Benefit of Constant Momentum Propulsion for Large ΔV Missions – Application in Laser Propulsion

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Outline

Tour of White Sands Laboratory (HELSTF/PLVTS) and Video of Flight Testing.

Comparison of Constant Momentum Mission and Constant Specific Impulse Mission. $\Delta v$, $v_{\text{jet}}$, $f$, $m_o$, $v_o$, $P_{\text{jet}}$, $m/E_{\text{jet}}$

Efficiency of conversion of laser energy to propellant kinetic energy, $\alpha \beta$.

Upper limit to conversion of laser energy to jet kinetic energy from energy conservation and definitions: $C_{v_{\text{jet}}} = \alpha \beta \Phi < 1$.

Comparing momentum quantities to energy quantities. The “Phi Factor” $\Phi = \langle v^2 \rangle / \langle v^2 \rangle$ and velocity distributions in propellant jet. $\Phi$ values for delta function, Maxwellian, Gaussian, Chunks and gas, supersonic expansion, etc.

Upper limits to performance based on chemical thermodynamics. Blowdown from defined equilibrium state $(u, \rho)$ of known volume.

Conclusions

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Pulsed Laser Vulnerability Test System (PLVTS)

- Original Performance
  - 800 joules/pulse
  - 10 Hz
  - 30μsec pulses
- Modified Performance
  - 1998
    - 400 joules/pulse
    - 28 Hz
    - 18μsec pulses
  - 1999
    - 150 joules/pulse
    - 30 Hz
    - 5μsec pulses

Optical Bench Set Up At 500-Ft Mark
Optical Power vs Time:
a) 2.5 ns; b) 5 ns; c) 18 ns; d) 35 ns

Field Test Telescope (FTT)
- 50 cm
- Cassegrainian
- Dynamic Focusing
- Minimum Acquisition
Distance is 200 m

Laser Beam Handoff to This Telescope Should Allow Altitudes of ~300 m (1,000 ft)
Laboratory Telescope Burn Patterns

Near Field
At ~10 Ft

5 cm Ref.

500 Ft

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FTT Beam Burn Patterns

500 Ft

1,000 Ft

11 cm Ref.

1,500 Ft

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Figure 2. Cross-sectional view of Myrabo Laser Lightcraft, Model 200-3/4. The maximum diameter of the test article at the shroud is \( \sim 10 \) cm. The indicated ring of Delrin weighs \( \sim 10 \) g and has a volume of \( \sim 7 \) cm\(^3\) and a surface area \( \sim 25 \) cm\(^2\). The idealized maximum plug nozzle exit area is \( \sim 350 \) cm\(^2\).
Overall Energy Conversion in Laser Propulsion Mission

\[ E_r = \frac{1}{2} m_r v_r^2 = \eta \alpha \beta \gamma \delta E_{\text{wall}} \]

\[ \eta = \text{propulsion efficiency (jet kinetic energy to vehicle kinetic energy)} \]
\[ \alpha = \text{expansion efficiency (internal propellant energy to jet kinetic energy)} \]
\[ \beta = \text{absorption efficiency (laser energy at vehicle to internal propellant energy)} \]
\[ \gamma = \text{transmission efficiency (laser energy at ground to laser energy at vehicle)} \]
\[ \delta = \text{laser efficiency (electric energy to laser energy at ground)} \]

***** Issue: separability of \( \eta \alpha \beta \gamma \delta \) *****

"$500 worth of electricity to put 1 kg into LEO." 

At $0.10/KWH, $500 buys 18,000MJ (1 KWH = 3.6 MJ); 1 kg at 10 km/s \( E_r = 50 \text{ MJ} \), so \( \eta \alpha \beta \gamma \delta \geq 0.0028 \)


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Definitions and Energy Conservation

Propellant Kinetic Energy: \[ E_p = \frac{1}{2} m_p \langle v_e^2 \rangle = \alpha \beta E_L \]
\[ \langle v_e^2 \rangle = \frac{\rho_e}{\rho_f} \int \frac{d(\rho v_e^2)}{d\rho} \]

Propellant Impulse: \[ I = m_0 \langle v_e \rangle \]
\[ \langle v_e \rangle = \frac{\rho_f}{\rho_e} \int \frac{d(\rho v_e)}{d\rho} \]

Coupling Coefficient: \[ C = \frac{I}{E_L} \]
\[ C = 2\alpha\beta \left[ \frac{\langle v_e \rangle^2}{\langle v_e^2 \rangle} \right] = \frac{2\alpha\beta \Phi}{\langle v_e \rangle} \]

Energy Conservation: \[ \alpha \beta \Phi = \frac{1^2}{2 m_p E_L} = \frac{CI}{2 m_p} = \frac{C \langle v_e \rangle}{2 E_L} = \frac{I \langle v_e \rangle}{2 E_L} \leq 1 \]

Propellant Internal Energy: \[ Q^* = \frac{u_c - u^0}{m_p} = \frac{\beta E_L}{m_p} \]
\[ C = \frac{\beta < v_e >}{u_c - u^0} \]

Propellant with added chemical energy, \( \Delta u \): \( (\alpha \beta \Phi)_{\text{apparent}} = \alpha \Phi (\beta + m_p \Delta u / E_L) \)

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**Delrin**

- Exhaust Velocity: \( v_e = I / m \)
- Ablation Efficiency: \( \alpha \beta \Phi = \frac{I^2}{2 m_e E_L} \)
- Mass Ablated per Shot: \( m = -0.0261 + 0.161E_L - 0.000133E_L^2 \)
- Impulse per Shot: \( I = 0.12 \)
INSTANTANEOUS PROPULSION EFFICIENCY

\[ \eta_i = \frac{2(v/v_{jet})}{1 + (v/v_{jet})^2} \]

\[ \eta_i < 1 \text{ if } v < v_{jet} \]

\[ \eta_i > 1 \text{ if } v > v_{jet} \]

CONSTANT MOMENTUM COMPARED TO CONSTANT SPECIFIC IMPULSE MISSION

The Constant Specific Impulse Mission

\[ \int \frac{dm}{m} = -\frac{1}{v_{jet}} \int dv \]

\[ f = \frac{m}{m_0} = \exp \left(-\frac{v - v_0}{v_{jet}}\right) = \exp \left(-\frac{\Delta v}{v_{jet}}\right) \]

The Constant Momentum Mission

\[ \int \frac{dm}{m} = \int \frac{-dv}{v} \]

\[ f' = \frac{m}{m_0} = \frac{v_0}{v} = 1 - \frac{\Delta v}{v_0} = \left(1 + \frac{\Delta v}{v_0}\right)^{-1} \]

\[ m_0 v_0 = m v \]
Figures of Merit for Laser Propulsion: \( m/E_{\text{jet}} \)

The Constant Specific Impulse Mission

\[
E_{\text{jet}} = -\frac{1}{2} \int \frac{m}{m_o} v_{\text{jet}}^2 \, dm = \frac{1}{2} (m_o - m)v_{\text{jet}}^2
\]

\[
B = \frac{m}{\frac{1}{2} (m_o - m)v_{\text{jet}}^2} = \frac{2x^2}{(e^x - 1)[\Delta v]^2} = \frac{2f(\ln f)^2}{(1 - f)[\Delta v]^2}
\]

The Constant Momentum Mission

\[
E_{\text{jet}}' = -\frac{1}{2} \int \frac{m}{m_o} v^2 \, dm = -\frac{1}{2} (m_o v_o)^2 \int \frac{m}{m_o} \, dm = \frac{1}{2} m v^2 - \frac{1}{2} m_o v_o^2 = \frac{1}{2} m v \Delta v
\]

\[
B' = \frac{m}{\frac{1}{2} m v \Delta v} = 2(1 - f') \frac{1}{[\Delta v]^2}
\]

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Mass accelerated per unit energy for constant momentum mission and constant specific impulse mission

\[
\frac{m}{E_{\text{jet}}} (x0.5(\Delta v)^2)
\]

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benefit3.aux July 29, 2003 9:11:47 AM
**Mission Time for Constant Specific Impulse and Constant Momentum Missions**

### Constant Specific Impulse

\[
P_{\text{jet}} = \frac{1}{2} v_{\text{jet}}^2 \frac{dm}{dt} = \frac{1}{2} F v_{\text{jet}}
\]

\[
f = \frac{m}{m_0} = 1 - \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} t
\]

\[
t = \frac{m_0}{2P_{\text{jet}}} \left( \frac{\Delta v}{\ln f} \right)^2
\]

\[
\Delta v = v_{\text{jet}} \ln \left( 1 - \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} t \right) = \sqrt{\frac{2P_{\text{jet}}}{m_0}} \left( \frac{\ln f}{1 - f} \right) t = \ln \left( \frac{B P_{\text{jet}}}{m_0} \right) \sqrt{\frac{2P_{\text{jet}}}{m_0}} \left( \frac{1}{1 - B P_{\text{jet}} \frac{1}{m_0}} \right)^{\frac{1}{2}}
\]

### Constant Momentum

\[
P_{\text{jet}} = \frac{1}{2} v_{\text{jet}} \frac{dm}{dt} = \frac{1}{2} F v_{\text{jet}}
\]

\[
f' = \frac{m}{m_0} = \left[ 1 + \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^2} t' \right]^{-1}
\]

\[
t' = \frac{m_0}{2P_{\text{jet}}} \left( \Delta' v \right)^2 \frac{f'}{1 - f'}
\]

\[
\Delta' v = \frac{2P_{\text{jet}}}{m_0 v_{\text{jet}}^0} t' = \sqrt{\frac{2P_{\text{jet}}}{m_0} \frac{(1 - f')}{f'}}
\]

---

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Conclusions

When $P_{\text{laser}} / m_0 \sim 0.05$ MW/kg small payloads (2 to 4 kg) may be launched into low earth orbit, $\Delta v \sim 10,000$ m/s.

At the same mass fraction, $f = 0.2$, $m/E_{\text{jet}}$ for constant momentum mission is 23% greater than for constant specific impulse mission.

For $\Delta v = 10,000$ m/s, $m_0 / P_{\text{jet}} = 20$ kg/MW, $f = 0.2$, $v_0 = 0$, the mission time for constant specific impulse propulsion is $\sim 315$ sec.

For $\Delta v = 10,000$ m/s, $m_0 / P_{\text{jet}} = 20$ kg/MW, $f = 0.2$, $v_0 = 2000$ m/s, the mission time for constant momentum propulsion is $\sim 155$ sec.

At the same $m/E_{\text{jet}} = 0.013$ kg/MJ and $\Delta v$, $f(\text{constant momentum}) = 0.35$, and $f(\text{constant specific impulse}) = 0.20$.

Based on measured $I$, $E_L$, and ablated mass, overall energy conversion efficiencies (laser energy to jet kinetic energy) of $\alpha \beta \sim 50\%$ were obtained with Delrin propellant in the laser lightcraft.

Jet exit velocities of $\sim 2000$ m/s with Delrin (based on measured mass) and $\sim 3000$ m/s with air (based on estimated mass).

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THE COUPLING COEFFICIENT AND THE SPECIFIC IMPULSE

\[ Q^* = \beta E_L / m \]

\[ E_{\text{jet}} = \frac{1}{2} m \langle v^2 \rangle = \alpha m Q^* = \alpha \beta E_L. \]

\[ I = m \langle v \rangle \]

\[ C = \frac{I}{E_L} \]

\[ \frac{1}{2} C \langle v \rangle = \alpha \beta \Phi \leq 1 \]

\[ P_L = \omega E_L. \]

\[ F = \omega E_L C \]

\[ \frac{1}{2} F \langle v \rangle = \alpha \beta \Phi P_L. \]

\[ P_{\text{jet}} = \frac{1}{2} \frac{F \langle v \rangle}{\Phi} = \alpha \beta P_L. \]

\[ (\alpha \beta \Phi)_{\text{apparent}} = \alpha \Phi (\beta + m \Delta u_{\text{chem}} / E_L) \]

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Laser Lightcraft Flights with Air: \[ h = \frac{1}{2} t^2 (C_{E_L} w / M g) = \frac{1}{2} g t^2 (T/W - 1) \]

Model 200-3/4, M = 0.04 kg, 10 kW at \( w = 25 \) Hz, \( E_L = 400 \) J/pulse
Table 1. Normalized absorption volume for air at 1.18 kg/m^3 as a function of internal energy and laser energy.

| u/Mg | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| h/MJ | 20 | 40 | 60 | 80 | 100 | 120 | 140 | 160 | 180 | 200 | 220 | 240 | 260 | 280 | 300 | 320 | 340 | 360 | 380 | 400 | 420 | 440 | 460 | 480 | 500 | 520 | 540 | 560 | 580 | 600 |
| V_{abs} | 42.3 | 21.6 | 14.1 | 21.7 | 16.9 | 8.47 | 7.06 | 6.05 | 5.30 | 4.71 | 4.26 | 3.82 | 3.41 | 3.06 | 2.82 | 2.62 | 2.44 | 2.26 | 2.12 | 1.95 | 1.82 | 1.71 | 1.61 | 1.52 | 1.44 | 1.37 | 1.31 | 1.25 | 1.20 |

Figure 1. Cross-sectional view of Mykrobo Laser Lightcraft, Model 200-3/4. The maximum diameter of the test article at the shroud is ~ 10 cm. The indicated ring of Delrin weighs ~ 10 g and has a volume of ~ 7 cm^3 and a surface area ~ 25 cm^2. The idealized plug nozzle exit area is ~ 350 cm^2.

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Model 200-3/4 Lightcraft with Air and Delrin

- Delrin with tight focus
- Air with tight focus
- Air with loose focus

Coupling Coefficient (N-s/MJ)

E_{laser} (J)
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Air Equilibrium (N2, O2, N, O, NO, NO2, no ions)
Ideal mixture of ideal gases

Equilibrium Blowdown

specific entropy, J/kg K
Thermodynamic properties of Mach 5 air at stagnation density, 
\( \rho = 5.90 \text{ kg/m}^3 \).

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Figure 10. Comparison of Equilibrium expansion and frozen expansion of air. The circles and nearby crosses represent the blowdown quantities obtained from initial \( (u\_v\_n)\) states of 2E5, and 1E4 J/kg for the frozen expansion and 2E3, 6E3, 1E4, and 4E4 J/kg for the equilibrium expansion. The results of the two frozen blowdown integrations to \( P_\text{in} = 1 \text{ bar} \) are plotted with those of the equilibrium blowdown to show that the differences in alpha are small, i.e., at low energy (2E3) 0.30 and 0.29 and at high energy (1E4) 0.32 and 0.27 for equilibrium and frozen blowdown, respectively.

Figure 11. Comparison of Equilibrium expansion from laser heated STP air (1.18 kg/m\(^3\)) and Mach 5 air at stagnation density (5.9 kg/m\(^3\)). In the STP air diagram (on left), the circles and nearby crosses represent the blowdown quantities obtained from initial \( (u\_v\_n)\) states of 2E5, and 1E4 J/kg for the frozen expansion and 2E3, 6E3, 1E4, and 4E4 J/kg for the equilibrium expansion.