Abstract  The conventional approach to GMTI uses narrowband signals and a short coherent processing interval (CPI). In this talk, we examine some of the fundamental theoretical issues involved in GMTI with wideband signals and long CPIs (WL-GMTI). The possibility of wideband, long CPI GMTI has received some attention in recent years, and there are a number of potential benefits:

1) Improved minimum detectable velocity (MDV).
2) Detection of targets with zero radial velocity (but non-zero tangential velocity).
3) Better fit with dual-use SAR/GMTI architectures.
4) Less demanding array requirements (shorter and/or sparser arrays).
5) Greater robustness to clutter internal motion.

The most convenient framework for WL-GMTI is a “post-SAR” architecture, where each spatial channel is pre-processed with synthetic aperture radar (SAR) image processing. The post-SAR architecture is the natural generalization of post-Doppler STAP to the wideband, long-CPI case.

Exact steering vectors in the post-SAR framework are computed analytically for constant-velocity targets, assuming a calibrated array. The steering vectors can be used with algorithms such as the GLRT or AMF to perform adaptive detection on the post-SAR data. We also derive a simple, exact expression for SINR loss when the covariance is known exactly. The loss is a two-dimensional function of both target velocity components, indicating the capability to detect both radial and non-radial target motion.

The final section of this talk examines WL-GMTI performance bounds based on optimal Bayesian detection. In particular, we study how detection performance varies as a function of the number of pixels that the moving target "smears" over in the SAR image. There is a surprising improvement in detection performance when the clutter has strong non-Gaussian tails. In at least some cases, it appears that much of the performance can be achieved with a simple sub-optimal detector.
Long CPI Wideband GMTI

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See also, ADM001741 Proceedings of the Twelfth Annual Adaptive Sensor Array Processing Workshop, 16-18 March 2004 (ASAP-12, Volume 1)., The original document contains color images.
Wideband, Long-CPI GMTI

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Wideband, Long-CPI GMTI (WL-GMTI)

Premise:
Combine physical aperture of GMTI with bandwidth and integration time of high-resolution SAR.

Purpose:
Supplement traditional SAR and GMTI modes by detecting slow and/or low RCS moving targets.

Potential Benefits:
- Lower minimum detectable velocity (MDV)
- Detection of targets with zero radial velocity (but non-zero tangential velocity)
- More efficient use of radar resources in multi-mode SAR/GMTI platforms
- Reduced array requirements (sparser/shorter)
- Greater robustness to clutter internal motion
Background and References


Purpose of this Talk

- Develop a basic framework for discussing and analyzing WL-GMTI
- Show how some of the basic tools of adaptive processing translate to WL-GMTI
  - Steering vectors
  - SINR loss
  - Detection
- Stimulate further interest and work!
Outline

- SAR as the WL-GMTI pre-processor
- WL-GMTI steering vectors
- SINR loss prediction for WL-GMTI
  - Theory
  - Examples
- Detection
- Summary
SAR Pre-processing

Data Cube

Vectorized Data Cube

Linear transformation to SAR basis

Multi-channel SAR image

Space (phase centers)

Fast time (range)

Slow time (pulse)

$z_1 ightarrow s_1 = M_1 z_1$

$z_2 ightarrow s_2 = M_2 z_2$

$z_3 ightarrow s_3 = M_3 z_3$

$z_4 ightarrow s_4 = M_4 z_4$

Stagger data in slow time to meet DPCA condition

SAR processor knows locations of phase centers

Each SAR resolution cell is automatically “phased up” for stationary clutter at that location

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WL GMTI-6

AFY 7/7/2004
Properties

- SAR is the wideband, long-CPI generalization of the Doppler processor in ordinary post-Doppler STAP
- Linear transform of input data cube
  - Annihilates the exoclutter subspace
  - Invertible transformation of the endoclutter subspace
- “Freezes” clutter into SAR resolution bins
  - Clutter in one bin is well-decorrelated from other bins multiple resolution cells away.
  - Stationary targets have trivial steering vectors
  - Stationary clutter has trivial covariance
- Moving targets smear over multiple resolution cells
Benefits of High Resolution for GMTI

- Target-to-clutter and target-to-noise ratio improve with increasing resolution
  - Improvement holds at least until target is resolved
  - Improvement can continue further if target contains small dominant scatterers
- High spatial resolution provides more clutter per unit area for training adaptive processor
  - Can train in both range and azimuth
- Abundance of training data facilitates more powerful adaptive processing methods
  - Algorithms with more adaptive DOFs
  - Automated data editing to eliminate potential movers from training data
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SAR Steering Vector*  
Constant Velocity Point Target

Image Domain

Spatial Frequency Domain

\[
\frac{1}{\varepsilon} (x - x_s)^2 + y^2 = y_s^2
\]

\[
G(k_x, k_y) = \frac{k_y}{\gamma \sqrt{k_y^2 + \varepsilon k_x^2}} e^{-ix_s k_x - iy_s \sqrt{k_y^2 + \varepsilon k_x^2}}
\]

\[
\gamma = \left| \frac{V_{\text{target}} - V_{\text{platform}}}{V_{\text{platform}}} \right|, \quad \varepsilon = 1 - \frac{1}{\gamma^2}
\]

WL-GMTI Steering Vector
Constant Velocity Point Target

\[ x_s(d) = x_s(0) + \frac{d}{2} \left( 1 - \frac{1}{\gamma} \cos \psi \right) \]
\[ y_s(d) = y_s(0) - \frac{d}{2} \sin \psi \]

\[ \gamma = \frac{\| \mathbf{v}_{\text{target}} - \mathbf{v}_{\text{platform}} \|}{\| \mathbf{v}_{\text{platform}} \|} \]
\[ \varepsilon = 1 - \frac{1}{\gamma^2} \]
\[ \cos \psi = \frac{\left( \mathbf{v}_{\text{platform}} - \mathbf{v}_{\text{target}} \right) \cdot \mathbf{v}_{\text{platform}}}{\| \mathbf{v}_{\text{platform}} - \mathbf{v}_{\text{target}} \| \| \mathbf{v}_{\text{platform}} \|} \]

\[ G(k_x, k_y; d) = \frac{k_y}{\gamma \sqrt{k_y^2 + \varepsilon k_x^2}} e^{-i x_s(d) k_x - i y_s(d) \sqrt{k_y^2 + \varepsilon k_x^2}} \]
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Simplifying Assumptions

- Large clutter-to-noise ratio
- Elements are mutually calibrated
- No internal clutter motion, crab, unmeasured aircraft motion and vibration
- No jammers and other interference
- Isotropic element patterns
- Optimal AMF processing with perfect knowledge of steering vectors and clutter covariance
Clutter-Limited SINR Loss Calculation

Target steering vector:

\[ u = \sum_{n=1}^{N} v(n) \otimes e(n) \]

Vectorized target image from \( n \)-th channel

Element space basis vector

Clutter + noise covariance:

\[ R = \sigma_n^2 I \otimes I + R_c \otimes ee^H \]

Noise power

Clutter pixel-space covariance

Inverse covariance:

\[ R^{-1} = \frac{1}{\sigma_n^2} I \otimes I - \frac{1}{\sigma_n^4} \left( R_c \left( I + \frac{N}{\sigma_n^2} R_c \right)^{-1} \right) \otimes ee^H \]

Large clutter-to-noise limit:

\[ \sigma_n^2 R^{-1} \xrightarrow{\sigma_n^2 \to 0} I \otimes I - \frac{1}{N} I \otimes ee^H \]

SINR loss:

\[ \sigma_n^2 \frac{u^H R^{-1} u}{u^H u} = 1 - \frac{\left\| \sum_{n=1}^{N} v(n) \right\|^2}{N \sum_{n=1}^{N} \left\| v(n) \right\|^2} \]
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SINR Loss Example

- Carrier = 1 GHz
- Bandwidth = 10 MHz
- CPI = 50 ms
- Range = 10 km
- Airspeed = 180 m/s
- Phase center locations = [0,1] m
SINR Loss Example

- Carrier = 1 GHz
- Bandwidth = 10 MHz
- CPI = 50 ms
- Range = 10 km
- Airspeed = 180 m/s
- Phase center locations = [0, 30] m

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SINR Loss Example

- Carrier = 1 GHz
- Bandwidth = 10 MHz
- CPI = 15 s
- Range = 10 km
- Airspeed = 180 m/s
- Phase center locations = [0,30] m

"Differential SAR" zone:
Target moves > 2 SAR resolution cells over time it takes platform to travel the length of the real aperture
SINR Loss Example

- Carrier = 1 GHz
- Bandwidth = 300 MHz
- CPI = 15 s
- Range = 10 km
- Airspeed = 180 m/s
- Phase center locations = [0,30] m

"Differential SAR" zone:
Target moves > 2 SAR resolution cells over time it takes platform to travel the length of the real aperture
SINR Loss Example

- Carrier = 1 GHz
- Bandwidth = 300 MHz
- CPI = 15 s
- Range = 10 km
- Airspeed = 180 m/s
- Phase center locations = [0,7,30] m

"Differential SAR" zone:
Target moves > 2 SAR resolution cells over time it takes platform to travel the length of the real aperture
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SINR Loss vs. Detection Performance

- SINR loss is a useful diagnostic, but it does not always translate directly into what we care about: detection performance
  - Case in point: SINR loss for stationary targets is infinite, but SAR detects stationary targets quite well!
- WL-GMTI straddles the regime between GMTI-type detection and SAR-type detection
- Need to consider detection theory to understand true capabilities of WL-GMTI
Illustrative Toy Model

- Radar: single-phase center (pure SAR)
- Moving point target with Rayleigh fading
- 15 dB mean target-to-clutter (for focused stationary target)
- Two clutter models:
  - Rayleigh (unrealistic, weak tails)
  - Log-normal (more realistic, heavier tails)
Stationary Target Detection

Probability of Detection vs. Probability of False Alarm

- Rayleigh clutter
- Log-normal clutter
Moving Target: Matched Filter Detector

Probability of False Alarm

Rayleigh clutter

Log-normal clutter, moving target smeared over $N$ pixels

$N=20$
$N=5$
$N=2$
$N=1$

Probability of Detection

More Gaussian

$N=20$
$N=5$
$N=2$
$N=1$

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Moving Target: Bayes-Optimal Detector

- Rayleigh clutter
- Log-normal clutter, moving target smeared over $N$ pixels

Probability of Detection vs. Probability of False Alarm for different values of $N$.
Moving Target: ALMF Sub-Optimal Detector

- Rayleigh clutter
- Optimal detector
- ALMF Detector (amplitude limiter + matched filter)
Summary

- Wideband, long-CPI methods offer the promise of detecting slow, low-RCS targets not detectable with traditional GMTI methods.
- This talk has explored some basic building blocks for analysis:
  - Wideband, long-CPI data model and steering vectors
  - SINR loss analysis
- It appears that the detection capability of WL-GMTI straddles the SAR and GMTI domains: SINR loss alone is not a reliable metric of performance:
  - Smearing of target over many pixels can enhance detection in strong-tailed clutter
  - Sub-optimal detector can approach optimal bound
- Many other aspects of the problem are ripe to be explored.