Parametric Filters For Non-Stationary Interference Mitigation in Airborne Radars

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<td>See ADM001263 for entire Adaptive Sensor Array Processing Workshop., The original document contains color images.</td>
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Standard Form 298 (Rev. 8-98)  
Prescribed by ANSI Std Z39-18
Motivation: Non-Stationary Interference

- Rapidly changing clutter locus with a circular array or bistatic radar system
- Presence of hot clutter due to an airborne jammer
- Use model of non-stationary interference to derive new filter
- Use small sample support to reduce effect of non-stationary interference
• M antennas, N pulses
• Target in primary range bin $p$

$$x_p(t) = ba(\theta)e^{j\omega t} + c_p(t), \quad t = 0, 1, 2, \ldots, N - 1$$

• Space-Time Slice

$$X_p = [x_p(0) \quad x_p(1) \ldots \quad x_p(N - 1)]$$

$$= ba(\theta)v^T(\omega) + C_p$$

$$= \begin{bmatrix} 1 & e^{j\omega/T_s} & \ldots & e^{j(N-1)\omega/T_s} \end{bmatrix}$$

spatial steering vector

clutter, jammer, noise, etc.

temporal steering vector
Data Model (cont.)

- Vectorized Forms

1. \( \mathbf{x}_p = \operatorname{vec}(\mathbf{X}_p) \)
   \[ = \mathbf{b}\mathbf{v}(\omega) \otimes \mathbf{a}(\theta) + \mathbf{c}_p \]

2. \( \mathbf{x}_p = \operatorname{vec}(\mathbf{X}^T_p) \)
   \[ = \mathbf{b}\mathbf{a}(\theta) \otimes \mathbf{v}(\omega) + \mathbf{c}_p \]

- Secondary Data

\[ \{ \mathbf{c}_k \} \ k = 1, \ldots, N_s \quad k \neq p \]
\[ E(\mathbf{c}_k) = 0 \quad , \quad E(\mathbf{c}_k\mathbf{c}_k^*) = \mathbf{R} \]
Space-Time Autoregressive Modeling

- Define \( H(z^{-1}) = \sum_{i=0}^{L-1} H_i z^{-i} \)

- Model: for some \( L \),

\[
H(z^{-1})c_k(t) = H_0 c_k(t) + H_1 c_k(t-1) + \ldots + H_{L-1} c_k(t-L+1)
\]

\[
= \varepsilon_k(t)
\]

is spatially and temporally white

- To estimate \( H(z^{-1}) \), solve

\[
\min_{H_0, \ldots, H_L} \sum_{k=1}^{N_s} \sum_{i=L}^{N} \left\| H(z^{-1})c_k(i) \right\|^2
\]
Filtering the Primary Data

STAR filter attempts to minimize clutter power:

\[
\begin{bmatrix}
H_{L-1} & H_{L-2} & \cdots & H_0 & 0 & \cdots & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & 0 & H_{L-1} & H_{L-2} & \cdots & H_0
\end{bmatrix}
\]

\( \varepsilon_k = \begin{bmatrix} e_k \\ \vdots \\ e_k \end{bmatrix} \),

\( c_k = H c_k \)

Span(\( H \)) orthogonal to clutter subspace if it dominates white noise:

\[
R = H^\perp Q H^\perp + \sigma^2 I
\]

clutter & jamming

so we project onto the orthogonal subspace using a matched subspace filter:

\[
x'_p = H^* \left( H H^* \right)^{-1} H x_p
\]

banded block Toeplitz

Dimension \( M'(N-L+1) \times 1 \)
1. Use SVD on secondary data to solve for $[H_0 \ H_1 \ \ldots \ H_{L-1}]$

   computational order: $O(N_M M^2 L^2 (N - L + 1))$

2. Form $\mathcal{H}$ and filter data: $P_{\mathcal{H}} \cdot x_p$

   computational order: $O(M' M^2 L^2 (N - L + 1))$

3. Perform regular beam and Doppler filtering for detection

   computational order: negligible

Resultant test statistic is $(v \otimes a)^* P_{\mathcal{H}} \cdot x_p$ not $(v \otimes a)^* R^{-1} x_p$
Prior Work

- Vector AR models used previously for clutter modeling by Michels, Rangaswamy, etc.

- Standard STAP filters extended to handle range-varying and hot clutter models by Zatman, Rabideau, etc.

- Matched subspace detectors used for subspace interference by Scharf

- Here, we extend the parametric model to handle the non-stationary interference
Range-Varying STAR Filter

- To improve performance at short ranges, use linearly varying matrix coefficients:

\[
\sum_{i=0}^{L-1} \begin{bmatrix} H_i & \Delta H_i \end{bmatrix} \begin{bmatrix} c_k(t - i) \\ \alpha k c_k(t - i) \end{bmatrix} = \varepsilon_k(t), \quad t = L + 1, \ldots, N
\]

- Analogous to ESMI technique of Hayward

- To normalized the noise subspace

\[
\alpha = \sqrt{\frac{12}{(N_s + 2)(N_s + 1)}}
\]
Range-Varying STAR Filter

- Minimize clutter power assuming linearly varying statistics

\[ e_k = \tilde{H} \begin{bmatrix} c_k \\ a_k c_k \end{bmatrix} \]

where

\[ \tilde{H} = \begin{bmatrix} H_{L-1} & H_0 & 0 \\ O & H_0 & 0 \\ 0 & H_{L-1} & H_0 \end{bmatrix} \]

- Filter data with matched subspace filter

\[ x'_p = \tilde{H}^* (\tilde{H} \tilde{H}^*)^{-1} \tilde{H} \begin{bmatrix} x_p \\ 0 \end{bmatrix} = P_{\tilde{H}} \begin{bmatrix} x_p \\ 0 \end{bmatrix} \]
• Define

\[ C_k = \begin{bmatrix}
    c_k(L+1) & c_k(N) \\
    \vdots & \vdots \\
    c_k(1) & c_k(N-L)
\end{bmatrix} \]

• Estimate filter coefficients:

\[
\begin{bmatrix}
    H_0 & \cdots & H_{L-1} & \Delta H_0 & \cdots & \Delta H_{L-1}
\end{bmatrix}
\]

as the left singular vectors with the \( M' \) smallest singular values of the extended data matrix:

\[
\begin{bmatrix}
    \frac{C_{-N_s/2}}{2} & \cdots & \frac{C_{N_s/2}}{2} \\
    -\frac{\alpha N_s}{2} & \cdots & \frac{\alpha N_s}{2}
\end{bmatrix}
\]
**ESTAR Filter Example**

20 element circular array, 18 pulses
SCR = -58dB   SNR = 10dB

Primary data vector snapshot at 20 km

4 tap ESTAR filter, 20 secondary snapshots
Computational Comparison

Some typical numbers: $M = 20$, $N = 18$, $M' = 20$

- **STAR Filter (L=5):** $O(140,000N_s) + O(2,800,000)$

- **ESTAR Filter (L=4):**

$$O\left(4N_sM^2L^2(N - L + 1)\right) + O\left(M'M^2L^2(N - L + 1)\right)$$

$$= O(384,000N_s) + O(1,920,000)$$

- **Extended PRI staggered algorithm:**

$$O\left(4N_sM^2K^2(N - K + 1)\right) + O\left(4\rho M^2K^2(N - K + 1)\right) = O(230,000N_s) + O(20,000,000)$$

# of sub-CPIs = 3

rank of sub-CPI covariance $\cong 90$
Average SINR Loss

Average SINR loss is the area between the curves.
Performance with Range-Varying Weights

20 km range

30 km range

\( N_s = 50 \) training vectors – 2 km training window
Performance with Range-Varying Weights

L=5 for STAR filter – L=4 for ESTAR filter
3-D STAR Filter for Hot Clutter

- Update filter for each new pulse received
  - Derive slow-time varying STAR filter
  - Can be used with intrinsic clutter motion
- Add fast-time matrix taps to exploit correlations across range bins
  - Additional filter taps help mitigate mainbeam jamming signals
Slow Time-Varying STAR Filter

- Same structure as the STAR filter but with new coefficients for each pulse

\[
H_{TV} = \begin{bmatrix}
H_{L-1}(1) & \cdots & H_0(1) & 0 \\
\vdots & & \vdots & \vdots \\
0 & & H_{L-1}(N-L+1) & \cdots & H_0(N-L+1)
\end{bmatrix}
\]

- Additional sample support required due to additional parameters to model slow-time variation
• Use slow-time varying STAR filter to model correlation across pulses

• For some fast-time filter order $J$, model the fast-time correlation as:

$$
\sum_{j=0}^{J-1} H_{TV,j} c_{k-j} = \varepsilon_k, \quad k = J + 1, \ldots, P
$$

- subscript denotes which fast-time sample $H$ is associated with
- number of fast-time samples used to whiten data

• Similar to a 2-D vector AR model with the slow-time taps changing with each pulse
Estimation of Parameters

- Define

\[
\tilde{H}(t) = \begin{bmatrix}
H_{0,0}(t) & H_{1,0}(t) & \cdots & H_{L-1,J-1}(t)
\end{bmatrix}
\]

\[
g_k(t) = \begin{bmatrix}
c_k(t + L - 1) \\
\vdots \\
c_k(t)
\end{bmatrix}, \quad
g_k(t) = \begin{bmatrix}
g_{k+J-1}(t) \\
\vdots \\
g_{k+P-1}(t)
\end{bmatrix}
\]

- Least squares solution:

\[
\min_{\tilde{H}(t)} \sum_{k=1}^{N_s} \left\| \tilde{H}(t)G_k(t) \right\|^2
\]

- New minimization for each slow-time step
Filtering the Primary Data

- 3D-STAR filter can be written as:

\[ H = \begin{bmatrix}
H_{TV,J-1} & \cdots & H_{TV,0} & 0 \\
\vdots & \ddots & \vdots & \vdots \\
0 & \cdots & H_{TV,J-1} & H_{TV,0}
\end{bmatrix} \]

- Project out the interference using 3D matched subspace filter

\[
\begin{bmatrix}
X_p^/ \\
\vdots \\
X_{p-P+1}^/
\end{bmatrix} = H^* (HH^*)^\dagger \begin{bmatrix}
X_p \\
\vdots \\
X_{p-P+1}
\end{bmatrix} = P_{\parallel} \begin{bmatrix}
X_p \\
\vdots \\
X_{p-P+1}
\end{bmatrix}
\]

Highly structured nature of subspace, small sample support make full 3-D STAR solution feasible.
Computational Comparison

Some typical numbers: $M = 20$, $N = 18$, $M' = 20$, $P = 3$

- **STAR Filter (L=7):** $O(235,000N_s) + O(4,700,000)$
- **3D-STAR Filter (L=2):** $O(N_s(MLJ)^2(N - L + 1)(P - J + 1))$
  $+$ $O(M'(MLJ)^2(N - L + 1)(P - J + 1))$
- **J=2:** $O(218,000N_s) + O(4,350,000)$
- **J=1:** $O(82,000N_s) + O(1,630,000)$

- **Optimized 3D-post-Doppler algorithm:**
  
  $O(N_s(MKP)^2(N - K + 1)) + O(\tilde{n}(MKP)^2(N - K + 1)) = O(518,000N_s) + O(70,000,000)$

  # of sub-CPIs = 3
  rank of sub-CPI covariance = 135
**Hot Clutter Examples**

Direct path jamming signal in mainbeam: $\text{JDOA}=1^\circ$

Only multipath component in mainbeam: $\text{JDOA}=-20^\circ$

$N_s=100$ training vectors – 4 km training window
Hot Clutter Examples

Direct path JDOA=1°

L=2, J=2 for 3D-STAR filter – L=7 for STAR filter

N_s=80 training vectors
3.2 km training window

note the narrow clutter notch of the 3D-STAR filter
Hot Clutter Examples

Direct path JDOA = -20°

L = 2, J = 1 for 3D-STAR filter – L = 7 for STAR filter

N_s = 80 training vectors
3.2 km training window

note the narrow clutter notch of the STAR filters
Conclusions

• STAR based filtering ideal for STAP problems that require small secondary sample support

• Easily extended to handle hot or range-varying clutter models

• Simulations with realistic circular array data show promising performance

• The structured nature of the filters leads to computationally efficient algorithms