Scaling in Computer Network Traffic

Darryl Veitch

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Department of Electrical & Electronic Engineering
The University of Melbourne
### Scaling in Computer Network Traffic

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<th>3. DATES COVERED</th>
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<th>13. SUPPLEMENTARY NOTES</th>
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<td>See also ADM001750, Wavelets and Multifractal Analysis (WAMA) Workshop held on 19-31 July 2004., The original document contains color images.</td>
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Networks ... Connect
Telecommunications Networks, Traffic, & Engineering

Networks: A Deep Hierarchy of Systems

Tele–Traffic: A Turbulent River of Myriad Data Sources

Engineering: Traffic over a Network: Designing, Managing, Optimising
Networks: A Deep Hierarchy of Systems

- Connectivity (node organisation, placement, topology).
- Physical layer technology (devices, timing, coding, error recovery..).
- Circuit vs Packet Switched Paradigms.
- Connection-oriented vs connectionless philosophy.
- Operation (routing, signaling, admission control, congestion control).
- Inter-Networking (Autonomous systems, domain routing, addressing, gateways, protocol translation).
Tele–Traffic: A Turbulent River of Myriad Data Sources

- ‘Geographic’ Complexity
  - At network edge (distribution of sources, destinations, nodes)
  - Internally (multiplexing and de-multiplexing of streams)

- Offered Traffic Complexity
  - User ‘sessions’ (durations, arrivals, number of clicks..)
  - Applications used (browser, napster..)
  - Underlying protocols (TCP, HTTP..)
  - Underlying data objects (files, video, audio..)
  - Shaping by network elements

- Temporal Complexity:
  - Human driven (diurnal, coffee breaks, think times..)
  - Source driven (real-time constraints, file sizes..)
  - Network driven (topology, protocols..)
  - Technology driven (link rates, switching rates)

  → Time scale rich: $ns$ to months, 1kbps to Terabits/s

  → Burstiness: temporal (scaling), amplitude (non-G), spatial (capacity diversity)
TeleTraffic Engineering: the traditional program

- Collect traffic measurements and measure characteristics
- Select traffic model
- Solve switch performance problems
- Solve network performance problems
- Design network to given quality of service at minimal cost
- Solve admission control problems
- Run network
- Solve congestion control and routing problems
- Improve network performance

← Iterate until paradigm shift

This ‘scientific method’ approach begins with measurements...
Traffic Modelling in 1988

Papers between 1966 and 1987  (P. F. Pawlita, ITC-12, Italy)

- Queueing theory: several thousand
- Traffic measurement: around 50.
Tele-Traffic Today

Measurement Practice

- Recognition of measurement, not just routine quantification, but discovery.
- Widespread monitoring of LAN’s, ISP networks, national infrastructures.....
- Large scale programs for Internet, from routine monitoring to ultra high resolution.
- Emergence of numerous small & large scale active probing efforts.
- Huge advances in measurement accuracy.

Renaissance in Modelling

- Return of science 101: Observation and inspiration before model.
- Return of verification: Evaluating usefulness against real data.
- New model paradigm as standard: Characteristic scale $\rightarrow$ scale invariance.
- New model classes: Source, link, network, and closed loop.
- New problems: Active, multi-route, and high resolution measurements.

Stimulus to Theory

- Statistical estimation: For time series with infinite moments and/or scaling.
- Queueing Theory: Dealing with sub-exponential (eg LRD) input processes.
- Analysis: Network feedback, and properties of new models.
Measurement Practice

- Recognition of measurement, not just routine quantification, but discovery.
- Widespread monitoring of LAN’s, ISP networks, national infrastructures.....
- Large scale programs, from routine monitoring to ultra high resolution.
- Emergence of numerous small & large scale active probing efforts.
- Huge advances in measurement accuracy.
- Router based measurements widely used.
Renaissance in Modelling

- Return of science 101: Observation and inspiration before model.
- Return of verification: Evaluating usefulness against real data.
- New model paradigm as standard: Characteristic scale $\rightarrow$ scale invariance.
- New model classes: Source, Link, and Network.
- New problems: Active, Multi-route, and High resolution measurements.
Stimulus to Theory

- Statistical estimation: For time series with infinite moments and/or scaling.
- Queueing Theory: Dealing with sub–exponential (eg LRD) input processes.
- Analysis: Properties of new models, especially mixed open and closed loop.
Passive and Active Measurement

Typical Passive Aims

- “At-a-point” or “Network core”.
- Backbone link utilisation, Link traffic patterns, Server workloads.
- Engineering view: Network performance.

Typical Active Aims

- “End-to-End” or “Network edge”.
- End-to-End Loss, Delay, Connectivity, “Discovery” and “Tomography”.
- Long and short term monitoring, “Network health” and “Route state”.
- Internet view: Application performance and robustness.
### Major Measurement Programs

<table>
<thead>
<tr>
<th>Passive</th>
<th>Active</th>
<th>Tools</th>
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<tr>
<td>• ♥ NLANR</td>
<td>(National Laboratory for Applied Network Research).</td>
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<td>♠ CAIDA</td>
<td>(Cooperative Association for Internet Data Analysis).</td>
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<td>♥ ♠ WAND</td>
<td>(Waikato Applied Network Dynamics [DAG cards]).</td>
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<td>♥ PingER</td>
<td>(Ping End-to-end Reporting).</td>
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<td>♥ Surveyor</td>
<td>(From Advanced Networks and Services).</td>
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<td>(Network Coordination Centre).</td>
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Flows and Packets

Flows are sets of packets associated to the same data transfer.

Flow arrivals: \( Y(t) \)

Packet arrivals: \( X(t) \)
Zooming on a Single Link

[Diagram showing a scatter plot with time in seconds on the x-axis and sequence number on the y-axis, with a highlighted area.]
Traffic Data → Timeseries

What are we Measuring?

- *Internet Protocol* (IP) packets, the unit of transport across networks.
- Data split into packets, with: header, payload.
- Payload carries higher layer protocols: TCP, UDP, ICMP.
- Protocols support services & applications, more protocols:
  - **TCP**: HTTP, FTP, SNMP, ... (for reliable data)
  - **UDP**: VoIP, DNS, NTP,... (for real time)
- For each packet:
  - Could filter based on criteria  (address, type, size, ...)
  - Could capture all or part  (e.g. just the header).
  - Must *timestamp*.
- Key concept, a *flow* (collection) of packets.
Two Point Processes from Traffic

Flow arrivals: \( Y(t) \)

Packet arrivals: \( X(t) \)

Most common time series extracted are packets or bytes per bin
Prelude to a Paradigm

**Seeing Packet Traffic**

Believing Packet Traffic

- 1994: CCSN/SS7 (Duffy, McIntosh, Rosenstein, Willinger) – Self-Similar.
- 1994: Internet (Paxson, Floyd) – Failure of Poisson modelling.
- 1994 → ··· LAN’s across the world see – Self-Similarity.
- 1994: Web docs (Cunha, Bestavros, Crovella) – Heavy tailed file sizes.
The Self-Similarity of Ethernet Traffic

The reference Bellcore trace, ‘pAug’, is close to Fractional Gaussian Noise.

\[ \delta = 12 \text{ms} \]
\[ \delta = 12 \times 8 \text{ms} \]
\[ \delta = 12 \times 8 \times 8 \text{ms} \]
\[ \delta = 12 \times 8 \times 8 \times 8 \text{ms} \]
Measuring the Exponent using Wavelets

Wavelet coefficients of our traffic process:

\[ d_X(j, k) = \langle X, \psi_{j,k} \rangle. \]

Spectral definition of LRD is

\[ \Gamma_X(\nu) \sim c_f |\nu|^{-\alpha}, \ |\nu| \to 0, \ \text{with} \ \alpha \in (0, 1). \]

In this case, provided \( N \geq 1 \) vanishing moments:

\[ \mathbb{E}|d_X(j, k)|^2 \sim c_f C(\alpha) 2^{\alpha j}, \ j \to +\infty \]

Estimating the LHS from data using

\[ S_2(j) = \frac{1}{n_j} \sum_k |d_X(j, k)|^2, \]

where \( n_j \) is the number of \( d_X(j, k) \) available at octave \( j \) (scale \( a = 2^j \)),

We use ‘Logscale Diagram’ to refer to the log-log plot:

\[ \text{LD} : \ \log_2 S_2(j) \text{ vs } j, \]

in which straight lines are evidence for scaling, slope \( = \alpha \).
Analysis of Trace ‘pAug’

ETHERNET: bytes per 12ms bin.

Logscale Diagram, $N=2$ \[ \begin{align*}
(j_1, j_2) &= (3, 15), \\
\alpha_{est} &= 0.59, \\
Q &= 0.011384 \end{align*} \], $D_{init}$
An Old Story for Science and Nature

What does it mean?

- No characteristic time-scale controlling the dynamics / statistics.
- Statistically, All scales (in a range) are equivalent, under a renormalisation.
- Radical temporal burstiness: no natural burst scale.
- Key parameters no longer special scales, but relations across scales.
- Absolute quantities → scale dependent quantities.
- Affects system description, behaviour, measurement ....
The Self-Similar Traffic Model

Fractional Gaussian Noise (fGn) and Fractional Brownian Motion (fBm)

The unique Gaussian processes which are

\[
\text{Stationary; } \text{Cov} \left[ X_H \right](k) = \frac{1}{2} \left[ (k - 1)^{2H} + 2k^{2H} + (k + 1)^{2H} \right]
\]

\[
\text{Stationary Increments; } \text{Var} \left[ Z_H(k) \right] = k^{2H}
\]

Corresponding traffic models:

Rate: \( R(t) = \mu + \sigma X_H(t) \)

Total Traffic: \( W(t) = \mu t + \sigma Z_H(t) \)

where \( Z_H(t) = \sum_{i=1}^{t} X_H(i), \quad W(t) = \sum_{i=1}^{t} R(i). \)
The Long–Range Dependent (LRD) Traffic Models

LRD definition: a slowly decaying covariance

\[ \Gamma_X(k) \sim c_r k^{-\beta}, \quad 0 < \beta < 1, \]

where \( \beta = 2 - 2H \).

Corresponding traffic rate model:

\[ R(t) = \mu + \sigma X_{\beta,c_r,k^*}(t). \]

LRD more general than H-SS:

- **Second order** description only, not necessarily Gaussian!
- Has been called **second order asymptotically self-similar** (but careful!)
- Heavy tail ‘begins’ only after some cutoff scale \( k^* \).
- Tail may be ‘thin’, low mass (small \( c_r \)), independent of variance!
- Small scale structure **unspecified**.
- At a minimum, **three** correlation parameters, not just \( H \).
Non-Gaussian LRD – the On/Off Source

• Alternating renewal process: \( \{A_i\} \) i.i.d. \( \{B_i\} \) i.i.d.

• LRD if \( A \) or \( B \) heavy tailed:
  • If \( \Pr(B > x) \sim cx^{-\alpha} \), \( \beta_{\text{LRD}} = 3 - \alpha \), \( c_r = \frac{c(1-\lambda)^3}{(\alpha-1)\mathbb{E}[A]} \).
  • Efficient generation (\( O(1) \) computation and state)

Corresponding traffic rate models:

• as active/silence sources.

• as building blocks for a compound source, eg.
  • \( N \uparrow, p = \Pr(\text{On}) \rightarrow 0, \lambda = \text{const}, h = \text{const}: \rightarrow \text{M/G/}\infty \)
  • \( N \uparrow, p = \text{const}, \lambda = \text{const}, h \rightarrow 0: \rightarrow fGn \)
Impacts on Traffic Modelling

Statistics

• Difficult estimation of almost everything.
• Difficult model choice: Non-stationarity confusions and others.
• Large confidence intervals (need more data).
• Wavelets can help kill LRD, mitigate NS, in many cases.
  Complexity only $O(n)$ and can be done on-line.

Queueing

• Larger buffer sizes, lower utilisation at given QoS.
• ‘Buffer insensitivity’ (bigger won’t help much).
• Impact depends a lot on the amplitude burstiness:
  • fBm storage (Norros) – Weibullian tail.
  • large peak On/Off superpositions – Regularly varying tails, infinite mean
    (Cohen; Boxma; Whitt).
• Service discipline is important – can reinstate finite mean (Boxma et al.)
• Critical time scales can exist, but value depends on full structure.
Lessons for Network Performance

- Heavy tails can be thinned, but not cut off.
- Smoothing the beast will not tame it.
- **Statistical Multiplexing** works as ever – recommended.
- Processor Sharing helps the individual, but not the society.
- Hi Peak rates can kill.
- Power-laws are very persistent, if you can’t kill it:
  - Make sure it is irrelevant, or
  - Make it work for you, or
  - Understand it, calm it down, then live with it.
- LRD not everything, obviously: variance, *marginal.*
The ‘Self-Similarity’ model has limits

- Only really asymptotic: true for ‘large scales’, $\approx > 1$ second.
- Really requires Gaussianity – often far from the case!
- Range of scales where valid may not be crucial for engineering purposes.
- At small scales, feedback can mold the beast.
High Amplitude Burstiness at Small Scale

So small scales are much harder ...
Examples of Scaling in Traffic: 2nd Order Wavelet Analysis

**ETHERNET**: bytes per 12ms bin.

**INTERNET**: new connections per 10ms bin.

Logscale Diagram, $N=2$  \[ \left( j_1, j_2 \right) = (3,15), \quad \alpha_{\text{est}} = 0.59, \quad Q = 0.011384 \],  \[ D_{\text{init}} \]

Logscale Diagram, $N=2$  \[ \left( j_1, j_2 \right) = (8,19), \quad \alpha_{\text{est}} = 0.59, \quad Q = 0.81665 \]
Biscaling in TCP Arrivals

All Connections

TCP

UDP

HTTP

SMTP

Collaboration with Patrice Abry and Nicolas Hohn
Evidence for Multifractality

Wavelet $q$th order moments: $\mathbb{E}|d_X(j, k)|^q \sim C 2^{\alpha_q j}, \; j \to 0.$

Estimating the LHS from data using

$$S_q(j) = \frac{1}{n_j} \sum_k |d_X(j, k)|^q,$$

and measure the slopes $\hat{\alpha}_q = \zeta(q) + q/2$ in a log-log plot (the ‘MD’).

Instead of testing for linearity of $\zeta(q)$ vs $q$, look for $\zeta(q)/q$ vs $q$ constant.
A Short History of Multifractal Traffic Modelling

- 1997: Ethernet (Taqqu, Teverovsky, Willinger) – not MF
time domain \(\delta = 10\)ms.
- 1997: TCP, LAN gateway (Riedi, Véhel) – MF at ‘high freq’
iincrements \(\delta = 150\)ms.
- 1998: Ethernet (Abry, Veitch) – not MF
wavelet distributions \(\delta = 10\)ms.
wavelet domain; discrete packet counts \(\delta = 10\)ms large & small regimes
- 1998: TCP, WAN (Feldmann, Gilbert, Willinger) – MF at small, Mono large
wavelet domain; discrete packet counts \(\delta = 10\)ms.
- 1999: ISP and simulated (Feldmann, Gilbert, Huang, Willinger) – MF at small, but dirty
wavelet domain; discrete packet counts \(\delta = 10\)ms.
wavelet distributions \(\delta = 10\)ms CI’s used.
- 2001: TCP, WAN (Roux, Veitch, Abry, Huang, Flandrin, Miechel) – IDC, but \(\approx\) mono.
wavelet distributions \(\delta = 10\)ms CI’s used.
- 2003: TCP, WAN, high rate (Zhang, Ribeiro, Moon, Diot) – Mono everywhere
wavelet domain; discrete byte counts \(\delta = 10\)ms CI’s used
- 2003: TCP, WAN, high rate (Hohn, Veitch, Abry) – Empirical scaling misleading?
wavelet domain; continuous packet counts \(\delta = 5\mu s \rightarrow 5\)ms CI’s used \(q = 2\)
Similar Data, A Less Flattering View

\[ h_q = \frac{\zeta_q}{q} \]

AUCK–d1

Fine Scales

Coarse Scales

Order q
Semi-Experiments on Packet Arrivals

Time
Original TCP Data
Permutation of Flows [A-Perm]
Permutation of Flows [A-Perm]
Original Order Poisson Arrivals [A-Pord]

(a smooth form of internal shuffling)
Original Order Poisson Arrivals [A-Pord]
Original Order Poisson Arrivals [A-Pord]
Permuted Poisson Arrivals [A-Pois]
Permuted Poisson Arrivals [A-Pois]
From simple (Semi-)Experiments, we learn a lot

From these flow arrival *manipulations*:

- Correlations between flows can be neglected
  - No need for session level hierarchical models
  - TCP dynamics between flows can be neglected
- For IP modelling, flow arrivals can be modelled as Poisson
  - Justifies an assumption commonly used in traffic modelling.

**Note:** true flow arrival process is LRD.
Semi-Experiment Outcomes

Flow Arrival Manipulation

- Correlation between flows can be neglected,
- For IP modelling, flow arrivals can be modelled as Poisson,
- [A-Clus]: knee of $Y$ affects knee of $X$!

Packets within Flow Manipulation

- LRD not caused by arrival process of packets within flows,
- Small scale behaviour governed by structure within flows,
- Finite Poisson process a reasonable 0-th order model,
- [P-Pois]: indistinguishable from [P-Uni],
- [P-ConstR]; [P-ScaledR]: rate acts like a scale parameter,

Flow Selection

- Observed LRD caused by heavy-tailed flow volumes,
- [S-Thin]: random thinning consistent with independent flows,
- [S-Dur]: also kills LRD (flow duration slaved to volume),
- [S-Pkt]: LRD still present even without heavy tail!

Flow Truncation Manipulation

- [T-Pkt]: also kills LRD, makes $X$ tend to $Y$, 
Poisson (Barlett-Lewis) Cluster Processes

Definition

- A Poisson process of seeds (flows), initiating independent groups of points (packets):
  \[ X(t) = \sum_{i} G_i(t - t_F(i)) \]

- Group: a finite renewal process with \( P \) points and inter-arrival distribution \( A \):
  \[ G_i(t) = \sum_{j=1}^{P(i)} \delta \left( t - \sum_{l=1}^{j-1} A_i(l) \right) \]

Parameters

- Flow arrivals: constant intensity \( \lambda_F \)
- Flow structure:
  - Packet arrivals: \( A, \frac{1}{E_A} = \lambda_A < \infty \), cf: \( \Phi_A(\omega), \omega > 0 \)
  - Flow volume: \( P, E P = \mu_P < \infty \), pgf: \( G_p(z) = \sum_{j=0}^{\infty} p_j z^j, |z| \leq 1. \)
Fourier Spectrum

\[ \Gamma_X(\nu) = \lambda_F \left( \frac{\mu_P}{\lambda_A} \Gamma_g(\nu) + (S_\psi(\omega) + S_\psi(-\omega)) \right), \]

\( \Gamma_g(\nu) \): spectrum of stationary renewal process with inter-arrivals \( A \), and

\[ \mathcal{R}(S_\psi(\omega)) = \frac{\Phi_A(\omega)}{(1 - \Phi_A(\omega))^2} \left( G_P(\Phi_A(\omega)) - 1 \right). \]

Verify LRD:

\[ \mathcal{R}(S_\psi(\omega)) \quad \omega \to 0 \sim LB(\beta)(2\pi\lambda_A)^{2-\beta} \omega^{-(2-\beta)} \to \infty \]

\[ \omega \to \infty \sim -\cos(c\pi/2) / (b\omega)^c \to 0 \]

where \( B(\beta) = \psi(1 - \beta) \cos(\pi\beta/2) / (2\pi)^{(2-\beta)} > 0 \)

Properties

- \( \lambda_F \) just a variance multiplier: ‘more of same’
- has scale parameter \( 1/\lambda_A \) if \( A \) has: \( \Gamma_X(\omega; \lambda_F, \lambda_A, c, F_P) = \Gamma_X(\omega/\lambda_A; \lambda_F, 1, c, F_P) \)
- Two terms dominating small-large scales
  - First term (small scales): scaled renewal process, no detailed \( P \) dependence
  - Second term (large scales): LRD, no \( A \) dependence beyond \( \lambda_A \)
Modelling a Typical Auckland IV Trace

\[ j = \log_2 (a) \]
\[ \log_2 \text{Var}(d_j) \]

\[ \lambda F \mu P \]
\[ c \]

\[ \lambda F \mu P \]

\[ \lambda F \mu P \]

\[ \log_2 \lambda_A + \log_2 (\pi^2 (c + 1)/(3\epsilon c^2)) \]

\[ \log_2 \lambda_A + \frac{1}{2 - \beta} \left( \log_2 \mu_p - \log_2 (2LB(\beta)) - \log_2 c \right) \]
Comparison between data and fitted Cluster Model

$h_q = \frac{\zeta_q}{q}$

AUCK–d1

Fine Scales

Coarse Scales

AUCK–d1 PCP

Fine Scales

Coarse Scales