TABLES OF NATURAL FREQUENCIES AND NODES FOR TRANSVERSE VIBRATION OF TAPERED BEAMS

by Han-chung Wang and Will J. Worley

Prepared under Grant No. NsG-434 by
UNIVERSITY OF ILLINOIS
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for

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FOR TRANSVERSE VIBRATION OF TAPERED BEAMS

by

Han-chung Wang and Will J. Worley

Department of Theoretical and Applied Mechanics
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Urbana, Illinois

SUMMARY

The natural frequencies, nodal points and mode functions for transverse vibration of tapered beams are presented in this report.

The beams considered have the cross-sectional area bounded by the curve

$$\frac{Y}{h}^\beta + \frac{Z}{b}^\gamma = 1$$

with the thickness \( h \) and width \( b \) varying along the beam according to the relations

\[ h = h_0 \left( \frac{X}{L} \right)^\phi \quad \text{and} \quad b = b_0 \left( \frac{X}{L} \right)^\psi \]

where \( \beta, \gamma, \phi \) and \( \psi \) are positive constants not necessarily integers.
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INTRODUCTION

1. Statement of the Problem

The geometrical properties of solid bodies generated by revolving the line defined as

\[ \left| \frac{X}{Y} \right|^\alpha + \left| \frac{Y}{h} \right|^\beta = 1 \]

and of bodies bounded by the surface defined as

\[ \left| \frac{X}{Y} \right|^\alpha + \left| \frac{Y}{h} \right|^\beta + \left| \frac{Z}{b} \right|^\gamma = 1 \]

have been published in the first and second reports \([1, 2]\)** under this grant. The current report treats the dynamic response of a general class of tapered beams. The shape of beams includes the bodies generated by Eqs. (4.1) and (4.2) with the parameter \(\alpha = 1\). Special cases of bodies of the above shapes have been tested widely in NACA and NASA reports as well as in the technical journals, \([3, 4, 5, 6, 7]\). Beams of special shapes also have been applied as designs for high speed machine guns as reported in a recent paper by Elder \([8]\).

To achieve more nearly complete information on the applications of this class of bodies, the results for the mode functions and for the natural frequencies of vibration of tapered beams are presented in this report. This report includes much of the existing data as special cases. The results appear in tables and in graphical form. The complete and detailed derivations are reported by Wang \([9]\).

2. Profiles of the Beams

The beams whose flexural rigidity and mass per unit length vary according to two arbitrary powers of the longitudinal coordinate are considered in this report. The relationship of the variations may be written as

\[ EI = E_0 I_0 \left( \frac{X}{Y} \right)^m \]

\[ \rho A = \rho_0 A_0 \left( \frac{X}{Y} \right)^n \]

* The notation (4.1) is adopted to aid in cross-referencing equations from the first three reports under this grant.

** Number in brackets refer to the References.
where $E_o I_o$ is the bending rigidity and $\rho_o A_o$ is the mass per unit length at the larger end of the beam where $X = \ell$.

The relations of Eq. (4.3) can be considered as a homogeneous beam with the moment of inertia and cross-sectional area varying with powers $m$ and $n$ respectively. The relations can be applied for a general class of cross-sections with varying thickness and varying width. An important group of beam shapes can be considered as shown in Fig. 1. The cross-section of the beam is symmetrical and its first quadrant is bounded by the curve of the equation

$$
\left( \frac{Y}{Y} \right)^{\beta} + \left( \frac{Z}{Z} \right)^{\gamma} = 1
$$

(4.4)

where $b$ represents half of the width and $h$ represents half of the thickness of the beam. These parameters vary according to the relations

$$
 b = b_o \left( \frac{X}{\ell} \right)^{\psi}, \quad h = h_o \left( \frac{X}{\ell} \right)^{\phi}
$$

(4.5)

The constants $\psi$ and $\phi$ are positive but not necessarily integers.

The selection of different values for the parameters $\gamma$ and $\beta$ in Eq. (4.4) permits the cross-section of the beam to be varied from the diamond shape, $\gamma = \beta = 1$, through the elliptical shape, $\gamma = \beta = 2$, to the rectangular shape, $\gamma$ and $\beta \gg 1$. The moment of inertia and the area for this group of cross-section may be expressed in terms of $\gamma$ and $\beta$ [10], which gives

$$
I = \frac{4}{3} b_o h_o \left( \frac{X}{\ell} \right)^{\psi + 3\phi}
$$

(4.6)

$$
A = 4b_o h_o \left( \frac{X}{\ell} \right)^{\psi + \phi}
$$

Comparison of Eqs. (4.6) with Eqs. (4.3) yields the relationships

$$
m = \psi + 3\phi, \quad n = \psi + \phi
$$
The beam described by Eqs. (4.3) can also be considered as a nonhomogeneous beam with uniform cross-section, provided the modulus of elasticity and the density vary as powers $m$ and $n$. The longitudinal vibration of beams of this type has been treated by Lindholm and Doshi [11].
3. Symbols

- $X$: horizontal coordinate along the length of the beam
- $y$: vertical coordinate perpendicular to $X$
- $z$: horizontal coordinate perpendicular to $X$
- $\ell$: reference length of the beam
- $L$: actual beam length
- $b$: half the width of the beam
- $h$: half the thickness of the beam
- $b_o$: the width of the beam at the larger end
- $h_o$: the thickness of the beam at the larger end
- $x$: dimensionless abscissa, $X/\ell$
- $\alpha$: exponent of $(X/\ell)$
- $\beta$: exponent of $(y/h)$
- $\gamma$: exponent of $(z/b)$
- $\psi$: exponent of $x$ for width variation
- $\phi$: exponent of $x$ for thickness variation
- $m$: exponent of $x$ for area moment of inertia variation
- $n$: exponent of $x$ for cross-sectional area variation
- $EI$: bending rigidity of the beam (elastic modulus times moment of inertia of the cross-section)
- $\rho A$: mass per unit length of the beam
- $\theta$: $= 4 - m + n$
- $u$: $x^\theta$
- $\delta_x$: $x \frac{d}{dx}$
- $\delta_u$: $u \frac{d}{du}$
- $\omega$: circular frequency
- $p$: eigenvalue, $\rho A_0 \ell^4 \omega^2 / (EI_0)$
- $s$: indicial root
- $\theta F_3$: generalized hypergeometric function
- $y(x)$: mode function
$U(x, \xi)$ influence function for beam deflections

$K(x, \xi)$ kernel of homogeneous integral equations

c truncation of the beam, dimensionless; see Fig. 1

$a_r$ coefficients of the series

$P_u Q_v \delta^u P \delta^v Q - \delta^v P \delta^u Q$

$r$ radius of gyration
4. Acknowledgement

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The investigation was part of the work of the Engineering Experiment Station of which Professor Ross J. Martin is Director and was conducted in the Department of Theoretical and Applied Mechanics of which Professor Thomas J. Dolan is Head. The authors wish to acknowledge the assistance of Mr. Tom E. Breuer, an undergraduate student, for contributions to various phases of the project.

Both the ILLIAC II and the IBM 7094 computer facilities were used. The ILLIAC II was constructed in the Digital Computer Laboratory, now known as the Department of Computer Science, University of Illinois with support from the Atomic Energy Commission, grant USAEC AT(11-1)-415, and from the Office of Naval Research, grant NONOR-1832 (15). The IBM 7094 computer facility is partially supported by the National Science Foundation under NSF GP700.
SOLUTIONS OF MODE FUNCTIONS

Neglecting rotatory inertia, the differential equation for the mode functions of vibrating beams is given as

\[ \frac{d^2}{dx^2} EI(X) \frac{d^2y}{dx^2} - \rho \omega^2 A(X) y = 0 \]  \hspace{1cm} (4.7)

For a homogeneous beam, having the profile described by the relations

\[ I = I_o \left( \frac{X}{L} \right)^m \] \hspace{1cm} and \hspace{1cm} \[ A = A_o \left( \frac{X}{L} \right)^n \]  \hspace{1cm} (4.8)

Eq. (4.7) can be expanded as

\[ x^4 \frac{d^4y}{dx^4} + 2m x^3 \frac{d^3y}{dx^3} + m(m-1)x^2 \frac{d^2y}{dx^2} - p x^\theta y = 0 \]  \hspace{1cm} (4.9)

where \( x = (X/L) \), \( p = \rho A_o L^4 \omega^2 / (EI_o) \) and \( \theta = 4 - m + n \).

Upon introducing the differential operator \( \delta_x \) to represent \( x \frac{d}{dx} \), Eq. (4.9) may be written as

\[ \delta_x (\delta_x - 1) (\delta_x + m - 2) (\delta_x + m - 3) y - p x^\theta y = 0 \]  \hspace{1cm} (4.10)

Let \( u = x^\theta \) and let \( \delta_u = u \frac{d}{du} \), thus \( \delta_x y = \theta \delta_u \delta_x y \) and Eq. (4.10) yields

\[ \delta_u (\delta_u - \frac{1}{\theta}) (\delta_u + \frac{m-2}{\theta}) (\delta_u + \frac{m-3}{\theta}) y - \frac{p}{\theta^4} u y = 0 \]  \hspace{1cm} (4.11)

Equation (4.11) is a type of generalized hypergeometric equation [12], which possesses a general solution with linear combinations of four generalized hypergeometric functions. It can be written as

\[ y = A_1 y_0 + A_2 y_1 + A_3 y_{2-m} + A_4 y_{3-m} \]  \hspace{1cm} (4.12)
where each series, in hypergeometric series notation, is defined as

\[
\begin{align*}
y_0 &= {}_0F_3 \left( \begin{array}{c} - \frac{1}{\theta} \end{array} \left| \begin{array}{c} 1 + \frac{m-2}{\theta}, 1 + \frac{m-3}{\theta} \\ \theta^4 \end{array} \right. \right) \\
y_1 &= u \frac{\theta^5}{\theta^2} {}_0F_3 \left( \begin{array}{c} - \frac{1}{\theta} \end{array} \left| \begin{array}{c} 1 + \frac{m-1}{\theta}, 1 + \frac{m-2}{\theta} \\ \theta^4 \end{array} \right. \right) \\
y_{2-m} &= u \frac{2-m}{\theta^3} {}_0F_3 \left( \begin{array}{c} - \frac{1}{\theta} \end{array} \left| \begin{array}{c} 1 - \frac{1}{\theta}, 1 - \frac{m-1}{\theta}, 1 - \frac{m-2}{\theta} \\ \theta^4 \end{array} \right. \right) \\
y_{3-m} &= u \frac{3-m}{\theta^4} {}_0F_3 \left( \begin{array}{c} - \frac{1}{\theta} \end{array} \left| \begin{array}{c} 1 + \frac{m-2}{\theta}, 1 + \frac{m-3}{\theta} \\ \theta^4 \end{array} \right. \right)
\end{align*}
\]  
(4.13)

provided that \( m \) is not an integer. When \( m \) is an integer, then logarithmic terms appear in the general solution.

The solutions of the differential equation (4.10) also can be obtained by the standard series method, the method of Frobenius. The series in Eq. (4.12) will have the form

\[
y_s = x^{s+\infty} \sum_{r=0}^{\infty} a_r x^{r} \theta^r
\]  
(4.14)

with the recurrence formula of the coefficients being

\[
a_r = \frac{p}{(s+r\theta)(s+r\theta+1)(s+r\theta+m-2)(s+r\theta+m-3)} a_{r-1}
\]  
(4.15)

where \( s = 0, 1, 2-m \text{ and } 3-m \) are the roots of the indicial equation.

When the parameter \( m \) is an integer, then two equal roots of \( s \) may exist for the indicial equation or the denominator of the recurrence formula of Eq. (4.15) becomes zero. When equal roots occur, the series solutions of Eq. (4.13) are dependent on each other. When the denominator of the recurrence formula is zero, the series solutions are meaningless. In these cases, by applying the theorems due to Frobenius [13], the independent series solution for the repeated root, \( s = s_0 \), is

\[
y_{s_0} = \log x \cdot (y)_{s=s_0} = -x^{s_0} \sum_{r=1}^{\infty} \frac{1}{\eta^r-1} \left( \frac{1}{s_0 + \eta \theta} + \frac{1}{s_0 + \eta \theta - 1} + \frac{1}{s_0 + \eta \theta + m-2} + \frac{1}{s_0 + \eta \theta + m-3} \right) a_r x^{r} \theta^r
\]  
(4.16)
The series solution for the root, \( s = s_1 \), which makes the recurrence formula of the coefficients undefined, is given by

\[
y_{s_1} = \log x \cdot (y)_{s=s_1} + x^s \left\{ \sum_{r=1}^{\infty} \left[ \frac{1}{s-\eta} \frac{1}{s+\eta\theta+1} \frac{1}{s+\eta\theta+2} \frac{1}{s+\eta\theta+3} \right]_{s=s_1} a_r \right\}^{r=0}
\]

(4.17)

Again, when \( \theta = 0 \), the series of Eq. (4.14), does not apply. For this case, the recurrence formula of the coefficients is again undefined; however, Eq. (4.10) is then of the Euler-Cauchy type for which a general solution always exists of the form

\[
y = A_1 x^{s_1} + A_2 x^{s_2} + A_3 x^{s_3} + A_4 x^{s_4}
\]

(4.18)

where \( s_1, s_2, s_3, \) and \( s_4 \) are the roots of the auxiliary equation

\[
s(s - 1) (s+m - 2) (s+m - 3) - p = 0
\]

(4.19)

Vibration can occur only when Eq. (4.19) has two or four non-real roots. This condition introduces the trigonometric function of \( \log x \) into the solution for \( y \).

Suppose \( s_1, s_2 \) are two real roots and \( s_3, s_4 \) are two complex roots represented by \( \alpha_1 \pm i\alpha_2 \), then the solution for the normal function has the form

\[
y = A_1 x^{s_1} + A_2 x^{s_2} + x^{s_1} \left[ A_3 \cos(\alpha_2 \log x) + A_4 \sin(\alpha_2 \log x) \right]
\]

(4.20)
FREQUENCY EQUATIONS AND NODAL POINTS

To establish the frequency equations for tapered beams with different end conditions, two separate geometrical categories of beams are treated. First, the complete tapered beam, which is gradually narrowed toward a point, is considered. The end coordinates of the beam are 0 and \( \ell \). Next, the truncated beam, with end coordinates c \cdot \ell \) and \( \ell \), as indicated in Fig. 1, is considered. The origin, for the truncated beam lies beyond the end of the beam and serves as a reference point for the variation of the area moment of inertia and for the cross-sectional area of the beam.

1. Complete Tapered Beams

Since one end of the beam is at the origin, the arbitrary constants \( A_3 \) and \( A_4 \) in the general solution, Eq. (4.12), must vanish if finite values exist for the deflection, moment and shear at the end \( x = 0 \). Hence, the general solution of the mode function for beams with any combination of parameters \( m \) and \( n \) can be written as

\[
y = A_1 P + A_2 Q \tag{4.21}
\]

where \( P \) and \( Q \) are the series of \( y_0 \) and \( y_1 \) as defined in Eq. (4.13). The frequency equation is then obtained using the boundary conditions at \( X = \ell \), that is, at \( x = 1 \). Three different cases are considered below.

A. Cantilever Beams

The base of the beam is fixed and the tip is free. Both the deflection and slope vanish at \( x = 1 \). Application of the boundary conditions to Eq. (4.21) and elimination of the constants \( A_1 \) and \( A_2 \) yields the frequency equation

\[
\frac{dQ}{dx} - \frac{dP}{dx} = 0 \quad \text{at } x = 1 \tag{4.22}
\]

Representation of \( \delta^u P \cdot \delta^v Q - \delta^v P \cdot \delta^u Q \) in symbolic form as \( P_u Q_v \) yields the relations \( P_{u} Q_{v} = -P_{v} Q_{u} \) and \( \delta (P_{u} Q_{u+1}) = P_{u} Q_{u+2} \). Using this notation, the frequency equation yields

\[
\frac{1}{x} P_{Q_1} = 0 \quad \text{at } x = 1 \tag{4.23}
\]
Since P and Q are in series form, as in Eq. (4.14), their derivatives and products are in series form as well.

Let the series be defined as $U = PQ_1$. Then $U$ is a solution of a fifth order differential equation

$$(6 - 1) (6 + m - 2) (6 + m - 3) (6 + m - 4) (6 + 2m - 5) U + 2 (25 + \theta + 2m - 6) \frac{p}{n} U = 0 \quad (4.24)$$

Solving Eq. (4.24) and comparing the corresponding terms of $U$ with Eq. (4.23) the series which satisfies both Eq. (4.24) and (4.23) is established as

$$U = \sum_{r=0}^{\infty} a_r r^x 1 + r \theta \quad (4.25)$$

Substitution of $U$ into Eq. (4.23) yields the frequency equation as a polynomial of $p$

$$\sum_{r=0}^{\infty} a_r r^r = 0 \quad (4.26)$$

with the recurrence formula of coefficients

$$a_r = \frac{-2 (2r \theta + \theta + 2m - 4)}{r \theta (r \theta + m - 1) (r \theta + m - 2) (r \theta + m - 3) (r \theta + 2m - 4)} a_{r-1} \quad (4.27)$$

B. Antisymmetrical Mode Vibration of Free-Free Beams

Tapered beams with both ends free are considered to consist of two equal halves fitted together at their large ends as shown in Fig. 2. Each half is a section of a solid which has the profile shown in Fig. 1. When the beam vibrates in the antisymmetrical mode, the deflection and the bending moment are zero at the middle section where the two halves are joined together, that is

$$y = 0 \quad \text{and } EI(x) \frac{d^2 y}{dx^2} = 0 \quad \text{at } x = 1$$

Substitution of these conditions into Eq. (4.21) and applying the differential operator $P_{u}Q_{v}$, the frequency equation becomes

$$\frac{1}{x^2} U = 0 \quad \text{at } x = 1 \quad (4.28)$$

where $U = PQ_2 - PQ_1$. The function $U$ is a solution of a fifth order differential
equation

\[(5-\theta-1)(5+m-4)(5+m-3)(5+2m-5)(5+m-2)U+2(25+\theta+2m-6)px^\theta U = 0 \quad (4.29)\]

Hence, the frequency equation may again be represented by a polynomial of \(p\), of the form of Eq. (4.26), with the coefficients

\[a_r = \frac{-2(2r\theta+\theta+2m-4)}{r\theta(r\theta+\theta+m-1)(r\theta+\theta+m-2)(r\theta+\theta+m-3)(r\theta+\theta+2m-4)} a_r-1 \quad (4.30)\]

The locations of the nodes for the normal mode vibration, can be obtained by substituting the corresponding natural frequency, \(p_1\), into Eq. (4.21) and solving for \(x\) with \(y\) equal to zero. Since \(P\) and \(Q\) are generalized hypergeometric functions, the equation, which gives the nodal points for cantilever beams and free-free beams of antisymmetrical mode, can be represented by

\[0^F_3\left(\infty, 1+\frac{1}{\theta}, 1+\frac{m-1}{\theta}, 1+\frac{m-2}{\theta}; \frac{p_1}{\theta^4}\right) 0^F_3\left(\infty, 1+\frac{1}{\theta}, 1+\frac{m-2}{\theta}, 1+\frac{m-3}{\theta}; \frac{p_1}{\theta^4}\right) 0^F_3\left(\infty, 1+\frac{1}{\theta}, 1+\frac{m-1}{\theta}, 1+\frac{m-2}{\theta}; \frac{p_1}{\theta^4}\right) = 0 \quad (4.31)\]

C. Symmetrical Mode Vibration of Free-Free Beams

For the symmetrical mode vibration of the free-free beam, Fig. 2, the slope and shear are zero at the middle section of the beam, hence

\[\frac{dy}{dx} = 0 \quad \text{and} \quad \frac{d}{dx}EI(x)\frac{d^2y}{dx^2} = 0 \quad \text{at} \quad x = 1\]

Upon substituting the end conditions into Eq. (4.21) and using the notation \(P_uQ_v\), the frequency equation may be written as

\[\frac{d}{dx}(x^{m-3}U) = 0 \quad (4.32)\]

at \(x = 1\), where

\[U = PQ_3 + (m-3)PQ_2 - (m-2)PQ_1\]
As discussed in the previous sections, $U$ may be represented by a series which is a solution of a sixth order differential equation

$$(\delta - \theta - 1) (\delta - \theta + m - 2) (\delta - \theta + m - 3) (\delta + m - 4) (\delta + m - 3) (\delta + 2m - 5) U$$

$$+ 2 (2\delta + \theta + 2m - 6) (\delta + m - 3) px^\theta U = 0$$

(4.33)

Substituting the solution $U$ into Eq. (4.32) and evaluating at $x = 1$, gives the equation for the eigenvalues $p$ as

$$\sum_{r=0}^{\infty} (r\theta + \theta + m - 2) a_r p^r = 0$$

(4.34)

with the recurrence formulas for the coefficients

$$a_r = \frac{-2(2r\theta + \theta + 2m - 4)}{r \theta(r\theta + m - 1)(r\theta + \theta + m - 2)(r\theta + \theta + m - 3)(r\theta + \theta + 2m - 4)} a_{r-1}$$

(4.35)

The locations of the nodes for the normal mode vibration in this case can be solved from the equation

$$\left[ - \frac{p_i}{(\theta + 1)(\theta + m - 1)(\theta + m - 2)} \right]_0 F_3 \left( \begin{array}{c} -; 1 + \frac{1}{\theta}, 1 + \frac{m - 1}{\theta}, 1 + \frac{m - 2}{\theta} \\ \frac{p_i}{\theta^2} \end{array} \right)$$

$$+ \left[ \frac{p_i}{\theta (\theta + m - 2)(\theta + m - 3)} \right]_0 F_3 \left( \begin{array}{c} -; 1 + \frac{1}{\theta}, 1 + \frac{m - 2}{\theta}, 1 + \frac{m - 3}{\theta} \\ \frac{p_i}{\theta^4} \end{array} \right)$$

$$- \left[ \frac{p_i x}{(\theta + 1)(\theta + m - 2)(\theta + m - 3)} \right]_0 F_3 \left( \begin{array}{c} -; 1 + \frac{1}{\theta}, 1 + \frac{m - 2}{\theta}, 1 + \frac{m - 3}{\theta} \\ \frac{p_i}{\theta^4} \end{array} \right)$$

$$0 F_3 \left( \begin{array}{c} -; 1 + \frac{1}{\theta}, 1 + \frac{m - 1}{\theta}, 1 + \frac{m - 2}{\theta} \\ \frac{p_i x^\theta}{\theta^4} \end{array} \right) = 0$$

(4.36)
2. Truncated Tapered Beams
   
   A. Exact Solutions

   A beam which is gradually reduced to a small cross-section instead of to a point, may be considered as a beam truncated at the location \( x = c \) as shown in Fig. 1. The total length of the beam is \((1 - c) \cdot \ell\), where \( \ell \) is a reference length for the tapering. Since the beam does not start from the origin, the general solution of the mode function must be written in the form of Eq. (4.12) with four series. By substituting the end conditions into the mode function, a fourth order determinant equation of the eigenvalues \( p \) may be obtained.

   Consider the case for a cantilever beam; the end conditions are

   \[
   y = y' = 0 \quad \text{at } x = 1
   \]
   and \( EI y'' = (EI y'')' = 0 \quad \text{at } x = c \)

   which gives the frequency equation

   \[
   \begin{bmatrix}
   [y_0]_{x=1} & [y_1]_{x=1} & [y_{2-m}]_{x=1} & [y_{3-m}]_{x=1} \\
   [y'_0]_{x=1} & [y'_1]_{x=1} & [y'_{2-m}]_{x=1} & [y'_{3-m}]_{x=1} \\
   [y''_0]_{x=c} & [y''_1]_{x=c} & [y''_{2-m}]_{x=c} & [y''_{3-m}]_{x=c} \\
   [(x^m y'_0)']_{x=c} & [(x^m y'_1)']_{x=c} & [(x^m y''_{2-m})']_{x=c} & [(x^m y''_{3-m})']_{x=c}
   \end{bmatrix}
   = 0 \quad (4.37)
   \]

   where \( y_0, y_1, y_{2-m} \) and \( y_{3-m} \) are defined by Eqs. (4.13), (4.16), or (4.17).
B. Approximate Solutions

The calculation of the natural frequencies from the characteristic equation, Eq. (4.37), involves tedious numerical computations since each element in the determinant is a combination of generalized hypergeometric series. For practical purposes and in order to supply results to check the exact solutions, two approximate methods are introduced for calculating the upper bound and the lower bound of the approximate fundamental frequencies.

The Ritz method is one of the approximate methods which has been widely used to determine the upper bound of the natural frequencies for elastic systems. The frequency of the fundamental mode is calculated by minimizing the expression for the energies, which, for beams having the prescribed profile is

$$ p = \int_{c}^{1} x^m \left( \frac{d^2 y}{dx^2} \right)^2 dx / \int_{c}^{1} x^n y^2 dx $$

(4.38)

As an example, consider a cantilever beam. In order to satisfy the boundary conditions at free end, a one term approximation is assumed for the second derivative of the beam deflection, as

$$ y'' = 12a (x - c)^2 $$

(4.39)

Integration of Eq. (4.39) gives the deflection curve

$$ y = a \left[ (x - c)^4 + C_1 x + C_2 \right] $$

where $C_1 = -4 (1 - c)^3$ and $C_2 = 4(1-c)^3 - (1-c)^4$ in order to satisfy the boundary conditions $y = y' = 0$ at $x = 1$.

The lower bound approximate frequency of the fundamental mode is obtained from the expression
where

$$K(x, \xi) = \frac{E I_o}{\ell^3} \xi^n U(x, \xi)$$

is the kernel of a homogeneous integral equation for the beam profile of current interest. The influence function for beam deflections, $U(x, \xi)$, is the static deflection of the beam at $x$ with a unit load applied at a distance $\xi$ measured from the origin. For a cantilever beam, $U(x, \xi)$ can be expressed as

$$U(x, \xi) = \frac{\ell^3}{E I_o} \left[ \frac{1}{(2-m)(3-m)} x^{3-m} - \frac{\xi}{(1-m)(2-m)} x^{2-m} 
+ \frac{(2-m)\xi - (1-m)}{(1-m)(2-m)} x - \frac{(3-m)\xi - (2-m)}{(2-m)(3-m)} \right]$$

for $m \neq 1, 2$ and $3$, and

$$U(x, \xi) = \frac{\ell^3}{E I_o} \left[ \frac{x^2}{2} - \xi x (\log x - 1) - x - \xi + \frac{1}{2} \right], \quad \text{for } m = 1$$

$$U(x, \xi) = \frac{\ell^3}{E I_o} \left[ x (\log x - 1) + \xi (\log x - 1) + \xi + 1 \right], \quad \text{for } m = 2 (4.44)$$

$$U(x, \xi) = \frac{\ell^3}{E I_o} \left[ x (1 - \frac{\xi}{x}) - \log x - \frac{\xi}{2x} + \xi - 1 \right], \quad \text{for } m = 3$$
NUMERICAL RESULTS AND DISCUSSION OF TABLES

1. Natural Frequencies and Nodal Points for Complete Tapered Beams

The natural frequencies for different vibrational modes of complete tapered beams can be calculated by solving for the roots of polynomials of Eqs. (4.26) and (4.34). The coefficients of the polynomials, for cantilever beams, for free-free beams executing antisymmetrical vibrational modes and for free-free beams executing symmetrical vibrational modes, can be generated from Eqs. (4.27), (4.30) and (4.35) respectively. The recurrence formulas indicate that the coefficients reduce rapidly as the number of terms increases. Therefore, in the numerical calculations, the first five frequencies are computed with the first sixteen terms of the series. Computer programs are written to generate the coefficients as well as to solve for the roots.

The exponents, for a uniform beam, are \( \psi = \phi = 0 \) or \( m = 0, \theta = 4 \), and the frequency equation for cantilever beams, Eq. (4.26), becomes

\[
\sum_{r=0}^{\infty} \frac{(-1)^r}{(4^r r!)} (4 p)^r = 0
\]

(4.45)

where \( p = \omega^2 \left( \frac{p A}{E I_o} \right) \xi^4 \) as defined earlier. Let the beam length be \( L \), which is equal to \( \xi^0 \) for the complete tapered beam, and let the frequency constant \( K \) be \( \sqrt{p} \). Then the natural frequency can be represented as

\[
\omega = K \sqrt{\frac{E I_o}{\rho A_o}} / L^2
\]

(4.46)

The first five frequency constants, \( K \), for uniform cantilever beams are obtained from Eq. (4.45) using the first 16 terms of the series. These frequency constants are

3.51601, 22.0345, 61.6972, 120.911, 199.860

The substitution of each frequency into Eq. (4.31) establishes the locations the nodes for the corresponding natural frequency, which are given in the following table.
Uniform Cantilever Beam

<table>
<thead>
<tr>
<th>ψ</th>
<th>φ</th>
<th>mode constant, K</th>
<th>locations of nodes, (X/L)</th>
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<td>.00</td>
<td>.00</td>
<td>1st 3.51601 1.00000</td>
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<td>2nd</td>
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<td>.21656 1.00000</td>
<td></td>
</tr>
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<td>3rd</td>
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<td>120.90191</td>
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<tr>
<td>5th</td>
<td>199.85953</td>
<td>.07345 .27678 .50009 .72125 1.00000</td>
<td></td>
</tr>
</tbody>
</table>

In the table, the first two columns display the combinations of the exponents ψ and φ, the third and fourth columns indicate corresponding modes and their natural frequencies. The remaining columns show the locations of the nodes for different modes.

For cantilever beams with other combinations of ψ and φ, the results for frequencies and nodes for the first five modes are listed in Table 1. Page one of Table 1 lists data for the combinations of ψ and φ corresponding to beams of constant thickness with width varying as $x^ψ$. Page two of the table lists corresponding data for beams of constant width with thickness varying as $x^φ$. The frequency data of these two cases are also plotted in Figs. 3 and 4 as the ratio of frequencies of tapered beams to those of uniform beams. Fig. 5 indicates the variation of the ratio of frequencies for the beams with taper both in width and thickness according to $x^φ$. The ratio of frequencies for the first three modes, for 81 combinations of ψ and φ, are plotted in Fig. 6 in three dimensional form. The figures reveal the variation of the natural frequencies of different modes as the taper of the beam varies. It is of interest to note that the frequencies of the fundamental mode increase when the beams taper either in width or in thickness. The frequencies of the higher modes increase as the taper of the width increases, that is, as ψ decreases. The higher mode frequencies decrease as the taper on the thickness increases, that is, as φ increases.

The shapes of the first four normal modes for the vibration of tapered cantilever beams are plotted in Figs. 7, 8 and 9. Fig. 7 shows the change of mode shapes and the shifting of the nodal points for constant thickness beams as the exponent ψ increases. Fig. 8 shows those for beams with constant width and varying thickness. Fig. 9 displays those for beams with both width and thickness varying as a same power of x, that is ψ = φ. The amplitudes of the deflections are normalized to the deflection at the free end.
For free-free beams of antisymmetrical and symmetrical mode vibration, the frequency constant $K$ and the nodal points are also computed. For a uniform free-free beam, vibrating in antisymmetrical modes, the frequencies can be obtained from Eq. (4.20) with coefficients given by Eq. (4.30) and with $m = 0, \psi = 4$, that is

\[
\sum_{r=0}^{\infty} \frac{(-1)^r}{(4r+3)!} (4p)^r = 0
\]  

(4.47)

The substitution of the first five roots of Eq. (4.47) into Eq. (4.31) gives the locations of the nodes for the corresponding mode. The results are listed as follows:

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\phi$</th>
<th>mode</th>
<th>frequency constant, $K$</th>
<th>locations of nodes (X/L)</th>
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<td></td>
<td>3rd</td>
<td>74.63888 .12020 .45291 .81825</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>4th</td>
<td>138.79131 .08814 .33213 .60005 .86666</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>5th</td>
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</tbody>
</table>

The results for other combinations of $\psi$ and $\phi$ are listed in Table 2 in the same order as in Table 1. The variation of the ratio of the frequencies of tapered beams to the frequencies of uniform beams are plotted in Fig. 10. The nodal points listed here are for one half of the beam length. The nodes of the other half of the beam are symmetrically located. The notation $L$ represents half the total length of the free-free beam.

For free-free beams of symmetrical mode vibration, the frequencies and nodal points are evaluated from Eqs. (4.34) and (4.36) respectively. The results are listed in Table 3. For a uniform beam, the frequency equation is given as

\[
\sum_{r=0}^{\infty} \frac{(-1)^r}{(4r+1)!} (4p)^r = 0
\]  

(4.48)

which gives the first five roots and the corresponding nodes as follows:
Uniform Free-Free Beam of Symmetrical Mode

<table>
<thead>
<tr>
<th>$\psi$</th>
<th>$\phi$</th>
<th>mode</th>
<th>constant, $K$</th>
<th>locations of nodes ($X/L$)</th>
</tr>
</thead>
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<td>4th</td>
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<td></td>
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<td></td>
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</table>

The above calculated results for frequencies and nodes for general tapered beams are checked with existing results of some special cases which appear in References [14, 15, 16, 17, 18, 19, 20].

2. Natural Frequencies of Truncated Beams

For truncated cantilever beams the frequency constants, $K$ in Eq. (4.46), are the roots of Eq. (4.37). The numerical results are obtained by using the method of regula falsi. The series involved in each of the elements of Eq. (4.37) are calculated using 16 terms. The frequency constants $K$ for the first two vibrational modes are presented in Table 4 for beams truncated at 0.2 $\ell$ and at 0.4 $\ell$. The combinations of the exponents $\psi$ and $\phi$ again include the beams with constant thickness, with constant width and with both thickness and width varying as the same power.

The upper bound and the lower bound approximations for the fundamental natural frequencies are calculated for six different degrees of truncation as well as for complete tapered beams. The values for the lower bound approximation are evaluated from Eq. (4.41) with the kernel defined in Eq. (4.42). The values of the upper bound approximation are evaluated from Eq. (4.38) with the deflection curve defined as Eq. (4.40). The approximate values of $K$ are listed in Table 5 with 84 different combinations of $\psi$ and $\phi$.

Comparisons of the correct frequencies of truncated beams with the approximate frequencies appear in Figs. 12, 13 and 14. The results for constant thickness beams with varying width appear in Fig. 12, while Fig. 13 displays results for constant width beams with varying thickness and Fig. 14 gives the results for beams for which both width and thickness vary. The frequencies of the upper bound approximation for constant thickness beams are closer to the correct results than those for constant width beams. This implies that the assumed deflection curve is more nearly correct for the constant thickness beams. The lower bound approximations also yield more nearly
correct results for beams with constant thickness than for beams with constant width. This is true because the frequencies of the higher modes for constant thickness beams are larger than those for constant width beams.

3. Radii of Gyration of Cross-Section

The evaluation of circular frequencies from the frequency constants $K$, as defined in Eq. (4.46), involves the calculation of the radius of gyration, $(I_o/A_o)^{1/2}$. For the beams with cross-sections bounded by Eq. (4.4), the area moments of inertia and the cross-sectional areas with different values of $\gamma$ and $\beta$ were listed in the first report [1]. The radius of gyration of the cross-section at the large end of the beam, calculated from Eq. (4.6), is

$$\frac{r_g}{h_o} = \frac{1}{3} \frac{\Gamma\left(\frac{3}{\beta} + 1\right) \Gamma\left(\frac{1}{\gamma} + \frac{1}{\beta} + 1\right)}{\Gamma\left(\frac{1}{\beta} + 1\right) \Gamma\left(\frac{1}{\gamma} + \frac{3}{\beta} + 1\right)}$$

(4.49)

The results of Eq. (4.49) are listed in Table 6 with the same combinations of $\alpha$ and $\beta$ as those used in the first report.
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<th>(\psi)</th>
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<th>MODE</th>
<th>FREQUENCY CONSTANT, K</th>
<th>LOCATIONS OF NODES (X/L)</th>
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Fig. 1 Cantilever Beam

Fig. 2 Free-Free Beam
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