteady flow. We note that in the experiments, the Reynolds number was much higher, the Mach was about 0.17 and the boundary layer ahead of the cavity was very thick (\( L/\theta \approx 10). \) Further analysis of parametric sensitivity of the present results is underway.

**Receptivity and control via adjoint Navier-Stokes**

We have modified the existing linearized Navier-Stokes code optionally solve the adjoint equations (defined with respect to a suitable inhomogeneous right-hand-side forcing or boundary condition). The adjoint equations provide a natural way to characterize receptivity of cavity to external disturbances, and can be used as part of an optimal control algorithm (see for example [17]). Preliminary tests involving active noise cancellation yielded good results and an effort is underway to characterize the receptivity of two-dimensional cavity oscillations and their optimal control.
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References


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**Transitions and New Discoveries**

None.