An Energy Budget Model to Calculate the Low Atmosphere Profiles of Effective Sound Speed at Night

by Arnold Tunick

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An Energy Budget Model to Calculate the Low Atmosphere Profiles of Effective Sound Speed at Night

Arnold Tunick
Computational and Information Science Directorate, ARL
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**14. ABSTRACT**

This report presents an alternate energy budget model to the one implemented in the Acoustic Battlefield Aid (ABFA) for generating low atmosphere profiles of effective sound speed at night. The alternate model is based on the solution of a quartic equation for surface temperature, which assumes a balance between the net (long wave) radiative flux and the turbulent fluxes of heat and moisture close to the ground. In contrast, the model in ABFA assumes that the net surface radiation is constant with time at night, reduced only in the presence of clouds, and is balanced entirely by the surface heat flux. Therefore, the alternate model may be an improved calculation for this application. However, some difficulties were encountered in the application of traditional Monin-Obukhov (M-O) profile theory for calculating nighttime turbulence structure from wind speed, temperature, and humidity difference data. Model breakdowns were problematic for a nighttime case when surface wind speeds were low, \( U < 4.0 \text{ ms}^{-1} \), and skies were clear. During such stable and very stable conditions, the M-O scaling length too often became small—small enough to cause a computational sensitivity. Thus, we recommend that the alternate model in its present form be applied only when surface wind speeds and surface roughness lengths (i.e., canopy heights) can affect values for the M-O scaling length larger than about 13 m.

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1. Introduction

Advanced sensors and computer models for the surveillance, detection, identification, and tracking of sound-emitting targets in combat are rapidly emerging (1,2). Most new Army acoustic systems use unattended microphone sensors to construct small ground-based, beam-forming arrays to determine line-of-bearing angles, e.g., the remote netted acoustic detection system (3). At the same time, point-to-point transmission of acoustic signals is greatly influenced by variations in the wind flow, temperature field, and turbulence over varying topography and surface energy budgets (4-6). Thus, the Army is looking to implement the best possible acoustic propagation models, particularly those that will incorporate the impacts of the environment (e.g., the atmosphere, turbulence, and terrain) on acoustic signatures.

A key element in determining point-to-point acoustic transmission is modeling the variation of the effective speed of sound through the lower atmosphere. The effective speed of sound ($c_{\text{eff}}$), which accounts for the bearing of the sound source from a given receiver and the compass bearing of the mean wind, is readily derived from measured or modeled estimates of air temperature, relative humidity, and wind velocity. It is useful to define the effective speed of sound ($c_{\text{eff}}$) from the following expression given by Noble (7) and Osteshev (5):

$$c_{\text{eff}} = c_o + \bar{u} \cos(\theta_w - \pi - \theta_R) ,$$

in which $c_o = \sqrt{\frac{R T}{M}}$, $R = 8314.32 \text{ J mol}^{-1} \text{K}^{-1}$ is the universal gas constant, $T$ is air temperature in Kelvin, $M$ is molecular mass, $\gamma_s = \frac{c_p}{c_v}$ is the ratio of specific heats, $\bar{u}$ is the mean of the horizontal wind, $\theta_w$ is the bearing of the wind from North, $\theta_R$ is the bearing of the receiver from the source, and $\bar{u} \cos(\theta_w - \pi - \theta_R)$ is the component of the sound speed along the direction of propagation from the source to the receiver. As discussed in Osteshev (5), the effective sound speed in equation 1 is valid only for nearly horizontal propagation angles (8). In addition, Wong and Embleton (9) have deduced the ratio of specific heats and molar mass $\left(\frac{\gamma_s}{M}\right)$ as a function of temperature and humidity in the following form:

$$\frac{\gamma_s}{M} = 0.04833 + (R_h - 0.023)A_t ,$$

in which $A_t = 9.2 \times 10^{-5} + 5.5 \times 10^{-6} t + 4.25 \times 10^{-7} t^2$, $R_h$, is relative humidity (in the range 0.0 to 1.0), and $t$ is air temperature ($^\circ$C). Then, for applications in outdoor acoustics, sound speed differences across vertical layers will generally cause acoustic waves to be refracted upward if the effective sound speed decreases with height, and they will be refracted downward if sound speed increases with height.
Monin-Obukhov (M-O) similarity models (10–14) have been very well suited for such acoustic applications. For example, the energy budget and meteorological models implemented in the Acoustic Battlefield Aid (ABFA) (15,16) are relatively simple and quite robust for generating low atmosphere profiles of effective sound speed, especially for daytime convective atmospheres. However, for the nighttime case, the model in ABFA assumes that the net surface radiation is constant with time, reduced only in the presence of clouds, and is balanced entirely by the surface heat flux. Therefore, in this report, we are interested in testing the effectiveness (for this application) of an earlier quartic solution for the surface temperature (17,18), i.e., an alternate energy budget model, which explicitly computes the net (long wave) radiative flux and the turbulent fluxes of heat and moisture close to the ground. Then, the primary energy budget model results can be used to estimate profiles for wind speed, air temperature, and relative humidity, from which, effective sound speed information can be generated.  

2. Model Equations

2.1 A Quartic Solution for the Surface Temperature

The alternate surface energy budget model described in this section is a semi-empirical, equilibrium model (17,18) that explicitly computes each energy balance component, except the flux of energy into and out from the ground, which is computed simply as a percentage, 10% daytime and 5% nighttime, of the computed net radiation (21). The model’s primary input are reference level (1 to 2 m) values for wind speed, air temperature, relative humidity, atmospheric pressure, surface thermal emissivity, soil water content, and cloud cover information. (For daytime calculations, the day of interest, longitude, and latitude of the site are required to determine the solar declination, hour, and zenith angles.) The model’s primary output are calculations of the net radiation, the effective surface temperature, and the soil, sensible, and latent heat fluxes.

In accordance with Carson (22), it is assumed that the energy budget close to the ground (as shown in Figure 1) can be written as

\[ R_{net} = H + L'E + G, \]

where \( R_{net} \) is the net radiative flux, \( H \) is the turbulent sensible heat flux, \( L'E \) is the latent heat flux attributable to surface evaporation, and \( G \) is the flux of heat into or out of the soil. Positive values of \( R_{net} \) and \( G \) are chosen to represent fluxes directed downward while positive \( H \) and \( L'E \) values represent fluxes directed away from the air-soil interface.

1Note that Swanson and Reichard (19) and Swanson (20) have also reported on the use of M-O similarity-type formulations to generate low atmosphere sound speed profiles. However, they based their estimates of the M-O scaling constants on data retrieved from a ground-based micrometeorological instrument rather than from energy budget principles, as discussed in this report. Nevertheless, their goal was very similar, i.e., to incorporate similarity-based estimates of effective sound speed within probability of detection (sensor performance) algorithms for unattended acoustic sensor arrays.
The formulations in the model used to compute the incoming short wave solar radiation ($R_{S\downarrow}$) for daytime cloudless skies are patterned after Meyers and Dale (23) and are augmented as needed with empirical results by Haurwitz (24) to account for cloudy skies, i.e.,

$$R_{S\downarrow} = I = I_o T R T P T W T A \cos Z,$$  \hspace{1cm} (2)

in which $I_o$ is the extra-terrestrial flux density at the top of the atmosphere on a surface normal to the incident radiation, $Z$ is the solar zenith angle, and $T_i$ are the transmission coefficients for Rayleigh scattering ($R$), absorption by permanent gases ($P$), water vapor ($W$), and absorption and scattering of aerosols ($A$). The empirically derived transmission coefficient for cloudy skies (24) is defined by the ratio of insolation with partly or completely covered sky to insolation of cloudless skies.

Gates (25) gives the expression for downward long wave radiation ($R_{L\downarrow}$) for clear skies in the model, i.e.,

$$R_{L\downarrow} = -170.9 + 1.195 \sigma T_r^4,$$  \hspace{1cm} (3)

in which $R_{L\downarrow}$ is in watts per meter$^2$, $\sigma = 5.6697 \times 10^{-8}$ W m$^{-2}$ K$^{-4}$, and $T_r$ the reference level (1 to 2 m) temperature in Kelvin. From Paltridge and Platt (26) an addition to equation 3 is made to account for clouds, giving the total expression as

$$R_{L\downarrow} = -170.9 + 1.194 \sigma T_r^4 + 0.3 \varepsilon_c \sigma T_c^4 (cc),$$  \hspace{1cm} (4)

in which $\varepsilon_c$ is the long wave flux emissivity of the cloud base, $T_c$ is the temperature of the cloud base in Kelvin, and $cc$ is the fractional cloud cover. Having an estimate of the cloud height, one can approximate $T_c$, assuming an average of the dry and moist adiabatic lapse rate.
The upward long wave radiative flux \( R_{L_{\uparrow}} \) is computed by a formulation from Yamada (27), i.e.,

\[
R_{L_{\uparrow}} = \varepsilon \sigma T_g^4 + (1 - \varepsilon) R_{L_\downarrow},
\]

(5)

To evaluate equation 5, one requires an estimate of the effective ground or surface temperature, \( T_g \). We achieve this through the application of M-O similarity formulations (10–14) and the flux-gradient hypothesis as advocated by Dyer (12) and Webb (14).

The similarity-based expressions for the sensible and latent heat fluxes are defined by the MARIAH algorithm (28) and are expressed as

\[
H = -c_p \rho u_* \theta_* = -\frac{c_p \rho k^2 V_r (T_r - T_g)}{\phi_m \phi_h \ln \left( \frac{z - d}{z_0} \right) \ln \left( \frac{z - d}{z_h} \right)},
\]

(6)

and

\[
L' E = -L' \rho u_* q_* = -\frac{L' \rho k^2 V_r (q_r - q_g)}{\phi_m \phi_q \ln \left( \frac{z - d}{z_0} \right) \ln \left( \frac{z - d}{z_h} \right)},
\]

(7)

in which \( c_p \) is specific heat; \( \rho \) is air density; \( k \) is von Karman's constant; \( V_r \) is the reference height wind speed; \( L' \) is the latent heat of vaporization; \( q_r \) and \( q_g \) are the reference height and effective surface values for specific humidity, respectively; \( u_* \) is the friction velocity (m/s); \( \theta_* \) is the potential temperature scaling constant (K); \( q_* \) is the specific humidity scaling constant (kg/kg); \( z \) is height above ground in meters; \( d \) is displacement height in meters (~70% of the principal roughness element); \( z_0 \) is the aerodynamic (surface) roughness height in meters (~14% of the principal roughness element); \( z_h \) is the thermal roughness height in meters (~13% of the aerodynamic roughness); and \( \phi_m, \phi_h, \) and \( \phi_q \) are the dimensionless wind shear, dimensionless temperature lapse rate, and dimensionless humidity lapse rate, respectively. Here, the dimensionless term for humidity is assumed to be equal to the dimensionless temperature lapse, i.e., \( \phi_q = \phi_h \), even though field observations have shown that this is not always correct. Also, for stable and very stable conditions at night, Mahrt (29) recommends setting \( z_0 = z_h \). Equations 6 and 7 can be evaluated in a straightforward manner, given expressions for the dimensionless shear and lapse rate terms.

Following the recommendations of Högström (30), we use

\[
\phi_m = [1 - 19(z / L)]^{-1/4} \text{ and } \phi_h = 0.95[1 - 11.6(z / L)]^{-1/2},
\]

(8)

for daytime unstable atmospheric conditions, and we use

\[
\phi_m = 1 + 5.3(z / L) \text{ and } \phi_h = 1 + 8(z / L)
\]

(9)
for nighttime stable atmospheric conditions\(^2\), where Busch (11) defines the M-O scaling ratio as

\[
\frac{z}{L} = k \frac{g}{\theta_v u^*} \frac{\theta_v}{\theta_v} \frac{z}{u^*},
\]  

(10)
in which \(\theta_v = T_v(1 + 0.61q_v)\) is the virtual potential temperature and \(\theta_v^* = \theta_v + 0.61T_vq_v^*\) is the virtual potential temperature scaling constant (\(q_v^*\) is the specific humidity scaling constant).

Here, reference level specific humidity, \(q_r\), is determined as a function of relative humidity (\(R_h\)), atmospheric pressure (\(P_r\)), and air temperature (\(T_v\)) with the formulation from Rogers (31), i.e.,

\[
q_r = \frac{3.8R_h}{P_r} \exp \left( 5.44 \times 10^3 \left( \frac{1}{2.73.15} - \frac{1}{T_r} \right) \right).
\]

(11)

In contrast, surface specific humidity, \(q_s\), is determined as a function of the following parameters (32): reference level specific humidity (\(q_r\)), saturation specific humidity at the surface (\(q_s\)) (i.e., substitute into equation 11 \(R_h = 100\%\) and \(T_v = T_g\)), the water capacity of the field (\(\theta_f\)), and finally, soil water content (\(w_s\)). As a result, surface specific humidity can be expressed as

\[
q_g = (1 - \delta) q_r + \delta q_s,
\]

(12)
in which

\[
\delta = 0.25 \left( 1 - \cos \left( \frac{\pi w_s}{\theta_f} \right) \right) \quad \text{for} \quad w_s < \theta_f
\]

(13)
and

\[
\delta = 1 \quad \text{for} \quad w_s > \theta_f.
\]

Next, we substitute equations 6 and 7 for \(H\) and \(L'E\) into equation 1 and define the soil heat flux (\(G\)) as \(G = \beta R_{net}\) (where \(R_{net} = R_{L\downarrow} - R_{L\uparrow} + R_{S\downarrow} - R_{S\uparrow}\) and \(R_{S\uparrow} = \alpha R_{S\downarrow})\). Then after some rearranging, we solve equation 1 for \(T_g\), which gives the quartic form for the equilibrium model as

\[
(1 - \beta) \left( \varepsilon \sigma T_g^4 - (1 - \alpha) R_{S\downarrow} - \varepsilon R_{L\downarrow} \right) + T_g \left( \frac{c_p \rho k^2 V_r}{\phi_m \phi_h \ln \frac{z - d}{z_o}} + \ln \frac{z - d}{z_h} \right) = 0
\]

(14)
in which \(\alpha\) is the surface short wave reflectivity, commonly called “albedo,” and \(\beta\) is the percentage (e.g., 10% daytime and 5% nighttime) of the computed net radiation used to calculate the soil (ground storage) heat flux. Naturally, at night, \(R_{S\downarrow} = 0\). The quartic equation is resolved numerically via a Newton-Raphson iterative scheme.

\(^2\)Alternately, Webb (14) suggests \(\phi_h = \phi_h = 1 + 5(z/L)\) for nighttime stable conditions.
2.2 The Energy Budget Model Implemented in ABFA

Wilson (15) and Wilson and Szeto (16) report on the development of the prototype model ABFA for the analysis of atmospheric refraction and turbulence effects on acoustical detection and tracking systems. Among the meteorological input capabilities embedded in ABFA is a surface energy budget model for generating low atmosphere profiles of effective sound speed. The ABFA energy budget model is based on the empirical formulations of Burridge and Gadd (33), as discussed in Stull’s (21) textbook (Chapter 7). For the nighttime case, the net long wave radiation is given by the following expression:

\[ R_{L\uparrow} - R_{L\downarrow} = (0.08 \rho c_p) \left( 1.0 - 0.1 \sigma_H - 0.3 \sigma_M - 0.6 \sigma_L \right), \]  

(15)

where \( \sigma_H, \sigma_M, \) and \( \sigma_L \) are the cloud cover fractions for high, middle, and low clouds, respectively. (In contrast, cloud cover is accounted for in the parameterization for \( R_{L\downarrow} \) shown in equation 4.) Clearly, this is a much simpler formulation than the solution of the quartic.

In ABFA, the net surface radiation is balanced entirely by the surface heat flux, \( Q \), so that the M-O temperature scaling constant is calculated as

\[ \theta_* = -\frac{Q}{\rho c_p u_*}, \]  

(16)

and the M-O scaling length is derived as

\[ L = -\frac{u_*^3}{k g \frac{Q}{\theta \rho c_p}}, \]  

(17)

where the friction velocity can be determined from the reference height wind speed \( V_r \) and values for roughness length \( (z_0) \) and displacement \( (d) \), i.e.,

\[ u_* = \frac{k V_r}{\phi_m \ln \left( \frac{z - d}{z_0} \right)}. \]  

(18)

Thus, the ABFA energy budget model output is used to estimate the similarity-based, log-linear profiles for wind speed and temperature extended upward through the surface layer, as discussed next.

2.3 The Similarity-Based Profiles

Given values for the M-O scaling constants from energy budget considerations, the formulations of Benoit (34) and Nickerson and Smiley (35) are applied to determine the similarity-based profiles for wind velocity, temperature, and specific humidity as follows:

\[ V(z) = V_r + \frac{u_*}{k} \left[ \ln \left( \frac{z}{z_r} \right) - \psi_m \left( \frac{z}{L} \right) + \psi_m \left( \frac{z_r}{L} \right) \right], \]  

(19)
\[ T(z) = T_r + \frac{\theta_e}{k} \left[ \ln \left( \frac{z}{z_r} \right) - \psi_h \left( \frac{z}{L} \right) + \psi_h \left( \frac{z_r}{L} \right) \right] \] , (20)

and

\[ q(z) = q_r + \frac{q_e}{k} \left[ \ln \left( \frac{z}{z_r} \right) - \psi_q \left( \frac{z}{L} \right) + \psi_q \left( \frac{z_r}{L} \right) \right] \] , (21)

where, for the nighttime case, the diabatic influence functions for momentum and heat, \( \psi_m \) and \( \psi_h \) respectively, are given by Webb (14), i.e.,

\[ \psi_m = \psi_h = -5 \frac{z}{L} . \] (22)

Here, we assume that \( \psi_q = \psi_h \).

For extended profiles of relative humidity, we use equation 11 and the following expression for atmospheric pressure given by Campbell and Norman (36), i.e.,

\[ P(z) = P_r \exp \left( \frac{-\left( z - z_r \right)}{8200} \right) \] (23)

which yields

\[ R_h(z) = 100.0 \frac{q(z)P(z)}{3.8} \exp \left( 5.44 \times 10^3 \left( \frac{1}{273.15} - \frac{1}{T(z)} \right) \right)^{-1} . \] (24)

### 3. Model Results

#### 3.1 A Hypothetical Case

A hypothetical nighttime case is outlined in Table 1. Input parameter values were chosen to represent low wind speeds and cool temperatures over a relatively smooth surface. For this example, the primary output parameters from the alternate energy budget model (described previously) are given in Table 2. First, it is shown that fairly good closure exists for this calculation, i.e., \( R_{net} - G \) is approximately equal to \( H + L' E \). This result helps to validate the model for partitioning the net radiation into its surface flux components. The small error (i.e., \( \Delta = 6.83 \text{ Wm}^{-2} \)) may be attributed to the approximation for soil heat flux. Second, the estimate for surface temperature, i.e., \( T_s = T_r - 2.25 \text{ K} \), appears to be quite reasonable for moderately stable conditions (i.e., \( L = 35 \text{ m} \)). Third, the M-O scaling constants appear to be valid and useful, so that applying equations 19 through 24 provides the nighttime profiles for wind velocity, air temperature, and relative humidity for the layer \( z \leq 60 \text{ m} \) (see Figure 2).
Table 1. Energy budget model parameters

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<thead>
<tr>
<th>Parameter</th>
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<tr>
<td>Reference atmospheric pressure</td>
<td>$P_r$</td>
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<tr>
<td>Reference relative humidity</td>
<td>$R_h$</td>
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<td>Reference air temperature</td>
<td>$T_r$</td>
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<tr>
<td>Reference wind velocity</td>
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<tr>
<td>Reference height above ground</td>
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<td>Principal roughness element or canopy</td>
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<td>height</td>
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<td>0.13 m$^3$m$^{-3}$</td>
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Table 2. Energy budget model results

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<thead>
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<th>Parameter</th>
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<td>Net radiative flux</td>
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<td>Soil (ground storage) heat flux</td>
<td>$G$</td>
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<td>Sensible heat flux</td>
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<td>Temperature scaling constant</td>
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<td>0.27 °K</td>
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<td>Humidity scaling constant</td>
<td>$q_*$</td>
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</tr>
<tr>
<td>Monin-Obukhov scaling length</td>
<td>$L$</td>
<td>34.64 m</td>
</tr>
<tr>
<td>Surface air temperature</td>
<td>$T_g$</td>
<td>280.90 °K</td>
</tr>
</tbody>
</table>

Figure 2. Typical similarity-based profiles for the nighttime case derived from the energy budget model: a) wind velocity (in units ms$^{-1}$), b) air temperature (in units C), and c) relative humidity (in percent).
Next, we apply equations 1 and 2 and the data shown before to derive the extended profiles for the effective speed of sound (see Figure 3). In Figure 3, the effective sound speed is determined for upwind propagation (left), i.e., the bearing of the receiver from the source and the direction of the wind from north are the same, and for downwind propagation (right), i.e., the bearing angles are 180° opposite. An asymmetry exists in the graph so that the profile for downwind propagation (on the right-hand side) is increasing with height above ground level, while the curve for upwind propagation (on the left side) is decreasing (slightly) with height. The effect that such profile variations in sound speed will have on short range acoustic attenuation is discussed next.

![Figure 3. Low atmosphere profiles of the effective speed of sound derived from the energy budget model for upwind propagation (dashed) and downwind propagation (solid).](image)

### 3.2 Approximation of Short Range Acoustic Attenuation

To briefly examine short range acoustic attenuation at night, we use the low atmosphere profiles of wind speed, temperature, and relative humidity (shown before) as input to a flat earth, non-turbulent acoustic propagation model called the Windows (version) Scanning Fast Field Program (WSCAFFIP). WSCAFFIP is a numerical code developed for assessing environmental effects on short range acoustic attenuation \((7,38)\). WSCAFFIP determines acoustic attenuation as relative sound pressure loss with range and azimuth for a given frequency and source-to-receiver geometry. WSCAFFIP contains propagation algorithms to represent the effects of atmospheric refraction, diffraction, absorption, and reflection (ground impedance) on acoustic transmission. Table 3 lists the model parameters for an initial approximation of short range acoustic attenuation over an open grass-covered \((h = 0.5 \text{ m})\) field. Figures 4 and 5 show the WSCAFFIP results corresponding to the modeled profiles of effective sound speed generated by the alternate (quartic) model.
Table 3. WSCAFFIP model parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance of receiver from source</td>
<td>$x$</td>
<td>1000 m</td>
</tr>
<tr>
<td>Range resolution</td>
<td>$\Delta x$</td>
<td>10 m</td>
</tr>
<tr>
<td>Bearing of receiver from source</td>
<td>$\theta_R$</td>
<td>90º</td>
</tr>
<tr>
<td>Bearing of the wind from North</td>
<td>$\theta_w$</td>
<td>90º (upwind) and 270º (downwind)</td>
</tr>
<tr>
<td>Source height above ground</td>
<td>$h_S$</td>
<td>2 m</td>
</tr>
<tr>
<td>Receiver height above ground</td>
<td>$h_R$</td>
<td>1 m</td>
</tr>
<tr>
<td>Frequency of interest</td>
<td>$f$</td>
<td>100 - 1000 Hz</td>
</tr>
<tr>
<td>Ground porosity (tall grass pasture)</td>
<td>$\Omega_p$</td>
<td>0.675</td>
</tr>
<tr>
<td>Flow resistivity (tall grass pasture)</td>
<td>$\sigma_r$</td>
<td>330 kPa s m$^{-2}$</td>
</tr>
</tbody>
</table>

Figure 4. Line plots of acoustic attenuation versus range for upwind propagation (dashed) and downwind propagation (solid), (i.e., WSCAFFIP model results for 300 Hz and 600 Hz, only).
Figure 5. Contour graphs of acoustic attenuation as a function of frequency and range for downwind propagation (top) and upwind propagation (bottom) (i.e., WSCAFFIP model results for 100 Hz through 1000 Hz, inclusive).
As stated earlier, acoustic waves will tend to be refracted downward as the effective sound speed increases with height and refracted upward as the effective sound speed decreases with height. Therefore, one might expect the amounts of attenuation (in part) to depend on the strength of the sound speed profile gradient between the ground and the model top. Because generally, the effective sound speed profile for upwind propagation has values decreasing with height, we would expect to see greater attenuation with range at each frequency for upwind propagation as a result. The line plots of relative sound loss with range in Figure 4 (for 300 Hz and 600 Hz only) and the contour graphs of attenuation as a function of range and frequency shown in Figure 5 (for 100 to 1000 Hz inclusive) show this result satisfactorily.

In contrast, the behavior of the downwind propagation attenuation curves can be understood in terms of “ducting” of acoustic modes. For this example, the sound speed profile forms an acoustic duct between the surface and 60 m. At lower frequencies (100 to 200 Hz), acoustic waves may be too long for any significant ducting to occur. At 300 Hz, there appear to be one or two trapped modes, while at 400 Hz (and greater) several modes appear to be trapped, creating the interference pattern. In this way, ducting of acoustic energy across the lower boundary layer at night is greatly affected by local meteorological profile structure.

3.3 Comparison to Observations

At the start of this project, it was thought feasible to present modeled profiles of effective sound speed for the nighttime case in comparison to values of effective speed of sound derived from selected micro-meteorological data. As an example, the tower data used to generate the three time-height series shown in Figure 6 were collected as part of the 1999 Cooperative Atmosphere-Surface Exchange Study (CASES-99) (39). CASES-99 was an extensive field experiment that focused on the nocturnal boundary layer (NBL) under mainly clear sky, light near surface wind conditions (this included occurrences of weak and intermittent turbulence). The experimental period was from October 1 to 31, 1999. The test area for CASES-99 was near Leon, KS (50 km) east of Wichita, KS. The test site was chosen (in part) because of its relatively flat terrain (average slopes were 0.5 degree). A large number of instruments were deployed in an area 4.8 × 3.2 km. These instruments included a heavily instrumented 55-m tower, numerous 10-m towers, multiple radar, lidar, scintillometers, tethersondes, rawinsondes, and research aircraft (see http://www.joss.ucar.edu/cases99).

The subset of CASES-99 data shown in Figure 6 is from the first intensive observing period (IOP) on 5 October 1999 and includes the measurements of wind velocity, air temperature, relative humidity (5 min averaged) taken on the 55-m main tower approximately 5 km southeast of Leon, KS (latitude: 37 deg 38.88' N, longitude: 96 deg 44.14' W, elevation 433.7 m above sea level) in an open field of mostly dry grasses, approximately 0.15 m in height (on average). Surface roughness height for the site was determined to be about 2 cm and thermal emissivity was estimated at 0.98 (40).
Figure 6. Time-height series of a) air temperature (in units °C), b) mean horizontal wind speed (in units ms⁻¹), and c) effective sound speed (in units ms⁻¹), generated from CASES-99 (IOP-1) field data.
However, we experienced some difficulties in the application of traditional M-O profile theory for calculating nighttime turbulence structure from temperature, humidity, and wind speed difference data (see equations 6, 7, and 14). As an example, Figure 7 shows a time series of the modeled surface temperature ($T_g$), the 5-m measured temperature ($T_r$), and the 5-m measured wind speed ($U_r$) for the first nighttime IOP. For this example, reference height is $z_r = 5$ m and the canopy height is $h = 0.15$ m. Certainly, the surface temperature estimates are unrealistically low. This “runaway cooling,” according to Mahrt (29), is a known defect in computer models that attempt to estimate meteorological gradients near the ground at night (during very stable conditions) with M-O similarity functions. To illustrate the problem further, Figure 8 shows a plot (at $t = 0.125$ hr past midnight) of the estimated surface temperature ($T_g$), the estimated M-O scaling length ($L$), and the estimated dimensionless wind shear ($\phi_m$) as a function of the 5-m reference height wind speed ($U_r$), as the wind speed is varied from $U_r = 1.0$ ms$^{-1}$ to $U_r = 10.0$ ms$^{-1}$. In addition, the data in Table 4 and Figure 9 show a more detail view of the results plotted in Figure 8. The data clearly show that the model calculation collapses at $U_r = 3.6 – 3.8$ ms$^{-1}$, where $T_g \leq 5.0$ C, $L \leq 8.0$ m, and $\phi_m \geq 1.6$. Then, for still lower wind speeds, $L$ becomes very small—small enough to cause a computational sensitivity, as evidenced in the plot for $\phi_m$. Given that both $\phi_m$ and $\phi_h$ are needed to define two principal components of the quartic, it can be reasoned that when $L$ is small and the $\phi$’s are large, the model defaults to a kind of radiative equilibrium, where little heat energy (sensible, latent, or released from the soil) is available to offset the radiative term. When this occurs, i.e., when turbulence collapses, the model results are invalid.

As a final point, we consider the effect on the model results brought about by varied surface roughness (see Figure 10). Recall that for this example, the canopy height is $h = 0.15$ m (short grass) and the roughness length is $z_0 = 0.02$ m. In Figure 10, when $U_r = 3.6$ ms$^{-1}$ and the canopy height < 0.25 m, the model again breaks down. Thus, for whatever the order of wind speed and roughness length, there appears be a derivable limit beyond which the calculation should be discounted. The arrow on Figure 9 suggests such a limit.

It is interesting to note that others had previously reported difficulties in the use of similarity-based models for the nighttime case (in the calculation of optical turbulence intensity [$C_n^2$]) (41–43). As an example, Figure 11 shows results from the $Cn2$ over-land optical turbulence model (42,43), derived from two levels (above ground) of conventional micro-meteorological information as input (i.e., wind speed, temperature, and humidity retrieved on farmland, over flat barren soil, under mostly clear skies). Although the $Cn2$ model estimates are in fairly good agreement with the observations during daytime hours, there are several instances at night when the model results show either a consistent bias (about a factor of 2 and mainly underestimated) in comparison to the measured turbulence data or the model results fluctuate toward extreme (minimum) values. Here, the largest errors occur when there is a computational sensitivity, as described before. Average surface layer wind speeds for the nighttime periods shown in Figure 11 were approximately 3 to 4 ms$^{-1}$. 
Figure 7. Time series of a) modeled surface temperature ($T_r$) and the 5-m temperature ($T_s$) and b) the 5-m measured wind speed ($U_r$) for the first nighttime IOP. (The dotted line in Figure 7b helps to show the large amount of data in this example where $U \leq 2.0$ m s$^{-1}$.)
Figure 8. A plot at \( t = 0.125 \) hr of a) the estimated surface temperature \( (T_g) \), b) the estimated M-O scaling length \( (L) \), and c) the estimated dimensionless wind shear \( (\phi_m) \) as a function of the 5-m reference height wind speed \( (U_r) \).

Table 4. Breakdown of M-O similarity models for the nighttime case

<table>
<thead>
<tr>
<th>( U_r ) (ms(^{-1}))</th>
<th>( T_g ) (C)</th>
<th>( L ) (m)</th>
<th>( \phi_m )</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2</td>
<td>-8.698</td>
<td>0.005</td>
<td>837.346</td>
</tr>
<tr>
<td>3.3</td>
<td>-8.698</td>
<td>0.008</td>
<td>547.225</td>
</tr>
<tr>
<td>3.4</td>
<td>-8.697</td>
<td>0.016</td>
<td>275.226</td>
</tr>
<tr>
<td>3.5</td>
<td>-8.682</td>
<td>0.066</td>
<td>71.424</td>
</tr>
<tr>
<td><strong>3.6</strong></td>
<td><strong>-8.447</strong></td>
<td><strong>0.275</strong></td>
<td><strong>18.173</strong></td>
</tr>
<tr>
<td><strong>3.7</strong></td>
<td><strong>2.579</strong></td>
<td><strong>4.914</strong></td>
<td><strong>1.964</strong></td>
</tr>
<tr>
<td>3.8</td>
<td>5.140</td>
<td>8.477</td>
<td>1.559</td>
</tr>
<tr>
<td>3.9</td>
<td>6.062</td>
<td>10.945</td>
<td>1.433</td>
</tr>
<tr>
<td><strong>4.0</strong></td>
<td><strong>6.647</strong></td>
<td><strong>13.245</strong></td>
<td><strong>1.358</strong></td>
</tr>
<tr>
<td>4.1</td>
<td>7.072</td>
<td>15.504</td>
<td>1.306</td>
</tr>
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<td>17.780</td>
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<td>7.671</td>
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<td>4.4</td>
<td>7.896</td>
<td>22.471</td>
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<td>4.5</td>
<td>8.088</td>
<td>24.908</td>
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</tr>
<tr>
<td>4.6</td>
<td>8.255</td>
<td>27.426</td>
<td>1.173</td>
</tr>
<tr>
<td>4.7</td>
<td>8.401</td>
<td>30.016</td>
<td>1.158</td>
</tr>
<tr>
<td>4.8</td>
<td>8.532</td>
<td>32.693</td>
<td>1.145</td>
</tr>
</tbody>
</table>
Figure 9. A more detailed view of Figure 8a.

Figure 10. A plot for $U_r = 3.6 \text{ ms}^{-1}$ of a) the estimated surface temperature ($T_g$), b) the estimated M-O scaling length ($L$), and c) the estimated dimensionless wind shear ($\phi_m$) as a function of the canopy height (in meters).
Figure 11. Time series of CN2 model output (dashed circle) compared to scintillometer data (solid square), determined from input data at 1 and 4 m above ground collected over a 450-m path on a) 8-9 July 1992 and b) 13-14 July 1992 (42,43).
4. Summary and Conclusions

The Army is looking to implement acoustic propagation models that will incorporate the impacts of the atmosphere, turbulence, and terrain on acoustic signatures. A key element in estimating outdoor acoustic attenuation with range (and frequency) is modeling the variation of the effective speed of sound through the lower atmosphere. M-O similarity models have been very well suited for this application, especially for daytime convective atmospheres, e.g., the energy budget and meteorological models in ABFA. However, for the nighttime case, we were interested in testing the effectiveness of an earlier quartic solution for the surface temperature, i.e., an alternate energy budget model, which explicitly computes the net surface radiation and the fluxes of heat and moisture close to the ground.

For a hypothetical nighttime case representing moderately stable atmospheric conditions, the alternate (quartic) model performed quite well. Model estimates for effective surface temperature, net radiation, sensible heat flux, soil heat flux, and latent heat flux were comparable to earlier published results (17,18). Also, for this case, energy budget closure was met satisfactorily. Thus, the primary energy budget model results could be used to predict profiles for wind speed, air temperature, and relative humidity, from which, effective sound speed (and ultimately acoustic attenuation) information could be generated.

However, we found that reasonable values for effective surface temperature were not obtainable from the alternate model to simulate the first nighttime IOP of the CASES-99 study. Computational difficulties were experienced because of low roughness height and low wind speed. We found that as the M-O scaling length becomes small, the energy budget model breaks down and the computation is not viable. Thus, until better solutions can be realized through future modeling or experimental works, we recommend that the alternate model be applied only when input canopy height and reference level wind speeds can affect values for the M-O scaling length larger than about 13 m.
5. References


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