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**14. ABSTRACT**
For several years, we have been accurately calculating the electronic structure of superlattices using a solution technique based on the Empirical Pseudopotential Method (EOM). In our method for forming the superlattice pseudopotential, the critical assumption is that the heterointerface charges are redistributed, making each constituent layer in the superlattice as bulk-like as possible. Here, we demonstrate that our technique for forming the superlattice pseudopotential is fundamentally different from the atomistic pseudopotential approaches that use a superposition of atomic pseudopotentials to represent the superlattice. We then present several applications of our method to InAsGaSb Type-II superlattices and, where possible, we compare our results to those calculated with an effective mass method, as well as to atomistic EPM methods. In all of these comparisons, our method provides excellent agreement with the measured data.

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Empirical Pseudopotential, superlattice, InAs, GaSb, type II interface

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Empirical Pseudopotential Modeling of Superlattices

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"Standard Model"

$\vec{k} \cdot \vec{P}$ perturbation theory

Envelope Function Approximation

More Exact Models (EPM)

Solve the Schroedinger Eqn.

for a more microscopically correct potential:

\[
\frac{-\hbar^2}{2m} \nabla^2 + V(\vec{r}) \Psi = E \Psi
\]

- We will concentrate on the EPM based models:

\[
\frac{-\hbar^2}{2m} \nabla^2 + V(\vec{r}) \Psi = E \Psi
\]
Physics of Bulk Solids with EPM

- Constructing the bulk potential:

\[ V(\vec{r}) = \sum_i \Omega_{\text{lattice}} \left( \vec{r} + \vec{t}_i \right) \]

\[ \text{Poisson Sum Formula} \quad \sum \vec{g} \quad V_{\vec{g}} e^{i\vec{g} \cdot \vec{r}} \quad V_{\vec{g}} \alpha \tilde{\Omega}(\vec{k}) \bigg|_{\vec{k} = \vec{g}} = \text{form factor} \]

in which \( \{ \vec{g} \} \) are the reciprocal lattice vectors such that \( \vec{g} \cdot \vec{t}_i = \text{Integer} \cdot 2\pi \).

This forces:

\[ V(\vec{r}) = V(\vec{r} + \vec{t}_i) \]

\[ \left( \frac{\hbar^2}{2m} \left| \vec{\xi} + \vec{g} \right|^2 + V_0 - E(\vec{\xi}) \right) b_{\vec{g}}(\vec{\xi}) + \sum_{\vec{h}} V_{\vec{g}-\vec{h}} b_{\vec{h}}(\vec{\xi}) = 0 \]
**Key Point**

- We do not need to know the full lattice potential:

\[
\text{Fourier Transform}\left\{\Omega(\vec{r})\right\} = V(\vec{k}) \quad \text{with} \quad -\text{Max} < k_x, k_y, k_z < \text{Max}
\]

We only need \( V(\vec{k} = \vec{g}) = V_g \) \( (\text{form factors}) \)

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**Optics Analogy:**

- Single Slit
- Array of Slits

\( \text{diffraction orders} \)
How do we form $V$ for the Superlattice ??

- There are at least two methods for representing the potential of a superlattice (W / B):

  $V(\vec{r}) = \sum W(\vec{r} + \vec{i}_W) + \sum B(\vec{r} + \vec{i}_B) + \text{Interface detail}$

  \[ W(\vec{r}) = \sum \Omega_W(\vec{r} + \vec{i}_W) \]
  \[ B(\vec{r}) = \sum \Omega_B(\vec{r} + \vec{i}_B) \]

  \text{Key Point: This construction requires functional fits to } \Omega_W(\vec{r}) \text{ and } \Omega_B(\vec{r}) + \text{others}

- Method (1): "Atomistic" EPM = AEPM

  $\text{Example: InAs / GaSb requires } \Omega_{In(As)}(\vec{r}), \Omega_{Ga(Sb)}(\vec{r}), \Omega_{Ga(As)}(\vec{r}), \Omega_{In(Sb)}(\vec{r})$

  $(8 \text{ ions} \times 5 \text{ parameters/ion} + 3 \text{ offsets}) = 43 \text{ parameters}$

  guess of potential + interface bond weightings + Temp. dependent segregation
• Method (2): Superimposed “Bulk” EPM $= \text{SEPM}$

\[ V_W(\vec{r}) = \sum_{\vec{g}} V^W_{\vec{g}} e^{i\vec{g} \cdot \vec{r}} \]

\[ V_B(\vec{r}) = \sum_{\vec{g}} V^B_{\vec{g}} e^{i\vec{g} \cdot \vec{r}} \]

\[ V(\vec{r}) = \text{rect} \left( \frac{z}{W} \right) \cdot V_W(\vec{r}) + \left[ 1 - \text{rect} \left( \frac{z}{W} \right) \right] \cdot V_B(\vec{r}) \]

in which:

\[ \text{rect} \left( \frac{z}{W} \right) = \sum_{n=-M}^{M} \frac{1}{\pi} \sin \left( \frac{\pi W}{P} \cdot n \right) e^{i \frac{2\pi n z}{P}} = \]

\[ \begin{array}{c}
\text{W} \\
\text{P}
\end{array} \]

Key Point: This construction only requires $V^W_{\vec{g}}$ and $V^B_{\vec{g}}$ (finite number of form factors)

that fit the bulk band diagram plus the offset.

(8 parameters for strained InAs + 7 parameters for GaSb + offset) = **16 parameters**
Graphical comparison in 1-dimension

(6ML / 12ML)

Barrier Lattice Potential

Well Lattice Potential

Method (1)
AEPM

Construction requires full knowledge of $V_W(\vec{k})$ and $V_B(\vec{k})$ for
- $\text{Max} < \vec{k} < \text{Max}$.

Method (2)
SEPM

Construction uses finite number of form factors,
$V_W(\vec{g})$ and $V_B(\vec{g})$.

Bulk Like

Interface complexity is covered by one parameter: Offset
• For both methods, the final potential can be written as a sum

\[ V(\vec{r}) \equiv \sum_{g_i, n} \overline{V}_{g_i, n} e^{i(g_i x + g_i y)} e^{i \frac{2\pi m}{P} z} \]  
**SEPM or AEPM**

but Fourier coefficients \( \overline{V} \) change.

• The Schroedinger Equation for the superlattice is:

\[
\frac{\hbar^2}{2m} \left[ \left( \frac{2\pi n}{P} + \xi_z \right)^2 \right] a_{g_i, n} + \sum_{g_i', n'} \overline{V}_{g_i - g_i', n - n'} a_{g_i', n'} = E(\vec{\xi}) a_{g_i, n}
\]

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Question for Discussion: *Which model, SEPM or AEPM, represents reality?*
Superlattice (InAs / GaSb) Blue Shift Results -vs- GaSb Thickness

- Fit #1 and Fit #2 both provide excellent fits to the InAs and GaSb band diagrams, but they interpolate differently.
Application of SEPM to Mid-IR Laser Tuning

SEPM model calculations, assuming 24 Å of $\text{In}_{0.4}\text{Ga}_{0.6}\text{Sb}$ hole well and thick $\text{In}_{0.2}\text{Ga}_{0.8}\text{AsSb}$ internal absorbers.