TECHNICAL REPORT 2001-003

Single Integrated Air Picture (SIAP)
Metrics Implementation

OCTOBER 2001

SINGLE INTEGRATED AIR PICTURE (SIAP)
System Engineering
Task Force (SE TF)
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FOREWORD

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The SIAP Metrics Implementation Technical Report is the result of collective efforts of members of the SIAP Attributes and Metrics Working Group, who drafted the content of the report through several face-to-face meetings, teleconferences, and electronic mail exchanges spanning the period from July to October, 2001. The following individuals contributed to the report through their participation in either live or virtual meetings of the Working Group:

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EXECUTIVE SUMMARY

PROBLEM

The SIAP Implementation Plan states that the overarching objective of the SIAP SE TF is "to identify incremental improvements in the … SIAP capability that will provide commensurate incremental improvements in warfighter capabilities." Metrics have been identified to quantify a SIAP and these incremental improvements (SIAP SE TF Technical Reports 2001-001 and 2001-002). Beyond the task of defining appropriate measures for quantifying the SIAP, however, it is essential to establish guidelines for implementation to ensure consistency in the evaluation and use of the measures. In particular, objective evaluation of the quantitative SIAP attribute measures, crucial to the process of assessing compliance with Theater Air and Missile Defense (TAMD) and Combat Identification (CID) Capstone Requirements Documents (CRDs), is dependent upon prior establishment of a track data base to measure and an rule for determining association of tracks with truth objects. The mathematical definitions of the SIAP attributes cannot be applied consistently without an understanding of the measurable data base and the criteria for tracks-to-truth "assignment." Decisions must be made as to how rigorously the assignment criteria are to be specified, how broadly they are to be applied, and how and under what conditions they should be modified. Some of these decisions involve considerations of possible computational approaches. There are related empirical issues, many specific to particular test environments, that may need to be addressed. For example, a live experiment may yield insufficient data to carry out an agreed-upon assignment rule. Some of these issues are presently known; others may arise as the SIAP assessment process evolves. *SIAP metrics implementation* is the term used in this report to encompass all of these assessment issues.

OBJECTIVES

This technical report has the following objectives.

1) Establish consensus on *which* tracks will be included in defining a SIAP (the measurability problem).

There are many track types that can be considered for scoring the SIAP metrics. It is paramount that there be consensus on which ones will be counted to ensure proper comparisons, and that the classes of tracks included (as well as excluded) be precisely defined.

2) Establish a tracks-to-truth assignment procedure applicable across a variety of testing environments, and address technical issues of algorithmic implementation (the assignment problem).

Justification of choices made and recommendations for later refinements are to be included. Technical issues include casting the procedure in a mathematically explicit form, and addressing data interpolation.

3) Define and address issues pertaining to live test evaluations in connection with the proposed measurability and assignment approaches (reconstruction problems).

As already suggested, the presently identified issues will be discussed here, but more issues are certain to arise as testing and evaluation gets underway. Thus, this objective can only be met in a provisional manner in this report.

4) Provide alternative track bases and excursions to scoring methods to allow comparison with the proposed approaches.

SIAP system engineering will face many assessment problems beyond the realm of current assessments, on which the measurability and assignment studies are focused. A framework should be provided for extending measurability to other track data bases, and for implementing alternative assignment and scoring procedures should the current proposals require reevaluation.

APPROACH

The approach to measurability is to adopt standardized or commonly accepted definitions of the track data bases to be considered, and to specifically identify those which constitute the information which an operator or commander can meaningfully act upon in the context of legacy SIAP-related systems. Link-16 is used as an initial context for most of the formal definitions, but the measurability recommendations are made with broader applicability in mind. Discussion of alternatives and justification of choices made are provided. The approach to assignment is to define a deterministic (algorithmic) assignment procedure which covers assignment concerns in both live evaluation and constructive simulation contexts. The assignment procedure is based on an optimization method (minimization of an appropriate assignment cost function), but the optimization criterion is combined with gating and nearest-neighbor multiple-track assignment possibilities in a way designed to capture the benefits of all of these approaches. Benefits and disadvantages are discussed. The cost function and related algorithmic issues are addressed in a mathematically explicit manner. The approach to reconstruction is to discuss the known issues and make recommendations for their resolution to the extent possible. Special cases such as formation tracking, time alignment, and test design issues are addressed and tentative recommendations are provided for each. Finally, alternatives to the proposed measurability and assignment approaches, providing a starting point for extending the results of this report to meet future assessment needs, are discussed.

FINDINGS

The track scoring approach presented identifies those measurable tracks that should be used in evaluating SIAP metrics. There is a brief discussion on how exceptions to the proposed scoring policy may be in order for ballistic missile tracks. A detailed assignment algorithm and its associated cost function are also presented along with examples and rationale for their implementation. Both proposals can be adapted to simulation as well as live exercise data reconstruction for defining a SIAP through the SIAP attributes. An appendix includes mathematical details on the theory by which the assignment cost function (for optimal assignment) was derived. Another appendix provides mathematical details on a truth data interpolation procedure for use in the assignment algorithm.

CONCLUSIONS

The recommended approaches to assignment and reconstruction provide a consistent preliminary basis upon which to evaluate SIAP metrics and quantify SIAP capabilities.

RECOMMENDATIONS

Institutionalize the approach defined herein as to the methodology for performing SIAP-related evaluation on all units where SIAP metrics are calculated and evaluate approach for applicability to other missions and domains. Explore the excursions and assignment alternatives identified herein for their potential applicability to future refinement and reevaluation of the methodology.

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1. INTRODUCTION

A methodology for evaluating the Single Integrated Air Picture (SIAP) by way of a hierarchy of well-defined SIAP metrics has been established in two related technical reports (SIAP SE TF Technical Reports 2001-001 and 2001-002). This methodology is intended to measure the degree to which the ability to build and maintain a SIAP is achieved – an objective which will in turn contribute to the assessment and forecasting of progress over time. In addition, the process of evaluating capability in this way gives significant insight into specific issues that must be resolved.

The SIAP assessment approach must be standardized to ensure uniformity and objectivity in the evaluation of capabilities. Ultimately, the ability to make good system engineering decisions towards the achievement of a SIAP capability will crucially depend upon this ability to make objective comparisons and assessments. This technical report is devoted to the standardization of the SIAP assessment methodology through the establishment of a self-correcting path towards uniform and objective evaluations of SIAP metrics.

A number of practical issues arise when one attempts to give objective values to SIAP metrics on the basis of data from live tests or high-fidelity simulations. The raw data collected generally does not yield the desired metric directly. "Implementation" of a SIAP metric (or class of metrics) encompasses all such issues of practical measurement, apart from the strict definition of the metric(s). The concern is primarily with the SIAP attributes, since these are linked to Theater Air and Missile Defense (TAMD) and Combat Identification (CID) Capstone Requirements Documents (CRDs) (2001), although many of the same issues will arise with metrics at other levels in the SIAP metrics hierarchy (cf. SIAP SE TF Technical Report 2001-002).

Issues identified so far have been grouped into the following three areas: measurability, assignment, and other reconstruction issues. The measurability issue concerns what track data is to be used for SIAP assessments. Track assignment concerns the procedure to be followed for determining which tracks are “assigned” to which truth objects; specifically, how and to what degree this procedure should be automated. Reconstruction comprises track assignment and other issues, mostly pertaining to live exercise evaluations, which involve the construction of track-to-object assignments from experimental data. Many of these issues are concerned with making allowance in test analysis for difficulties in obtaining complete track data or interpreting the track data that is obtained. It is the objective of this report to describe each of these implementation issues in detail, and make specific proposals and recommendations towards their resolution.

2. MEASURABILITY

The measurability problem refers to the problem of specifying a well-defined set of tracks on which to base SIAP assessments. The formal definition of a SIAP – the "product of fused, common, continual, unambiguous tracks of airborne objects in the surveillance area" (Theater Air and Missile Defense Capstone Requirements Document (TAMD CRD)) – does not define what tracks are to be considered. This is deliberate, allowing for evolution of the concept of a SIAP as network technology evolves.

There is understandably more interest in attaining a SIAP on that part of the aerospace picture which can be meaningfully acted upon – for example, tracks used for the purpose of allocating weapons, or at least for making a decision to fire (battle management). What this means for current assessments, though, may be quite different for what it may mean for long-term planning and forecasting. Even for current assessments, delineating precisely those tracks that are "actionable" when faced with multiple networks operating simultaneously on different principles involves some complications.

The approach of this report focuses on *current assessments*. Other classes of tracks identified outside this chapter will be considered "excursions" in Section 5 to be investigated for potential application to other aspects of SIAP system engineering analysis. The initial approach to measurability described in the following two sections has been guided by considerations of issues pertaining to tactical data links such as Link-16, but is expected to be applicable in other contexts in all essential respects. As the future analytical focus shifts to other kinds of networks, some refinement of terminology may be needed in order to maintain the intended emphasis on actionable information.

2.1 Definition of Terms and Assumptions

The following definitions of different track types are relevant to the discussion of track measurability issues.

> <u>Track</u> – "(1) the graphic and/or alphanumeric representation of an object, point, or bearing whose position and/or characteristics are collated from sensors and/or other data sources; (2) a collated set of data associated with a track number for the purpose of representing the position and/or characteristics of a specific object, point, or bearing" (MIL-STD-6016A, 1999). For SIAP assessment purposes, a track is understood as an actionable track, not to include tentative tracks or clutter tracks.
>
> <u>Local Track</u> – " A track established within an interface unit based on locally entered positional information. Amplifying data associated with the track may be derived locally, from supporting units, or from data links" (MIL-STD-6016A, 1999).

For the purposes of metrics implementation "positional information" is understood to consist of any subset of angle, range, and range-rate measurement components and their error covariance, as well as filtered kinematic state estimates and their error covariance. The term "interface unit" refers specifically to participants on a tactical digital information link (TADIL), but these definitions are understood to be generally applicable to any participant contributing to or using the SIAP. Local tracks include single-sensor tracks generated with local sensor data and single-platform multi-sensor composite tracks generated with data from multiple local sensors. Local tracks also include multi-platform, multi-sensor composite tracks, such as tracks generated by the Cooperative Engagement Capability (CEC) or “Joint Composite Tracking Network” (“JCTN”), provided the composite tracks are locally filtered/processed, i.e.; the positional information is locally entered (which need not always be the case when CEC or “JCTN” tracks are passed over Link-16, say).

> Remote Track – "A track established within an interface unit based upon positional information derived from a data link report or reports. Amplifying data associated with the track may be derived locally, from supporting units, or from data links" (MIL-STD-6016A, 1999).
>
> Local-Only Track – A local track that is not correlated to a remote track at the time of evaluation.
>
> Local-Mutual Track – A local track that is correlated to a remote track at the time of evaluation.
>
> Remote-Only Track – A remote track that is not correlated to a local track at the time of evaluation.
>
> Remote-Mutual Track – A remote track that is correlated to a local track at the time of evaluation.
>
> Pending Track – "A track which has not been subjected to the identification process" (MIL-STD-6016A, 1999).

Compliance of systems with SIAP-related requirements will be assessed on the basis of SIAP attributes. SIAP attributes are based on the actionable track data in the central track stores (CTS) of participants, where CTS is defined by the particular type of system. The CTS contains all of the track types defined in this section. In accordance with the definition of "track" cited above, a track is considered actionable once it has an associated track number in the form of a central track store locator (CTSL). Since this qualification implies nothing about the status of the identification process, actionable tracks include pending tracks as defined above. Pending tracks are, at least in some cases, "available for reporting" (MIL-STD-6011B, 1999). While many legacy systems are equipped with filters which prevent local pending tracks from being reported on Link-16, this is not a universal feature of legacy systems, and, even if pending tracks are not reported, as local tracks they are displayable and may influence an operator's decisions. Local pending data may also be subject to correlation attempts. Pending tracks, once they have acquired CTSLs, therefore count as actionable tracks for the purposes of this report and for all SIAP attribute evaluations.

However, mutual tracks, as well as certain other classes of correlated tracks, raise special issues, which require further restricting the class of measurable tracks.

2.2 Scoring Preferences

There are a number of issues remaining on the scoring of tracks. The first involves remote-mutual tracks. To avoid double-counting correlated tracks that are understood by a participant as containing two versions of the same information, it will be assumed for current assessments that one shall always use the local-mutual tracks. The scoring of local-mutual rather than remote-mutual tracks indicates preference for what are usually regarded as the more "actionable" tracks for most legacy air and missile defense systems. Furthermore, local tracks are always available in all legacy systems, whereas remote mutual track data is not always retained after the correlation process of some SIAP systems.[1]

The second issue addresses multiple local tracks from independent sources. This is not an issue for AEGIS and PATRIOT systems because these systems automatically filter tracks not meeting certain criteria *before* they enter the CTS. Not all SIAP systems have an automatic filtering capability; therefore, for these systems, a scoring precedence must be defined.

Other issues include local-to-local correlations, remote-to-remote, as well as network-to-network correlations. Again, these issues are system-dependent. Scoring preferences should ideally be based on models of the internal precedence logic for each type of correlation, but such models may be difficult to come by.

The range of possibilities makes it unlikely that a single precedence rule will be adequate for all purposes, but this issue is still under discussion. For present purposes, it is important that the precedence be clearly defined at least on a case-by-case basis whenever the SIAP attributes are applied to scoring of test or simulation results.

Table 1 gives an indication of the types of tracks that can be measured and the associated track data for each. In summary, the SIAP SE TF will initially base SIAP attribute and applicable MOP calculations on track data in participants' central track stores, excluding remote-mutual tracks. The information that will be used is marked in red (the bold 'X').

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[1] The assertions in the text refer to legacy U.S. systems. There are, however, exceptions involving certain legacy U.K. systems, some of which are to be included in live exercises of interest for near-term SIAP assessments (in particular, JCIET 02). For example, there are systems on some U.K. frigates which process and display remote-mutual tracks in preference to local-mutuals. For these exceptional systems, the remote-mutuals could be considered the more actionable tracks, and it would be consistent with the philosophy of this report to reverse the usual scoring precedence rule in just these cases.

	Local	Remote	Mutual
System track number	X	X	X
Link track number	X	X	X
Local position/velocity	**X**		**X**
Remote position/velocity		**X**	X

X - Track information available
X - Track information to be used for SIAP assessment

Table 1. Tracks to measure.

3. ASSIGNMENT

The computation of all SIAP attributes, as well as that of certain track-based SIAP MOPs, is dependent upon a prior knowledge of how tracks are assigned to objects. (cf. SIAP SE TF Technical Reports 2001-001 and 2001-002, and Appendix A of this report). Provided that such an assignment is specified, these track-based metrics are uniquely defined quantities, derivable from measurable characteristics. However, changing the assignment will generally change the outcome of the calculations, even if the track and object data sets used are unaltered. The specification of how the assignment should be done can thus be considered a major component of the task of providing clearly defined procedures for the computation of SIAP metrics. This section addresses assignment as an issue independent of other implementation questions. However, as with the measurability problem, the primary objective is to obtain a viable methodology for current and near-term SIAP assessments, for which some use of simplifying assumptions is probably unavoidable. Problems arising with the proposed approach will have to be dealt with on a case-by-case basis until initial assessment experience is sufficient to indicate more attractive options.

The assignment (or lack thereof, in the case of spurious tracks) of each track to a specified object will be referred to in the remainder of this report as a "tracks-to-truth" assignment, in keeping with the common use of that phrase in the testing and simulation communities. Recall that the term "object" as formally defined in SIAP SE TF Technical Report 2001-001, is only used in reference to what are commonly called "truth" objects; thus the use of the term "truth" does not imply any additional qualification.

3.1 Definition of Terms and Assumptions

In the following treatment of the tracks-to-truth assignment procedure, it is assumed that a suitable set of measurable tracks has been identified as per Section 2, and in particular that a set of tracks with scoring precedence has been identified from among the candidate tracks held in each participant's CTS. It is also assumed that a complete set of "truth" data (i.e., position, velocity, force membership, and other relevant characteristics for each object) is available as needed at every time throughout the evaluation period. These data may be fixed by prior determination (in some modeling environments), or obtainable through direct measurement or reliable interpolation between measurements. The assumption that truth is always available is a precondition that may be difficult to meet in the case of live test data (cf. Section 4.1 of this report), and it may be necessary to tailor evaluation intervals after the fact to ensure that it is satisfied. This assumption should not present any difficulty for simulations however.

The following specialized terminology is useful in discussing various approaches to assignment.

> Assignment Function – The formalized end result of a tracks-to-truth assignment. For each participant m and each scoring time t, the assignment function maps each track (from among those with scoring preference) held by m at time t, either to a particular object, or to no object (the latter case indicating that the track is spurious).

Since the assignment function generally represents a discrete data set, an explicit mathematical representation of the function is not essential for practical computations – the track-object associations may be represented by an array or any other convenient data structure.

> Assignment Procedure – Any process (whether explicitly formalized or not) by which an assignment function is derived, at least partly on the basis of track data and truth.

> Assignment Algorithm – An explicit, deterministic assignment procedure which relies exclusively on track data and truth.

An assignment algorithm is thus a special case of an assignment procedure which may be regarded as a suitable candidate for automation (through software coding). More generally, assignment procedures may involve elements of human decision-making (which are not easily encoded), may rely on information not strictly derivable from track data (for example, anecdotal information supplied by human participants, also not easily encoded), and may invoke probabilistic criteria.

> Gated Assignment – An assignment procedure which automatically prohibits any instantaneous track-to-object assignment if the track does not satisfy a pre-specified condition of proximity to the truth object ("gating constraint") in position, and possibly also in velocity or other amplifying data. If truth data is known for all (reportable) objects, then a track prohibited from all such assignments by gating constraints is regarded as

spurious, and an object similarly prohibited is regarded as untracked. (As presented below, an object may be regarded as untracked for additional reasons.) Gating criteria are re-evaluated at each instant in time for which the assignment procedure is carried out.

<u>Unique Assignment (UA)</u> – An assignment procedure which results in a one-to-one assignment between a subset of eligible tracks and a subset of eligible objects, where "eligibility" may be based on gating or other criteria. In other words, each track which is assigned to an object is assigned to a unique object, but tracks may be unassigned by failing to meet eligibility criteria, or because there are more tracks than objects. If there are fewer eligible tracks than eligible objects, then the assignment will cover no more objects than there are eligible tracks, and the remaining objects (formally, those outside the range of the assignment function) are regarded as untracked.

<u>Unique Optimal Assignment (UOA)</u> – A unique assignment procedure in which the assignment function is determined as that which optimizes (by convention, minimizes) the instantaneous value of a pre-specified assignment cost function over the set of all assignments satisfying UA eligibility criteria. The cost function may depend only on the instantaneous assignment, or may depend as well on assignments already made over some previous time interval (UOA with hysteresis). A gating constraint may also be imposed, resulting in a gated unique optimal assignment (GUOA).

Typically, the cost function specified for a UOA procedure is a weighted sum of all kinematic state estimate errors encompassed by a chosen assignment, possibly modified to include hysteresis. A UOA procedure with such a cost function is thus roughly a global error minimization procedure.

<u>Independent Nearest Neighbor Assignment (INNA)</u> – An assignment procedure which instantaneously assigns every track to the nearest object, possibly subject to gating constraints, but generally without regard for uniqueness or for how other tracks may be assigned.

Any procedure which leads to a definite assignment function can be used as a basis for evaluation of SIAP attributes (and other measures). Once the assignment function is determined, the extraction of a small (and definite) number of additional critical variables from the track and truth data completes the set of information required for the explicit computation of all SIAP attributes. Appendix A lists these additional variables, and demonstrates their sufficiency (taken together with the assignment function) for the evaluation of the SIAP attributes.

3.2 Proposed Approach to Assignment: Examples and Rationale

Briefly stated, the SIAP SE TF will implement a multi-step assignment algorithm for current SIAP assessments, incorporating the following features:

(1) assignment limited to tracks specified as having scoring precedence

(2) administrative assignment of remote-only self tracks, such as Link 16 J2.2 Air Precise Participant Location and Identification (PPLI) tracks

(3) a position/velocity and other set of gating constraints to rule out implausible assignments and identify spurious tracks

(4) for each track source independently, a first pass through a GUOA algorithm with all tracks and objects not excluded by gating considered eligible

(5) for each track source independently, a second pass through a gated INNA algorithm (with the same gating criteria as used in the first pass) for all tracks not assigned in the first pass

The proposed procedure is outlined in greater detail in Section (3.3). In the remainder of this section, the rationale underlying some of the elements of the procedure is discussed.

3.2.1 The Use of an Assignment Algorithm

First, the proposal to use an automated assignment algorithm, as opposed to a less formal assignment procedure, requires some discussion. Analysis of military field test data has typically relied on combinations of formal and informal track assignment procedures, with a strong contribution from the data analysts' experience and common sense judgements. These procedures have evolved over time and have proven to be useful. However, they are time and labor intensive.

On the other hand, Monte Carlo simulations designed to collect statistical distributions of track data from thousands of runs essentially demand the use of automated assignment algorithms. Assignment procedures which involve significant degrees of human intervention and judgement are unfeasible in such simulation environments.

The SIAP SE TF plans to pool data from a wide range of test exercises and simulations for use in SIAP assessment, and a consistent approach to measurement is desirable. The automated algorithms preferred in the simulation community can be adapted to both simulation and live testing environments, and represent the only viable option for assignment if simulations and live tests are to be scored on an equal footing. Hence, the use of an automated algorithm to compute SIAP attributes will be the standard for current SIAP assessments. This by no means implies that the SIAP SE TF discourages other assignment approaches, or will disregard data analyzed through other means. In fact, comparisons of different assignment procedures applied to the same data set will undoubtedly prove useful as the approaches described in this report are adapted to meet future needs. For this reason, multiple analyses using different assignment procedures are encouraged whenever this is a feasible option. However, until formally changed or modified, characterization of performance using "SIAP Metrics" must be reported using the procedures identified in this technical report as well as SIAP Technical Report 2001-001and 2001-002.

3.2.2 Gated Assignment

The computation of the SIAP attributes requires distinguishing between tracked and untracked objects, and between assigned and unassigned (spurious) tracks. This is most immediately obvious in the attribute of completeness (the ratio of tracked objects to all objects), and in the spurious track measure (the ratio of spurious tracks to all tracks) of clarity, but in fact most of the other attributes as well depend on a prior knowledge of which tracks are assigned. The assignment procedure, then, must make these distinctions.

As already suggested, the simplest approach to deciding whether a track is assigned or whether an object has a track assigned to it is to rule out implausible track-object pairings through gating. Current SIAP assessments will use a position/velocity gating criterion (details given in 3.3.5), together with a qualitative criterion prohibiting the assignment of a track to an object in a different environmental category (for example, an air track cannot be assigned to an exo-atmospheric object).

Figure 1 illustrates the use of position gating (a subset of position/velocity gating) to identify an untracked object (object C).

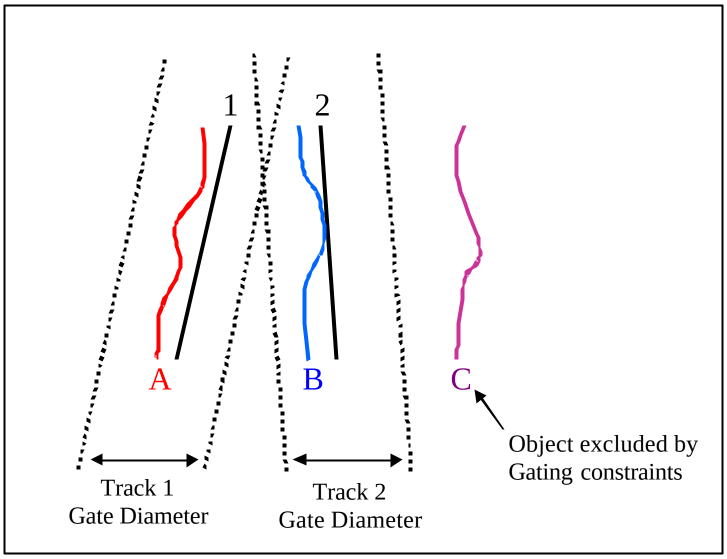

Figure 1. Use of gated assignment.

3.2.3 Unique Optimal Assignment

The need for unique (and in particular, unique optimal) assignment is more subtle. To illustrate this, Figure 2 depicts a typical situation that may arise with tracks from a single sensor with a slight systematic bias (a very common occurrence), in which gating alone does not resolve the assignment.

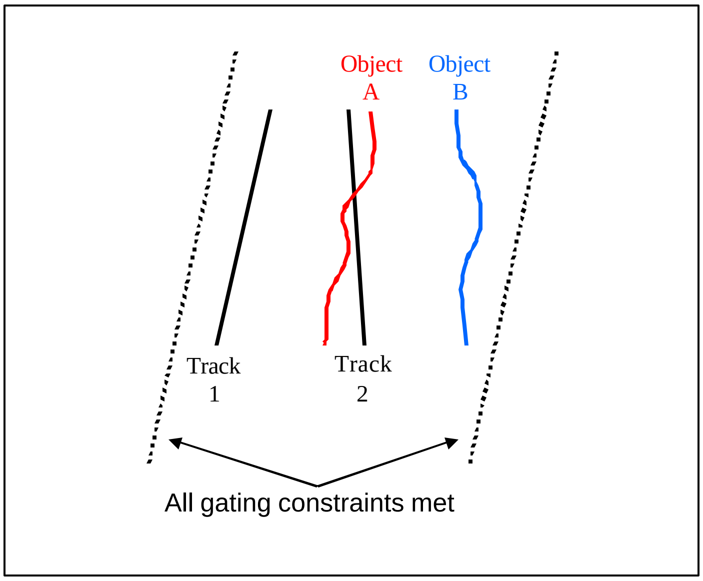

Figure 2. Assignment issue not resolved by gating.

Assume the use of an assignment algorithm, which does not account for the bias. A non-unique assignment would probably assign both tracks to object A (it is closer to each track considered independently), creating a "local dual." Local duals do occur, but under the hypothesis that they are far less likely on a single sensor than is a systematic offset, this assignment is less plausible than one in which the two tracks were assigned to distinct objects (this hypothesis, although apparently widely accepted, does need to be tested – hence the recommendation in Section 5.2 of this report for experimentation with alternative algorithms). The non-unique assignment algorithm would thus tend to perpetuate sensor biases through the assignment process, skewing attributes such as completeness and clarity in ways that might not be easily traceable.

A unique but non-optimal assignment, such as for example a unique nearest-neighbor assignment procedure, would avoid the problem just described, but has a different drawback. Unique nearest-neighbor assignment, if not otherwise qualified, depends on the order in which the assignment is done, which is an undesirable characteristic in an automated program. If qualified according to some arbitrary ordering, a nearest-neighbor assignment would be just as likely (in the example in Figure 2) to make the intuitively implausible assignment of 2-to-A, 1-to-B, as the more intuitive (and consistent with the presence of sensor bias) 1-to-A, 2-to-B. Furthermore, certain algorithms which are designed to process the closest assignments first (so-called "greedy" algorithms), would *always* make the less plausible choice with respect to track 1.

Imposing a criterion of global error optimization (that is, minimization of cumulative error over all assigned tracks) on a unique assignment algorithm is the easiest way to avoid these pitfalls. Of all of the options so far discussed, only a unique optimal assignment would consistently make the most plausible choice in the presence of small sensor bias, and thus tend to compensate automatically for systematic errors and not perpetuate them through the process. A UOA algorithm is thus proposed for assigning tracks from a single source (for example, local tracks held by a single participant, or remote tracks originating from the same source).

The situation is different, however, when grouping tracks originating from different sources (for example, when assessing local and remote tracks together). In this case, duals are frequent and need to be assessed accurately. Also, local duals which cannot be ruled out through a locally-applied unique assignment should of course be counted along with the other (local-remote) duals. This is the rationale for assessing tracks from separate source nodes independently, and for the "second pass" using a non-unique INNA procedure to assign the remaining tracks after all tracked objects are accounted for through the initial pass with UOA.

3.2.4 Time Aligning

Time-dependent SIAP metrics obtained from test data or simulations will be evaluated over a discrete set of predetermined "scoring times." Each scoring time is meant to be representative of a small interval of time within a longer period of evaluation. In addition, there may be certain set events particular to individual participants or systems at which evaluation is desired, regardless of the time at which these events occur (these are sometimes called "test article requested" scoring times). With the exception of the latter, it is recommended that scoring times be set in advance according to some randomized spread throughout the evaluation period, to avoid phasing bias. For example, a requirement could be imposed of one scoring time in each of a regularly spaced series of subintervals, but with the scoring time placed randomly within the subinterval. In the case of Monte Carlo simulations, once the schedule of scoring times is determined, the same schedule (that is, the same times, without additional randomization) should be used for all runs. This consistency between runs is important for statistical analysis.

The scoring times will not, of course, generally coincide with track reporting times. A convention is needed to match track data with truth data at a particular scoring time for purposes of assignment. The following procedure based on the interpolation of *truth data only* will be followed. For assignment of a local track at scoring time t_k, interpolate all truth object states to the last track update time before (or equal to) t_k, and assess the assignment costs for the track using track data at the update time. For the assignment of a remote track, follow an analogous procedure using instead the time stamp in the track (if available, as in the Link 16 J3.6 Space Track) or the report receipt time if there is no time stamp (as in the Link 16 J3.2 Air Track). If it is independently known that a particular object present at t_k was not present at the update (or time stamp or report receipt) time, then disregard that track-object pair for assignment purposes (i.e., gate out that track-object pairing). If an object existed at track update time (or time stamp or report receipt time) but does not exist at time t_k, then also disregard that track-object pair for assignment purposes.

If no information is available on track update or report receipt times, then there is no option but to assume that the track data is valid at the scoring time. This assumption is obviously less satisfactory than the approach just outlined, and should be invoked only as an alternative to discarding the track data entirely.

3.3 Proposed Assignment Algorithm

A complete description of the proposed tracks-to-truth assignment algorithm is now given. The steps of the process are outlined in Section 3.3.1. Section 3.3.2 addresses the assignment cost function, and Section 3.3.3 the gating criteria to be applied.

3.3.1 Detailed Outline of the Assignment Algorithm

This section will provide, in outline form, a detailed description of the proposed track assignment algorithm. It discusses each track type (local, remote, etc…) and how each track type is to be assigned.

Separately for node m at each scheduled scoring time, k,

Step 1: Down select to the set of current local tracks held by node m that have precedence for scoring.

Step 2: Discard from consideration at this scoring time all remote tracks held by node m which that node's track correlation function currently assesses to be mutual with node m local tracks.

Step 3: Down select to the set of current non-mutual remote tracks held by node m that have precedence for scoring.

Step 4: For J2.2 Air PPLI self tracks that are not mutual with local tracks, administratively assign the PPLIs to the corresponding truth file based on unit identification.

Step 5: Separately assign all retained local tracks to truth objects, using a first pass through a GUOA algorithm, followed by a second pass on local tracks *not yet assigned* through a gated INNA (tracks prohibited from all assignments by gating are declared to be spurious).

Step 6a: Separately assign retained non-mutual remote tracks from remote source node n using the same procedure and formulas as for retained local tracks.

Step 6b: If there are more sources of retained non-mutual remote tracks, increment n and return to Step 6a.

Step 7: Merge the separate assignments into a complete list of track-to-truth assignments for track stores at node m.

Repeat the above steps separately for each node being evaluated.

The assignment steps outlined are displayed pictorially in Figure 3.

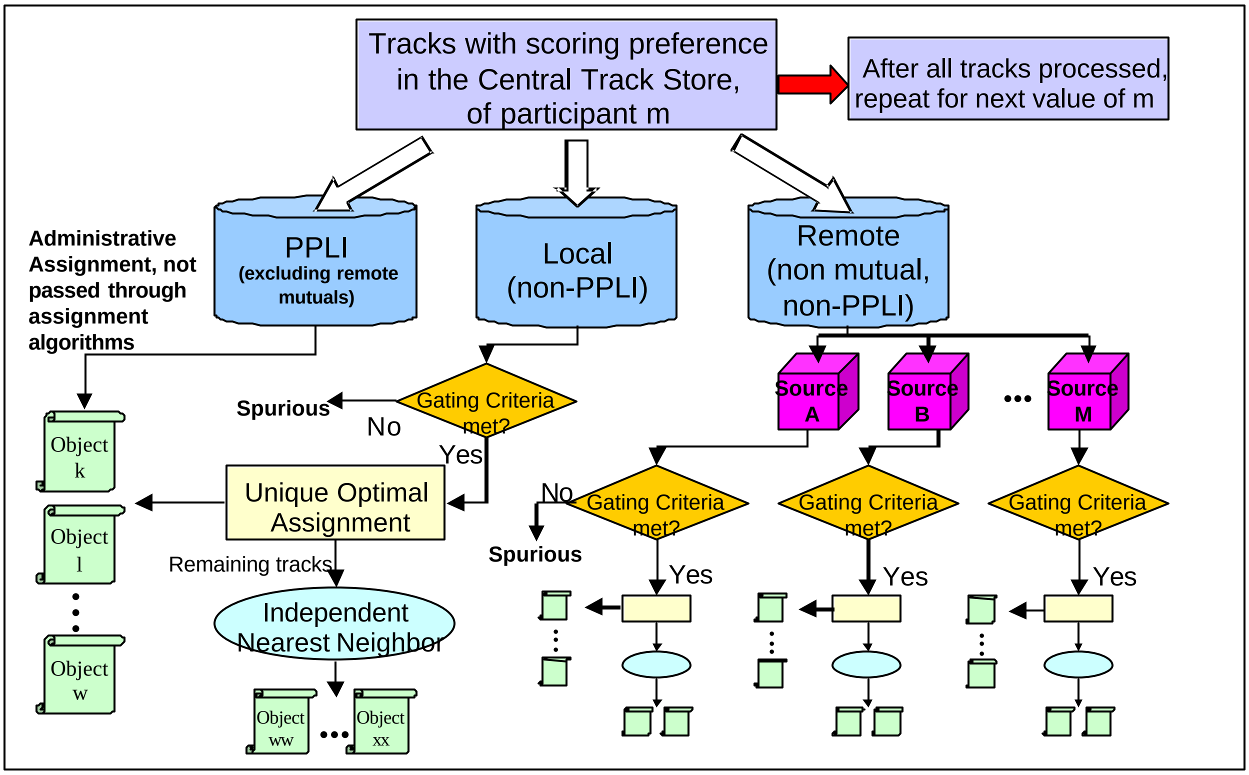

Figure 3. Track assignment procedure.

3.3.2 The Assignment Cost Function

This section describes the formulation of the assignment cost function to be used in the first pass GUOA algorithm of Step 5 of Section 3.3.1. Recall that this formulation applies separately to each participant (node) being evaluated, and in the case of remote tracks, to each source of tracks held at each node. However, so as not to encumber the notation with additional indices for the node and source, the formulae in this section will be understood to apply to tracks held by a single participant, originating from a single source.

The proposed UOA algorithm is based on a pairwise cost function that is effectively a weighted sum of squared components of two-dimensional track errors. It is the chi-squared (or Mahalanobis) distance for a two-dimensional track with a position/velocity error covariance corresponding to a case in which errors are independent but of equal magnitude in the two orthogonal directions. Consider the set of tracks held by the participant at the k^{th} scoring time (t_k) to be enumerated by an integral index i ranging from 1 to the number of tracks held (N_k). Consider the set of objects present at the k^{th} scoring time to be similarly enumerated by an integral index j ranging from 1 to the number of objects (J_k). The *unadjusted* (for hysteresis) cost function $C_k(i,,j)$ for assigning track i to truth object j at the kth scoring time is

$$C_k(i,j) = \frac{1}{\sigma_P^2(1-\gamma^2)} \Big\{ \left[(0-Px_{i,j})^2 + (0-Py_{i,j})^2\right] + \left(\frac{\sigma_P}{\sigma_V}\right)^2 \left[(S_i \sin\phi_i - Vx_{i,j})^2 + (S_i \cos\phi_i - Vy_{i,j})^2\right]$$
$$-2\gamma\left(\frac{\sigma_P}{\sigma_V}\right)\left[(0-Px_{i,j})(S_i \sin\phi_i - Vx_{i,j}) + (0-Py_{i,j})(S_i \cos\phi_i - Vy_{i,j})\right] \Big\} \quad (1)$$

where

$Px_{i,j}$, $Py_{i,j}$, $Vx_{i,j}$, and $Vy_{i,j}$ are the truth object horizontal position/velocity states in the track i-centered east-north-up (ENU) frame, interpolated to the last track update or report receipt time, as described in Section 3.2.4,
S_i is the track horizontal speed,
ϕ_i is the track heading measured clockwise from north,
σ_P is the position error standard deviation in an orthogonal direction,
γ is the correlation coefficient between the position and velocity errors in an orthogonal direction, and
(σ_P/σ_V) is the ratio of the position error standard deviation to the velocity error standard deviation in an orthogonal direction.

It should be noted that the truth object position/velocity coordinates $Px_{i,j}$, $Py_{i,j}$, $Vx_{i,j}$, and $Vy_{i,j}$ are *not* identical with the truth object data used in the definition of the SIAP attribute of kinematic accuracy, as the latter data are registered at the scoring time, not interpolated back to the update or receipt time.

The parameters γ and (σ_P/σ_V) weight the position and velocity errors. For steady state tracks these parameters are constant with typical values of $\gamma = 0.65$ and $(\sigma_P/\sigma_V) =$ 4.3 to 9.0 seconds. The actual values to be used may be modified after some initial experimentation. The quantity σ_P itself is an empirical parameter related to the sensor characteristics of the track source being assessed. It will generally be different for each sensor, though for initial assessment purposes the simplifying assumption may be made that σ_P is a constant for all sensors *of the same type*. Values to use for σ_P should be assessed from empirical track data.

Appendix B provides details as to how this unadjusted cost function was obtained, including a statistical interpretation of the parameters described above, and a recommendation for the estimation of σ_P from empirical data.

The cost function given in equation (1) is modified to include a single time step hysteresis effect, the purpose of which is to discourage rapid switches in assignment (though without eliminating them altogether). The *adjusted* cost function for assigning track i to truth object j at the kth scoring time is given by:

$$C_{ak}(i,j) = C_k(i,j)\, A_k(i,j) \quad (2)$$

where $C_k(i,j)$ is the unadjusted cost of assigning track i to truth object j at scoring time k, and

$$A_k(i,j) = \begin{cases} d \ \ (0.0 < d < 1.0) \text{ if i was assigned to j at scoring time } k-1 \\ 1.0 \text{ otherwise.} \end{cases} \qquad (3)$$

A default value of d = 0.15 has been shown through preliminary experimentation to produce the desired hysteresis effect without resulting in distorted assignments. Cost adjustment with this default value should be regarded only as a heuristic for use in the absence of further experimental data. Intuitively, the cost adjustment for hysteresis should depend on the scoring frequency. Further experimentation may show that it is appropriate to base the dependence of d upon the time interval Δt_k between the k^{th} and $k\text{-}1^{th}$ scoring times on an exponential relation or some similar functional form.

The total cost for an assignment at time step k is then obtained as follows. Consider first the case where there are at least as many truth objects as tracks. The *augmented cost matrix* $\overline{\mathbf{C}_{\mathbf{ak}}}$ is the $N_k X(J_k+1)$ matrix of extended real numbers, rows corresponding to tracks and columns to truth objects, whose (i,j) component is given by

$$\overline{C_{ak}}(i,j) = \begin{cases} C_g \,, \ \text{if } j = J_k + 1 \\ C_{ak}(i,j)\,, \ \text{if } j < J_k + 1 \text{ and gating criteria are satisfied for the pairing } (i,j) \\ \infty \ \text{ otherwise.} \end{cases} \qquad (4)$$

where C_g is a constant "guard cost" value representing the cost of not assigning a track to a truth object. Gating criteria are discussed in detail in Section 3.3.3. The function of the guard cost is to ensure that as many tracks as possible are assigned within the imposed gating criteria, without the need for an explicit representation of this condition. The guard cost value C_g is related to the gating criteria, as will be made explicit in the following section. An *assignment* is represented by an integer-valued, N_k-component vector $\mathbf{a_k}$ whose i^{th} component $a_k(i)$ is either the number (from 1 to J_k) of the truth object to which track i is assigned, or the number J_k+1 if the track is unassigned, with the constraint that each number from 1 to J_k may appear at most once as a component of $\mathbf{a_k}$ (unique assignment constraint). The *total cost* $CT_k(\mathbf{a_k})$ for the assignment $\mathbf{a_k}$ is then given by

$$CT_k(\mathbf{a_k}) = \sum_{i=1}^{N_k} \overline{C_{ak}}(i, a_k(i))\,. \qquad (5)$$

In the case where there are more tracks than truth objects, the roles of tracks and truth objects are reversed in this formulation. That is, truth objects are assigned to tracks (rather than tracks to objects), and the augmented cost matrix $\overline{\mathbf{C}_{\mathbf{ak}}}$ is a $J_k X(N_k+1)$ matrix with rows corresponding to truth objects and columns to tracks, and with the last column containing the guard costs for non-assignment of a truth object. Equation 5 is unaltered.

The optimal assignment is the assignment $\mathbf{a}_k$ which minimizes the total cost. The determination of the optimal assignment is thus an integer programming problem (the number of assignments being finite, with at least one assignment having finite cost), and this problem must be solved at each scoring time. Since there are no complicated constraints on the integral variables (the components of $\mathbf{a}_k$), the integer programming problem is algorithmically straightforward, and a number of standard routines are available for computing the optimal solution. The SIAP SE TF will recommend a specific solution program on the basis of some initial estimates of time and storage requirements, once the Common Reference Scenario (CRS) for SIAP evaluations has been established.

3.3.3 Gating Criteria

The exact gating criteria used may need to be modified after some initial experimentation. For initial assessments, the following criteria are recommended.

(1) A track must be for the same environmental category as the truth object:

An air track (J3.2 report) is only assignable to a truth object that stays aloft by aerodynamic lift and/or by virtue of having density lower than mean sea level air density, or to an air-to-air missile (AAM), an air-to-surface missile (ASM), or a surface-to-air missile (SAM).

A space or ballistic missile track (J3.6 report) is only assignable to a truth object that is in exo-atmospheric orbital, sub-orbital, or super-orbital trajectory, or to a ballistic missile or BMD interceptor object in any phase of powered, ballistic, or maneuvering flight, including a SAM that is being operated as a BMD interceptor.

(2) An air track i for which an altitude state is provided (and not declared to be unreliable) is only assignable to a truth object j for which $(Pzi - Pzj)^2/\sigma_{Pz}^2 < 19.5$, (i.e., a 0.99999 probability gate in altitude), where σ_{Pz}^2 is the typical variance of a track altitude state for a sensor of the type that generated track i. Quantify σ_{Pz}^2 empirically in the same step used to quantify $\sigma_{Px}^2 = \sigma_{Py}^2 = \sigma_P^2$ for the assignment cost function.

(3) An air track i is only assignable to truth object j for which the recommended cost function C(i,,j) < 28.5 (i.e., a 0.99999 probability gate in 4 horizontal states).

(4) The same cost value of 28.5 used for the horizontal gate should also be used as the guard cost C_g (see Section 3.3.2) for each unassigned track or untracked object.

3.4 Advantages and Disadvantages of the Proposed Procedure

This section will summarize the strengths and weaknesses of the proposed assignment algorithm. A primary advantage of giving local mutuals scoring precedence over remote mutuals is that local track data is always available in all legacy air defense systems, whereas remote mutual data is not always retained after correlation. Down selection of multiple local and remote tracks facilitates the scoring of tracks that operators are most likely to use. Administrative assignment of J2.2 Air PPLI tracks is based on accurate self-identification information. Another strength of the proposed algorithm is separate gated unique optimal assignments of tracks from the same source tends to remove source platform position bias and, to some extent, sensor bias from the tracks-to-truth reconstruction. Desirable assignment hysteresis across scoring times can be induced through appropriate design of assignment cost functions. Finally, gating criteria are easily modified so as to consider factors other than real-valued distance measures, offering greater flexibility to the assignment procedure.

There are two disadvantages of the proposed assignment algorithm. The first is, with separate assignment of tracks to truth at each node, the same network track number can end up being assigned to different truth objects at different nodes. Second, down selection of local and remote tracks to score in constructive simulations requires modeling of local-local track and remote-remote track precedence logic, which could be different at each type of node.

4. OTHER RECONSTRUCTION ISSUES

Reconstruction is a term used in the testing community that refers to all aspects of the process of constructing track-to-truth associations from data. The assignment procedure discussed in Section 3 is a major aspect of this process, but, as already discussed, involves something of an idealization. Many other problems arise in practice that may prevent a smooth application of this idealized assignment process. For the most part, these issues have not as of yet been specifically addressed by the SIAP SE TF. The following sub-sections provide a brief discussion of some of the reconstruction issues that may arise. One recommendation, based on input from representatives of the joint testing community, is described in Section 4.1. It may be necessary for the SIAP SE TF to provide additional specific recommendations on other points, if evidence of serious difficulties with test data should emerge as the assessment process matures.

4.1 Absence of Data

For whatever reason, the data for a particular participant in a test event or exercise may not be recorded or it may be lost. Because the calculations for the SIAP attributes are based on recorded track data from all participants, the absence of data is an issue, and how these cases are handled needs to be addressed. One does not simply want to discard the data from all of the units just because one participant's data cannot be included in the attribute calculations. The following presents a recommended procedure for computing averages of SIAP attributes without penalizing a participant for the lack of data. The variable defined as the number of objects at time t is allowed to depend on the participant m, and is now denoted $J_m(t)$. As long as data reported by participant m is recorded, $J_m(t)$ is the same number for all m, J(t). However, if the reporting of data from participant m is interrupted, $J_m(t)$ should be set to zero, producing an analytical situation in which it is as if participant m were not involved in the test at that time.

This convention attempts to offset data dropouts that may be encountered due to problems with the testing architecture or process, as opposed to problems with the network(s) being tested. Equations for the IADS roll-ups of the attribute measures can be adjusted accordingly.

As an example, consider the attribute of completeness, and assume that there are three participants in the current scenario with four objects in the AOI from time zero to time three. Participant 1 tracks and reports all four objects at each time period. Participant 2 tracks and reports three objects at each time period, and a fourth at times zero, one, and three. Participant 3 tracks and reports exactly three objects at each time period; however, no data is recorded at times two and three. Recall the completeness formula (SIAP SE TF Technical Report 2001-001,

$$C_m(t) = \frac{JT_m(t)}{J(t)} 100\% . \tag{6}$$

The IADS roll-up is adjusted from the usual average over time steps and participants to the modified average

$$C = \left[\frac{\sum_{m=1}^{M} \sum_{t_{start}}^{t_{end}} JT_m(t)}{\sum_{m=1}^{M} \sum_{t_{start}}^{t_{end}} J_m(t)} \right] 100\ \% \tag{7}$$

where $J_m(t)$ is the adjusted object count as defined above (for definitions of other variables see Appendix A). The roll-up completeness C following the unadjusted averaging formula (SIAP SE TF Technical Report 2001-001) for the example would be

$$C = \frac{(4+4+4+4)+(4+4+3+4)+(3+3+0+0)}{(4+4+4+4)+(4+4+4+4)+(4+4+4+4)} = 77.1\% , \tag{8}$$

thus reckoning completeness as if participant 3 were tracking no objects during the periods of data dropout. Using the proposed adjusted formula (5), completeness becomes

$$C = \frac{(4+4+4+4)+(4+4+3+4)+(3+3+0+0)}{(4+4+4+4)+(4+4+4+4)+(4+4+0+0)} = 92.5\% . \tag{9}$$

A similar adjustment can be made with any IADS-averaged metric. It is important not to overuse this approach, as there is a danger of skewing the averaging results to an extent that they can no longer be understood intuitively. If the absence of data is a persistent problem, then a safer approach would be to present a sample instantaneous data set in timeline form (such as depicted in Figure 4) and show where the problem lies.

A problem more difficult to remedy is the dropout of truth data for particular periods of time. SIAP Metrics cannot be calculated without accounting for truth objects. Unless an appropriate substitute can be found for the data intended to be collected (for example, perhaps a reliable sensor held good track data on all objects for the critical period), the only safe remedy is to restrict the period of evaluation to times for which the truth data is available. Fortunately, it appears that the use of PPLI data for truth in many recent joint exercises has reduced the severity of this problem (though without completely eliminating it), at least for friendly truth objects.

4.2 Time Aligning

Time aligning has already been discussed in the context of the tracks-to-truth assignment algorithm. However there may be other issues involving time that arise from the specific details of various live tests. If time offset and/or scale factor bias is evident in any metric data that are to be treated as truth data in the tracks-to-truth assignment, either those data can be discarded (and the consequences of their loss accepted) or attempts can be made to remove the time bias so that the data can still be used. For example, under certain circumstances this could be done by first estimating and compensating the time and measurement biases of one of the participating sensors using other truth data that can be trusted, and then solving for the time offset and scale factor bias in the corrupted truth data that minimizes the sum of distances between that data and the measurements or track state estimates of the registered sensor. Except as an indirect means to remove time bias in corrupted truth data (or as part of a separate assessment to characterize as-is sensor biases and the benefit of correcting them), usually no attempt should be made to remove time or other biases in track data that are to be scored.

4.3 Formation Tracking Considerations

The use of formation tracking introduces many complications, and an appropriate assessment procedure has not yet been worked out. Formation tracking and assessment issues are currently under study by a SIAP SE TF working group. The recommendations of this working group will be addressed in another technical report, and to the extent that they influence the near-term SIAP assessment approach, will be incorporated into future versions of this report.

There is, however, an immediate need to set down a convention for scoring SIAP attributes and track-based MOPs in tests in which formation tracking is used. U.S. Joint Forces Command (USJFCOM) has recently provided a clarification of TAMD CRD requirements for formation tracking, which sets the long-term objective that "each object should have a unique track identifier and associated characteristics" (CINCUSJFCOM J8 Memorandum of 29 June, 2001). In keeping with this long-term objective, the SIAP

SE TF will base SIAP attribute measures and appropriate MOPs on a one-object per track basis, with no special allowances made for formation tracks. Formation tracks will be treated as any other tracks in the assignment procedure of Section 3, and scored accordingly. Special MOPs have been introduced (cf. SIAP SE TF Technical Report 2001-002) to measure the extent of formation track use in a testing situation, and these MOPs provide a gauge of the effect of this one object per track scoring basis on assessment results for formation trackers.

Exceptions to this scoring policy may be in order for ballistic missile tracks in certain situations. USJFCOM explicitly recognizes this in the statement, "the unique issues of dealing with ballistic missile tracks will differ in this regard and may require further study" (CINCUSJFCOM J8 Memorandum of 29 June, 2001). The SIAP SE TF has elsewhere acknowledged (cf. SIAP SE TF Technical Report 2001-001) that ballistic missile defense (BMD) issues may require reconsideration of some elements of the proposed approach to SIAP metrics, which may include the implementation proposals made in this report. Recommendations pertaining to BMD have been deferred pending further study by the SIAP SE TF and other organizations involved.

4.4 Dimensionality in Accuracy Calculations

Not all tracking systems work in three dimensions. Even for systems capable of receiving three-dimensional information, it is still possible that only two-dimensional data is being recorded in a test situation (for example, some data collection formats do not use a height rate field). This presents some ambiguity in the definitions of kinematic accuracy measures, as already noted in SIAP SE TF Technical Report 2001-001.

The current definitions allow the test designer to weight the vertical position and velocity accuracy measures by some factor w between zero and one, to scale out vertical errors which a 2D system is intrinsically incapable of improving, and which perhaps cannot be reliably estimated anyway. In most cases, w will be set to exactly zero or exactly one, depending on the dimensionality of the current data. This allows a fair comparison among sensors of the same dimensionality, but it is not clear whether either choice will give useful accuracy measures when the sensors are mixed (which is in fact quite typically the case for current sensor networks). There is the possibility of using intermediate values of w to decrease the significance of the third dimension, while maintaining the third measure, but the SIAP SE TF has not agreed to any standard for setting the value.

There needs to be further exploration of the possible uses of the kinematic accuracy measure among joint systems subject area experts before this issue can be resolved entirely. For present assessments concerning only air vehicle tracks, the SIAP SE TF will compute the kinematic accuracy measure on a 2D basis (w=0 for all tracks and all participants), but recommends the use of 3D information where available and appropriate for the computation of MOPs in root cause analysis. Again, this approach is not expected to be adequate for the BMD component of the SIAP, for which full dimensionality (3D or 6D) in accuracy is essential. The approach will therefore be updated with future versions of this report as BMD recommendations are included.

The proposed assignment algorithm of Section 3 is basically two-dimensional, but allows for altitude gating on appropriate tracks. The dimensionality issue is raised here because the reconsideration of the kinematic accuracy definition may lead to a reconsideration of the assignment procedure in this regard.

4.5 Test Design Issues

The SIAP SE TF may need to look at specific problems associated with testing design and make recommendations to help bring test designs closer into line with SIAP assessment objectives.

5. EXCURSIONS AND OTHER APPROACHES

The proposals for current assessments do not represent all of the data that will have to considered throughout the SIAP system engineering process. This section provides other possible methods for treating SIAP data, including a list of alternative track bases from which to select candidate tracks as well as corresponding alternative scoring approaches. Experimentation with these approaches and excursions is encouraged. However, until formally changed or modified, characterization of performance using “SIAP Metrics” must be reported using the procedures identified in this technical report as well as SIAP Technical Report 2001-001and 2001-002.

5.1 Alternative Track Bases

Many aspects of the proposed metrics implementation approach can be easily adapted to any set of tracks. As mentioned in Section 2, the approach to measurability for current assessments is not the only one that should be examined. Any data collected from other track bases may be useful for SIAP system engineering in various ways, and in particular may reveal difficulties with or suggest improvements to the proposed approach. Furthermore, the relative importance of different track bases for warfighting capability assessments can be expected to change as SIAP system engineering evolves.

This following is a (nonexhaustive) list of alternative track data sets which are of interest in this regard.

- Platform track stores (current approach)
- Link 16 tracks only
- Link 11 tracks only
- Link 22 tracks only
- CEC/JCTN composite tracks only
- PPLI tracks only (subset of link tracks)
- Local sensor tracks

Since a number of excursions may be pursued simultaneously in different testing environments, it is important to record the track data sets actually used in each excursion. Therefore, the SIAP SE TF will require as a general policy that any assessments done under its sponsorship be submitted with a record defining the track data used at the participant level. Short descriptive statements (such as appear in the list just noted) are adequate, provided differences in scoring policy among different participants are noted as well.

Table 2 summarizes possible scoring excursions for each of these possible track bases, and the corresponding adjustments to the assignment procedure required.

Track Base	Scoring excursion
Platform track stores (current approach)	Local mutuals have precedence for scoring
	Remote mutuals have precedence for scoring
	A table defines local versus remote precedence for scoring.
Link 16 tracks	The set of local R2 tracks, remote J3.2 Air Tracks, and J2.2 Air PPLIs
	The set of local R2 tracks and remote J3.2 Air Tracks
	Exclusively J2.2 Air PPLIs
Link 11 tracks	Can include variants for separate assignment at each node and for global assignment of network tracks
Link 22 tracks	Can include variants for separate assignment at each node and for global assignment of network tracks
CEC/JCTN composite tracks	Can include variants for separate assignment at each node and for global assignment of network tracks
Local sensor tracks	Local tracks

Table 2. Alternative track bases and scoring excursions.

5.2 Alternative Approaches to Assignment

Recall in Section 3 that the SIAP SE TF encourages alternative approaches, partly as a way of gauging the effectiveness of the proposed procedure.

The procedure as outlined in Section 3 can easily be modified to accommodate other scoring preferences. For example, reasons have been given why local/mutuals have been given scoring precedence; however, it may be worthwhile to explore the impact of scoring remote/mutuals over local/mutuals. Remote track data will almost certainly increase in both accessibility and importance with future SIAP improvements. To accommodate this variant, it is only necessary to change the roles of local/mutual and remote/mutual tracks in the assignment procedure outlined in Section 3.3 (i.e., local/mutuals are omitted in the pass through local tracks, while remote/mutuals are assigned just as any other remote tracks).

It is also advisable to experiment with variants of the time aligning proposal of Section 3.2.4. The proposed procedure bases assignment on track and truth data at update (or report receipt) times. This approach obviates the need to extrapolate track data, and is thus expected to avoid the introduction of additional extrapolation errors into the assignment procedure. Furthermore, the use of the proposed time aligning has been implicitly assumed in the derivation of some details of the cost function (see Appendix B), and is convenient in that respect. However, there are some concerns over the algorithmic complexity of this approach, which may in principle require a truth data interpolation for every possible truth-track pairing at every time step. Also, there are concerns that anomalies in some of the attribute measures may result if there is significant "coasting" of tracks (retention of tracks in the absence of data updates), as coasted tracks will continue to be assigned as they would have been on the basis of their last update times. To trace the possible effects of track coasting, an alternative time aligning scheme based on extrapolation of track data to the current scoring time (rather than interpolation of truth data to update or receipt times) should be explored. Differences will very likely be observed in the measures of completeness and spurious tracks, since the alternative scheme will have a greater tendency to declare coasted tracks to be spurious. However, it is difficult to foresee, without experimentation, which time aligning scheme will yield more intuitively meaningful or analytically useful values for these metrics, or how great the differences are in practice. Some initial experimentation will also be needed to determine whether there are serious algorithmic complexity or inefficiency issues with the proposed approach. It should be kept in mind as well that the viability of any time aligning proposal for a live test evaluation is to some extent dictated by the quality and nature of the time data available, and by the way the data is formatted and stored. The method proposed in Section 3.2.4 is presently believed to be suitable, at least for initial experimentation, in the context of live tests and simulations planned over the next year or two, but future experience may lead to different recommendations.

One possible problem with the proposed assignment algorithm, noted in Section 3.4, is the possibility of a counter-intuitive assignment of the same track to different objects, depending upon the participant being assessed. Although this violates common sense, it is not clear whether these multiple assignments will be numerous, or whether they will skew the calculation of SIAP attribute measures significantly. One way of examining whether this issue represents a genuine drawback to the approach is to compare assessments based on the proposed assignment with re-calculations using a different approach – one for which a track is forced to be assigned to the same object for all participants. The latter condition is satisfied by another variant of the proposed assignment algorithm, in which remote tracks are *always* assigned according to the assignment selected at the source node. That is, all local tracks are assigned as usual, and remote tracks inherit the assignments. This variant carries its own disadvantages (possibilities for peculiar scoring in cases of poor data registration), but it would be instructive to set these against the disadvantages of the proposed procedure.

Finally, as mentioned in Section 3.2.1, automated assignment algorithms have not been used uniformly throughout the joint testing community, and it is reasonable to question whether the standardization of automated assignment proposed in this report offers any improvement over traditional procedures (many relying partly on human intervention). The best way to answer this question would be to pursue all approaches to the extent possible. Comparison in simple cases between manual or semi-automated procedures with proven reliability and fully automated algorithms such as the one proposed in this report are an excellent means of verifying the effectiveness of the latter in live test situations, as well as suggesting possible improvements. If frequent and severe discrepancies in assessments were obtained through different assignment procedures, this would naturally be grounds for serious reconsideration of the proposed approach.

6. CONCLUSIONS AND RECOMMENDATIONS

These proposals for SIAP metrics implementation, together with the detailed definitions of SIAP metrics provided in other SIAP SE TF publications, should satisfy the most important immediate objective for SIAP assessments – that of providing a consistent approach to measuring the quality of the current SIAP. Particular attention has been paid to establishing consistency in data assessment between live exercise and test evaluation on the one hand, and modeling and simulation on the other.

It is recommended that the implementation proposals in this report be applied to both simulation and live exercise data. It is recognized that the implementation strategies proposed will be modified as the methodology is used and improvements are identified.

7. REFERENCES

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APPENDIX A

Critical Variables for SIAP Attributes Implementation

This Appendix offers a systematic classification scheme for all variables that are directly involved in the *mathematical* derivation of the SIAP attribute measures. The assumption is made in this treatment that the *measurability* and *reconstruction* problems (including a complete tracks-to-truth assignment) have been resolved for a hypothetical situation under assessment.

One point of this exercise is to demonstrate exactly how the *assignment function* (the formal mathematical representation of the assignment, cf. Section 3.1 of the main text) enters into the derivation of the SIAP attribute measures. Another is to identify a minimal set of additional intermediate variables that must be extracted from track data and truth to complete the derivation. Finally, Appendix A concludes with a table (Table A-1) which can serve as a convenient reference for all formulae involved in the definitions of the SIAP attributes themselves (adapted from SIAP SE TF Technical Report 2001-001).

The overall scheme is suggested by Figure A-1.

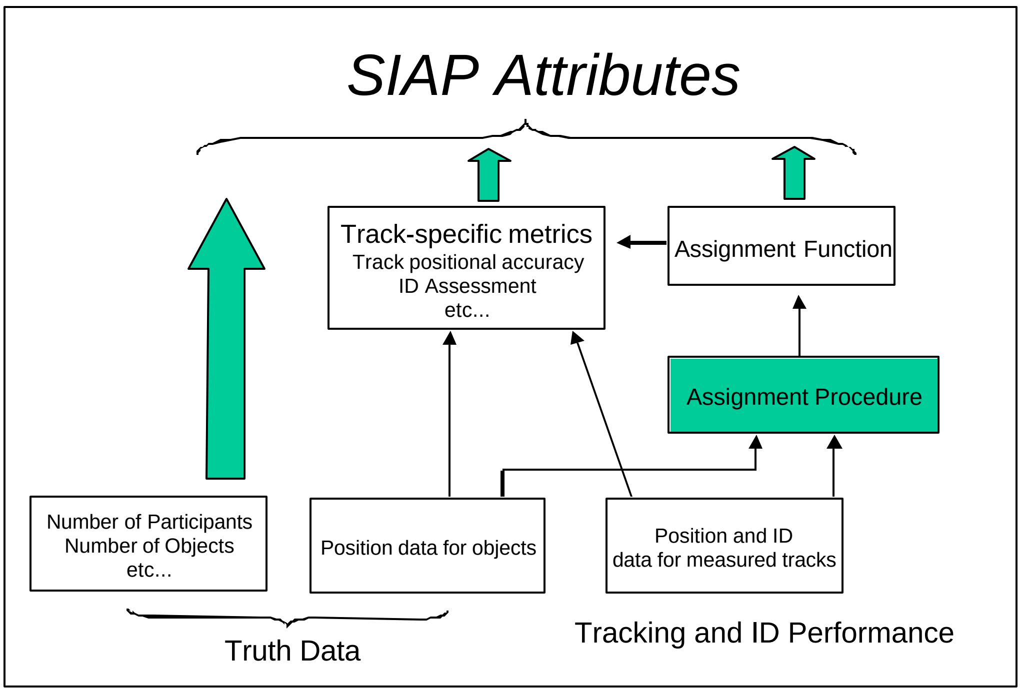

Figure A-1. SIAP attributes implementation.

All variables falling into the three "boxes" leading directly to the SIAP attributes will be explicitly identified, and the mappings suggested by the green (filled) arrows will be explicitly defined.

Critical Variables

The critical variables consist of the assignment function itself (including a specification of its domain, which effectively identifies the track data set), some truth variables not directly represented in the tracks-to-truth assignment, and a reduced list of MOPs and attributes (called "track-specific" metrics here) which are directly obtainable from position, velocity, and ID data on each track, the truth data, and the assignment function.

Truth Variables

$J(t)$ – total number of objects at time t

M – total number of tracking participants

$[t_{start}, t_{end}]$ – interval of evaluation

T_j – total time of flight of object j

Assignment Function

N – total number of track numbers (Central Track Store Locators, or CTSLs) ever occurring

$D \subseteq \{1 \ldots N\} X \{1 \ldots M\} X [t_{start}, t_{end}]$ – domain of assignment – the set of triples (n,m,t), where n is a track held by participant m at time t (each CSTL being indexed by a positive integer n from 1 to N)

F:D? Z

F(n,m,t) – assigns to each track at each time t, a positive value representing its assigned object j (or a value of 0 as a convention for indicating that the track is unassigned)

$f_{n,m}(t) = F(n,m,t)$

Track-Specific Attributes

Track Position Accuracy [$PA_{n,m}(t)$] – (possibly weighted) distance from position of track n to position of object j evaluated at scoring time t, where track n is assigned to object j at time t.

Track Velocity Accuracy [$VA_{n,m}(t)$] – (possibly weighted) difference of velocity of track n and velocity of object j evaluated at scoring time t, where track n is assigned to object j at time t.

Track-Specific MOPs

Track ID Assessment [$ID_{n,m}(t)$] – an indication of whether the ID is incorrect (-1), unknown (0), or correct (+1), for an assigned track n held by participant m, at scoring time t.

Track Commonality [$NC_n(t)$] – a boolean variable (=1 if yes, =0 if no), describing whether track n is held by all participants at scoring time t, within some given constraints.

Derived Variables

The derived variables include all variables used in SIAP SE TF Technical Report 2001-001, other than those representing truth data and the attribute measures themselves. They are explicitly derivable from the critical variables listed above, as the formulae show. Once these variables are determined, the attribute measures can be calculated directly from their definitions (which follow, in Table A-1).

Notation: $|A|$ = cardinality of set A
$m(A)$ = measure of set A

$N_C(t)$ – the number of assigned tracks held by all participants at time t $= \sum_{n=1}^{N} NC_n(t)$

$N_S(t)$ – the number of assigned tracks held by at least one participant at time t $= \left|\left\{ n \middle| \exists m, f_{n,m}(t) > 0 \right\}\right|$

$N_m(t)$ – the number of tracks held by participant m at time t $= \left|\left\{ n \middle| (n,m,t) \in D \right\}\right|$

$D_m(t)$ – set of assigned tracks held by participant m at time t

$$= \left\{ n \,\middle|\, f_{n,m}(t) > 0 \right\}$$

$NA_m(t)$ – the number of assigned tracks held by participant m at time t

$$= \left| D_m(t) \right|$$

$I_{j,n,m}$ – set of times for which track n held by participant m is assigned to object j

$$= \left\{ t \,\middle|\, f_{n,m}(t) = j \right\}$$

$T_{j,n,m}$ – the total time of track n on object j by participant m

$$= m\left(I_{j,n,m} \right)$$

$JT_m(t)$ – the number of objects being tracked by participant m at time t

$$= \left| \left\{ j \,\middle|\, j \neq 0 \wedge \exists n, f_{n,m}(t) = j \right\} \right|$$

$I_{j,m}$ – set of times for which a track held by participant m is assigned to object j

$$= \left\{ t \,\middle|\, \exists n, f_{n,m}(t) = j \right\}$$

$TT_{j,m}$ – the total time object j is tracked by participant m

$$= m\left(I_{j,m} \right)$$

$NU_{j,m}$ – the minimum number of tracks needed to cover object j over $TT_{j,m}$

$$= \min\left\{ K \,\middle|\, \exists A \subseteq (1 \ldots N), \left[|A| = K \right] \wedge \left[\bigcup_{n \in A} I_{j,n,m} = I_{j,m} \right] \right\}$$

$TL_{j,m}$ – the total time of the longest track on object j by participant m

$$= \max_n \left(T_{j,n,m} \right)$$

$R_{j,m}$ – number of excess tracks (track number changes)assigned to object j by participant m

$$= \frac{\left(NU_{j,m} - 1 \right)}{TT_{j,m}}$$

R_m – average number of excess tracks assigned by participant m

$$= \frac{\sum_{j=1}^{J} \left(NU_{j,m} - 1 \right)}{\sum_{j=1}^{J} TT_{j,m}}$$

R – roll-up average of R_m

$$= \frac{1}{M} \sum_{m=1}^{M} R_m$$

$D_{j,m}(t)$ – set of tracks held by participant m and assigned to object j at time t

$$= \left\{ n \,\middle|\, f_{n,m}(t) = j \right\}$$

$JC_m(t)$ – number of objects tracked by participant m with correct ID at time t

$$= \left|\left\{ j \middle| D_{j,m}(t) \neq 0 \wedge \left[\forall n \in D_{j,m}(t), ID_{n,m}(t) = 1\right]\right\}\right| *$$

$JI_m(t)$ – number of objects tracked by participant m with incorrect ID at time t

$$= \left|\left\{ j \middle| D_{j,m}(t) \neq 0 \wedge \left[\forall n \in D_{j,m}(t), ID_{n,m}(t) = -1\right]\right\}\right| *$$

$JU_m(t)$ – number of objects tracked by participant m with unknown ID at time t

$$= \left|\left\{ j \middle| D_{j,m}(t) \neq 0 \wedge \left[\forall n \in D_{j,m}(t), ID_{n,m}(t) = 0\right]\right\}\right| *$$

$JA_m(t)$ – number of objects tracked by participant m with ambiguous ID at time t

$$= JT_m(t) - JC_m(t) - JI_m(t) - JU_m(t)$$

*subject to modification in accordance with CRS

	Xm(t)	Xm	X	Range	Target
C*	$\frac{JT_m(t)}{J(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} JT_m(t)}{\sum_{t_{start}}^{t_{end}} J(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} JT_m(t)}{\sum_{t_{start}}^{t_{end}} J(t)}\right]$	0 = C = 100%	100%
A	$\frac{NA_m(t)}{JT_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} NA_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} NA_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}\right]$	A = 1	1
S*	$\frac{N_m(t) - NA_m(t)}{N_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} \left[N_m(t) - NA_m(t)\right]}{\sum_{t_{start}}^{t_{end}} N_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} \left[N_m(t) - NA_m(t)\right]}{\sum_{t_{start}}^{t_{end}} N_m(t)}\right]$	0 = S = 100%	0
LT	--	$\frac{1}{R_m}$	$\frac{1}{R}$	LT > 0	∞
LS*	--	$\frac{\sum_{j=1}^{J} TL_{j,m}}{\sum_{j=1}^{J} T_j}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{j=1}^{J} TL_{j,m}}{\sum_{j=1}^{J} T_j}\right]$	0 = LS† = 100%	100%
PA	$\frac{\sum_{n\in D_m(t)} PA_{n,m}(t)}{NA_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} \sum_{n\in D_m(t)} PA_{n,m}(t)}{\sum_{t_{start}}^{t_{end}} NA_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} \sum_{n\in D_m(t)} PA_{n,m}(t)}{\sum_{t_{start}}^{t_{end}} NA_m(t)}\right]$	PA = 0	0

Table A-1. Formulae for SIAP attributes.

	Xm(t)	Xm	X	Range	Target
VA	$\frac{\sum_{n\in D_m(t)} VA_{n,m}(t)}{NA_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}}\sum_{n\in D_m(t)} VA_{n,m}(t)}{\sum_{t_{start}}^{t_{end}} NA_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}}\sum_{n\in D_m(t)} VA_{n,m}(t)}{\sum_{t_{start}}^{t_{end}} NA_m(t)}\right]$	VA = 0	0
CID*	$\frac{JT_m(t)-JU_m(t)}{JT_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}}\left[JT_m(t)-JU_m(t)\right]}{\sum_{t_{start}}^{t_{end}} JT_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}}\left[JT_m(t)-JU_m(t)\right]}{\sum_{t_{start}}^{t_{end}} JT_m(t)}\right]$	0 = CID = 100%	100%
IDC*	$\frac{JC_m(t)}{JT_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} JC_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} JC_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}\right]$	0 = IDC = 100%	100%
IDA*	$\frac{JA_m(t)}{JT_m(t)}$	$\frac{\sum_{t_{start}}^{t_{end}} JA_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}$	$\frac{1}{M}\sum_{m=1}^{M}\left[\frac{\sum_{t_{start}}^{t_{end}} JA_m(t)}{\sum_{t_{start}}^{t_{end}} JT_m(t)}\right]$	0 = IDA = 100%	0
CM(t)* = $\frac{NC(t)}{NS(t)}$			CM* = $\frac{\sum_{t_{start}}^{t_{end}} NC(t)}{\sum_{t_{start}}^{t_{end}} NS(t)}$	0 = CM = 100%	100%

*expressed as a percentage

†LS is bounded above by C

Table A-1 (Continued)

APPENDIX B

Further Details of the Assignment Cost Function

This appendix provides a derivation of the assignment cost of Section 3.3.2. In particular, it is shown that equation 1 can be derived through the application of several simplifying assumptions to a more general (more idealized) formula. The more general cost function requires knowledge of covariance error data at each scoring time. Because such data may not always be reliable (or available), the idealized cost function is not as easily implemented as the practical cost function prescribed in Section 3.3.2.

The general cost function (and the approach) described in this appendix can serve as a starting point for experimentation with alternative assignment algorithms.

Near-Ideal Unadjusted Cost Function

If full, accurate, track state error covariance matrices were available for all tracks , then a very good unadjusted cost function would be

$$C_k(i,j) = \Delta\mathbf{x}_{i,j}{}^T \Sigma\mathbf{P}_{i,j}{}^{-1} \Delta\mathbf{x}_{i,j} - G(n,\alpha) \qquad \text{(B-1)}$$

where

$\Delta x_{i,j} = x_i - x_j$ for track state column vector x_i and truth state column vector x_j, both consisting of n comparable states,

$\Sigma\mathbf{P}_{i,j} = (P_i + P_j)$ and is an nxn matrix for track i error covariance P_i and truth j knowledge error covariance P_j, (in simulations where truth is known exactly, $P_j = 0$),

$G(n, \alpha)$ is the α percentile value of a chi-squared statistic with n degrees of freedom.

The gate value for the assignment is zero, because $G(n, \alpha)$ is in the cost.

This formulation would allow normalized assignment of tracks having different dimensionality, such as horizontal-only four-state tracks, full position/velocity six-state tracks, and full position/velocity/acceleration nine-state tracks. It would allow use of all the information in a local track file together with the more limited available information in a network track. Unfortunately, full error covariance information is not usually available for all tracks and, where it is be available (e.g., some local tracks), it may not be credible.

The adjusted cost $C_{ak}(i,j)$ at scoring time k for assigning track i to truth object j is

$$C_{ak}(i,j) = C_k(i,j)\, A_k(i,j) \qquad \text{(B-2)}$$

where $C_k(i,j)$ is the unadjusted cost of assigning track i to truth object j at scoring time k,

$$A_k(i,j) = \begin{cases} d_1 \ (0.0 < d_1 < 1.0) \text{ if } C_k(i,j) > 0 \text{ and i was assigned to j at scoring time k - 1} \\ d_2 \ (1.0 < d_2) \text{ if } C_k(i,j) < 0 \text{ and i was assigned to j at scoring time k - 1} \\ 1.0 \text{ otherwise.} \end{cases} \quad \text{(B-3)}$$

Practical Unadjusted Assignment Cost Function

When considering aerospace vehicle tracks, as already noted, full credible track error covariance will generally not be available. For single-sensor tracks for which the source sensor is collocated with the participant, the track error covariance matrix may be imputed from track quality on the basis of assumptions regarding the orientation of principal axes. For some remote tracks however, there is insufficient information from which to determine principal directions (e.g., the track is a composite track or the location of the source sensor is unknown), so it is not even possible to impute the full error covariance. Furthermore, remote tracks may only include latitude/longitude and track-centered local horizontal velocity states (altitude can be marked as unreliable and J3.2 has no altitude rate field).

On the other hand, because they may assume reporting responsibility, tactical command and control systems must be able to represent air vehicle local tracks states in latitude/longitude and track-centered local horizontal velocity states. Also, truth data can be represented in any convenient coordinate frame

Because for air vehicle tracks full error covariance is, in general, not available or not credible, and because some tracks contain only four horizontal states, it is recommended that track and truth be represented as state vectors in track-centered, stabilized east-north-up (ENU) Cartesian reference frames (a different frame for each track). For track-to-truth assignment, use only the horizontal position and velocity states, i.e.,

$$\begin{aligned} \Delta x_{i,j} &= [(Px_i - Px_{i,j}), (Py_i - Py_{i,j}), (Vx_i - Vx_{i,j}), (Vy_i - Vy_{i,j})]^T \\ &= [(0 - Px_{i,j}), (0 - Py_{i,j}), (Vx_i - Vx_{i,j}), (Vy_i - Vy_{i,j})]^T \\ &= [(0 - Px_{i,j}), (0 - Py_{i,j}), (S_i \sin \phi_i - Vx_{i,j}), (S_i \cos \phi_i - Vy_{i,j})]^T \end{aligned} \quad \text{(B-4)}$$

where $Px_{i,j}$, $Py_{i,j}$, $Vx_{i,j}$, and $Vy_{i,j}$ are the truth object horizontal position/velocity states in the track i-centered ENU frame, S_i is the track horizontal speed, and ϕ_i is the track heading measured clockwise from north.

For some local track databases, sufficient data would be available to completely characterize the near-ideal cost function in this four-dimensional simplification. However, this is not generally the case for all CTS tracks. Link 16

J3.2 air track reports, for instance, do not provide enough data to decompose position errors into orthogonal directions. For common treatment of remote and local tracks, then, it is recommended to model track error as independent and equal in the x and y directions of the track-centered ENU frame, i.e., the error covariance is modeled as

$$(\mathbf{P}_i + \mathbf{P}_j) = \Sigma \mathbf{P}i,j = \begin{bmatrix} \sigma^2_{Px} & 0 & \sigma_{Px,Vx} & 0 \\ 0 & \sigma^2_{Py} & 0 & \sigma_{Py,Vy} \\ \sigma_{Px,Vx} & 0 & \sigma^2_{Vx} & 0 \\ 0 & \sigma_{Py,Vy} & 0 & \sigma^2_{Vy} \end{bmatrix} \qquad \text{(B-5)}$$

where $\sigma^2_{Px} = \sigma^2_{Py} = \sigma^2_P$, $\sigma^2_{Vx} = \sigma^2_{Vy} = \sigma^2_V$, and $\sigma_{Px,Vx} = \sigma_{Py,Vy} = \sigma_{P,V}$ and are sums of the corresponding terms in the P_i and P_j error covariance matrices.

Substituting the covariance matrix modeled by B-5 into the general unadjusted cost function given by B-1 (without the gating term), the unadjusted cost becomes the following two-dimensional chi-square distance form:

$$\begin{aligned} C_k(i,j) = \frac{1}{\sigma_P^2(1-\gamma^2)} \Big\{ & \left[(0-Px_j)^2 + (0-Py_j)^2\right] + \left(\frac{\sigma_P}{\sigma_V}\right)^2 \left[(S_i \sin\phi - Vx_j)^2 + (S_i \cos\phi_i - Vy_j)^2\right] \\ & - 2\gamma \left(\frac{\sigma_P}{\sigma_V}\right) \left[(0-Px_j)(S_i \sin\phi_i - Vx_j) + (0-Py_j)(S_i \cos\phi_i - Vy_j)\right] \Big\} \end{aligned} \qquad \text{(B-6)}$$

where $\gamma = \sigma_{P,V}/(\sigma_P\,\sigma_V)$ is the position-velocity correlation coefficient. For steady-state air vehicle tracks, γ and (σ_P/σ_V) are approximately constant for zero predict ahead, with typical values of $\gamma = 0.65$ and $(\sigma_P/\sigma_V) = 4.3$ to 9.0 seconds. The experimental basis of these assertions is addressed in the next section of this appendix.

Given the assumptions, the only unknown value in the recommended unadjusted cost function $C_k(i,j)$ is σ_P. Options for treatment of σ_P include assuming σ_P is constant for all tracks and finding the best gate size through experimentation. The advantage of this approach is simplicity, for no further assumptions are needed. The disadvantage of this treatment is that it does not use all available information to achieve the best track-to-truth reconstruction.

Another option is to estimate $\sigma_P = f[TQ(t)]$, where TQ = track quality. For example, compute $TQ(t_u)$ for local tracks at their last update times t_u before the scoring time t_k. Use the TQ value in remote track reports, assuming it is valid at time t_r of receipt of the report just before t_k. Use a default gate size of 28.5 (about 0.99999 probability gate), or find the best gate size through experimentation. The advantage of this option is if TQ was generated in a universal way, this approach makes best use of remote track data. The disadvantage of this approach is, in the near term, there is no expectation that TQ will be calculated in a universal way.

It is therefore recommended that an empirically estimated σ_P be used for each track source in the scenario. That is, pick objects for which the assignment of local tracks is obvious and unambiguous, calculate the kinematic position accuracy attribute measure with the vertical state weight set to 0, divide the result by the square root of two, and average over such tracks. Use a default gate size of 28.5 (about 0.99999 probability gate) or find the best gate size through experimentation. The advantage of this option is that it provides about the best that can be done in the absence of credible covariance and universally calculated TQ. Unfortunately, this approach also requires more analysis and is process intensive.

The adjusted cost function presented in Section 3.3.2 of the main text is that represented by equations B-2, B-3, and B-6 above, along with the recommendation just described for empirical estimation of σ_P.

Steady-State Covariance Values

In this section, background details and experimental data are provided in support of the claims made earlier regarding the approximate constancy of certain steady-state covariance parameters.

The Kalman filter algorithm is commonly used to estimate the state and error covariance of a system from the measurements. The error covariance is a measure of the uncertainty associated with the state estimate. For target tracking, the error covariance is an $n \times n$ symmetric matrix, where n is the dimension of the state vector. The diagonal elements of the covariance matrix are the mean square errors associated with the state vectors. The off-diagonal terms of the covariance matrix are indicators of the cross-correlation between the corresponding errors of the state vector. Under certain circumstances, the error covariance can reach a steady state and become time-invariant. The steady-state error covariance is assumed in computing the assignment costs in the tracks-to-truth assignment step in computing SIAP attribute measures.

The error covariance for a conventional Kalman filter can be computed independent of the measurements, provided that measurement errors are known at each time and the state and measurement equations are implemented in the same coordinate frame. The time interval between updates and the errors for the state and measurement models must all remain constant in that coordinate frame to obtain a steady-state error covariance for a given set of track filter parameters. Not all of these assumptions are realized in air vehicle tracking situations, but making these assumptions permits the derivation of values needed in the tracks-to-truth assignment cost function.

A spherical shaped measurement error volume was employed to simplify derivation of the values need in the tracks-to-truth assignment cost function (this maximizes the amount of uncertainty applied to each coordinate state). Under this assumption, the state and measurement models are decoupled between each of the coordinates, and the measurement error is also independent of sensor position. The tracks-to-truth assignment cost function needs covariance values for position and velocity states of each coordinate, so a state model with constant velocity dynamics is employed. The Kalman filter with a constant velocity motion model that attains steady-state conditions is often referred to as an alpha-beta filter (Kalata, 1984). The alpha and beta values can be used to calculate the Kalman gain for the track filter. The closed-form solution is determined by computing the error covariance at the current time using the Kalman filtering equations, equating the current error covariance with the previous error covariance, and then solving for the values of alpha, beta, and the elements of the covariance matrix.

Each set of tracking parameters will determine unique values of alpha and beta. The tracking parameters needed to compute the error covariance are the measurement time interval and the errors for the state and measurement models. The measurement time interval and the measurement model error are functions of the sensor and usually cannot be adjusted by the tracking algorithm. The state model error accounts for the expected amount of mismatch that exists between the state model and the true model. The state model error is a design parameter which is usually chosen for a balance between the needs of providing very accurate state estimates when the target is not maneuvering and minimizing the amount of error during maneuvering periods.

The steady state error covariance values for a 1-sec measurement interval (T_m), a measurement error (σ_m) of 250 meters, and a state model error (σ_w) of 25 m/s$^{1.5}$ are presented in Figure B-1. Notice the covariance values obtain a steady-state condition within just a few of updates after initialization. The covariance value for position is indicated by σ_p^2, the value for velocity is indicated by σ_v^2, and the value for the cross-correlation value between position and velocity is indicated by σ_{pv}. The tracks-to-truth assignment cost function needs two quantities to be computed from the covariance values. One is the ratio between the position and velocity standard deviations and is computed as σ_p / σ_v. The second is the position-velocity correlation coefficient (γ) and it is computed as $\sigma_{pv} / \sigma_p \sigma_v$. The values of σ_p / σ_v and γ computed from the covariance values of Figure B-1 are presented in Figure B-2. These typical values of $\sigma_p / \sigma_v \approx 4.3$ sec and $\gamma \approx 0.65$ can be used directly in the tracks-to-truth assignment cost function, provided (1) the method of time aligning for computation of the cost function is to interpolate the truth states back to the time of the last update of the track and (2) the track is a local track (so the update time is the track filter update time) or the track a remote-only track in which the time stamp (often a receipt time stamp) is very close to time in which the track filter on the remote platform was updated with a sensor measurement.

If the track states are predicted for significant time (e.g., remote-only track states were predicted due to delay in being transmitted, or the alternative method of assignment time aligning is employed in which tracks are predicted to the scoring time), then data for the tracks-to-truth assignment cost function need to be predicted to the desired time and the values of σ_p / σ_v and γ computed for the assignment cost function. Figure B-3 presents these values computed as a function of the prediction time interval. Notice that the value of σ_p / σ_v increases in a near linear manner while the value of γ remains essentially constant for this set of tracking parameters. The steady-state error covariance elements and the computed and predicted values of σ_p / σ_v and γ are presented in Figures B-4, B-5, and B-6 for T_m= 10 sec, σ_m= 500 meters, and σ_w = 15 m/s$^{1.5}$. This set of results exhibits the same trend as the first set but the corresponding value of σ_p / σ_v for this set are larger due to the significantly increased measurement error and sample interval.

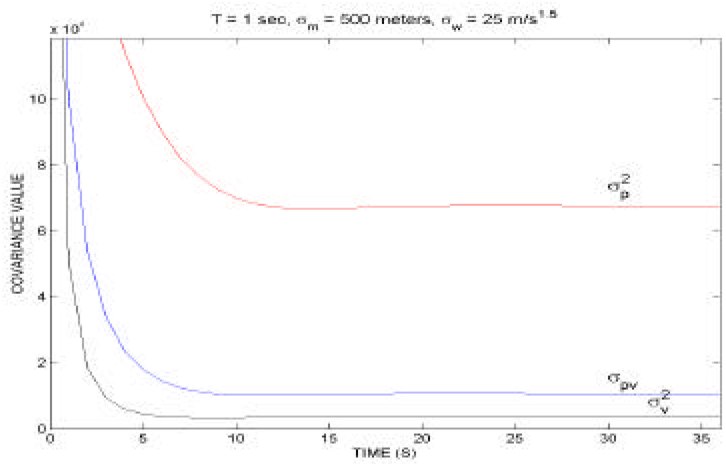

Figure B-1. Covariance values for parameter set 1.

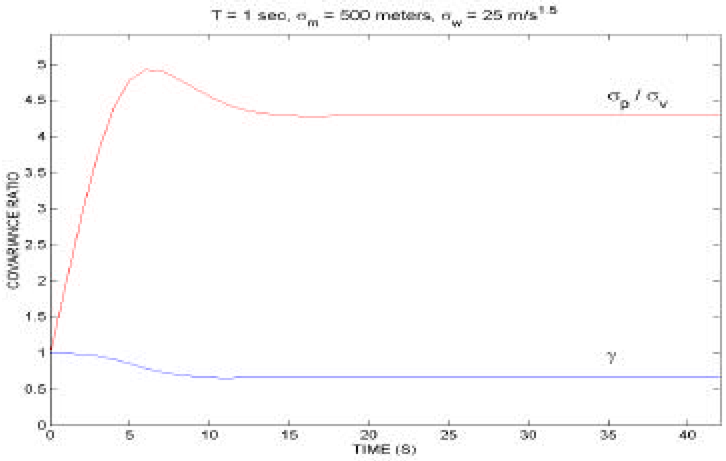

Figure B-2. Values of σ_p / σ_v and γ for parameter set 1.

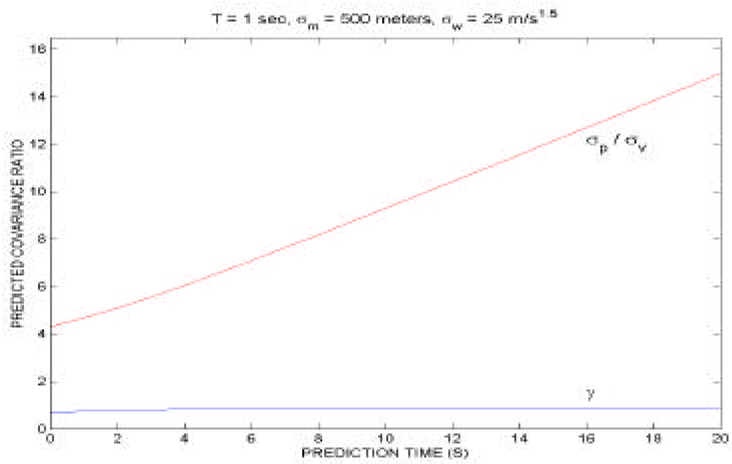

Figure B-3. Predicted values of σ_p / σ_v and γ for parameter set 1.

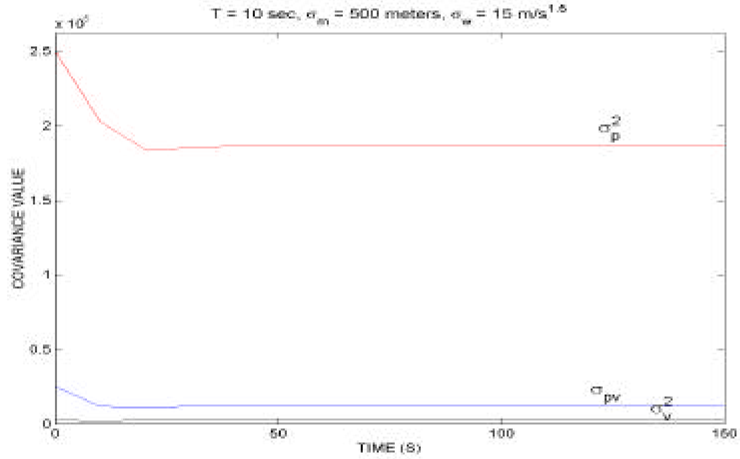

Figure B-4. Covariance values for parameter set 2.

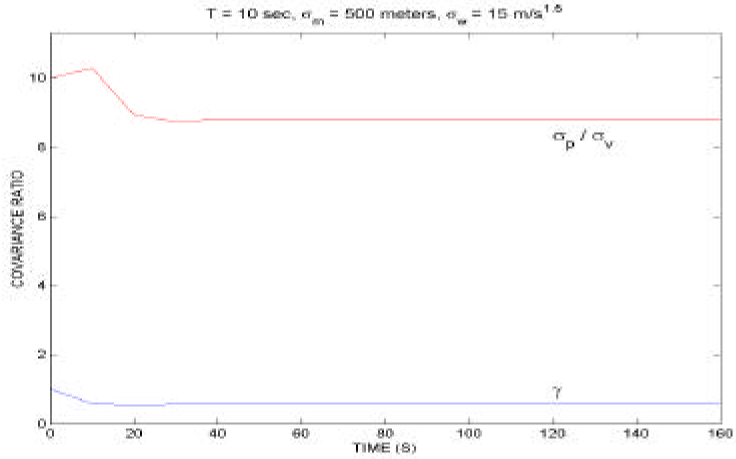

Figure B-5. Values of σ_p / σ_v and γ for parameter set 2.

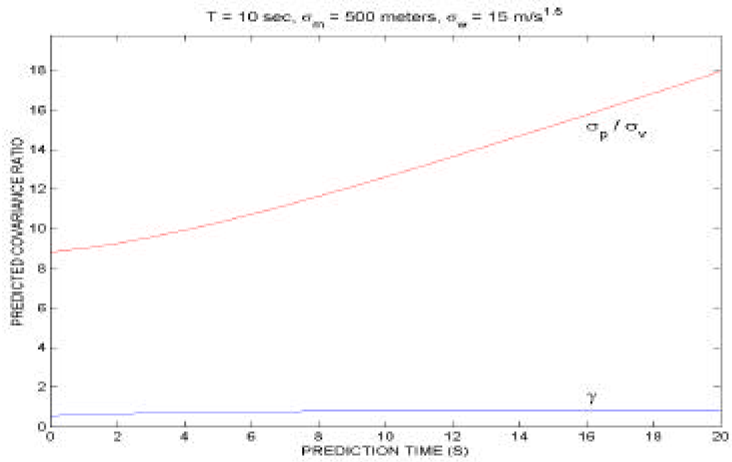

Figure B-6. Predicted values of σ_p / σ_v and γ for parameter set 2.

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APPENDIX C

Interpolation of Air Vehicle Truth Trajectory Data

Air vehicle truth trajectory data used for SIAP assessments is expected to be scripted or recorded at discrete times. To compute assignment costs for tracks-to-truth assignment and subsequently to score tracks for kinematic accuracy, as will as to compute several measures of performance, the air vehicle truth trajectories must be interpolated to particular times, such as track update times or scoring times, that fall between the discrete times at which the truth trajectory data are recorded. This appendix outlines practical recommended methods that have been used successfully for interpolation of truth trajectory data.

The problems of interpolating truth trajectory data are substantially different for environments such as simulations in which the truth state data are scripted versus empirical situations such as open-air exercises in which the truth data are collected from instrumentation or navigation subsystems on board the air vehicles or by designated sensors such as test range radars. Scripted truth data have no errors, except for round-off error from truncated precision, and the scripted data can include position, velocity, acceleration, and any other states that might be of interest for tracks-to-truth assignment or scoring. In most circumstances, all that are needed to interpolate between scripted truth data are polynomial fits to the truth data. In contract, empirical data always contains measurement or estimation errors, may only include position data, and sometimes include spurious data. For empirical data, some form of filtering and smoothing is usually warranted to reduce the position errors, estimate the velocity states, and gate out spurious data, and the solutions from these filtering and smoothing processes form the best basis for state interpolation. This appendix treats these two types of interpolation separately.

Interpolation of Scripted Truth Trajectory Data

Scripted air vehicle truth trajectory data are typically recorded at 1 to 10 Hz data rate (1.0 to 0.1 second intervals) and include a time and the air vehicle position, velocity, and acceleration in three orthogonal coordinates. (It may also include body attitude unit vectors and even body attitude angle rate data, but these are not of interest in this annex.) The formulas in this annex assume the truth trajectory state data are provided in, or have been converted to, a convenient Cartesian reference frame, such as the earth-centered, earth-fixed (ECEF) coordinates defined in the World Geodetic Standard 1984 (WGS 84) or an east-north-up (ENU) Cartesian frame centered on a fix latitude, longitude, zero altitude location on the WGS 84 ellipsoid. SIAP assessments with scripted truth data need air vehicle truth states at any time during the duration of a scenario, so interpolation of truth state data must be performed.

The interpolation approach in this section is based on using only the truth state data recorded at the bounding scripted times, referred to here as T_{last} and T_{next}. That is, for interpolation in an interval between T_{last} and T_{next}, the interpolation scheme will not use truth state data recorded at times before T_{last} or after T_{next}. This avoids complexity of having to invoke different interpolation methods at the beginning and end of scenario times, when for one side there is no additional state data recorded. The interpolation formulas also meet the requirement that, in the limit as time approaches one of the bounding times with recorded data, the interpolation scheme returns the truth state values exactly as recorded at the bounding time. This ensures that states will be continuous across the entire scenario time. Finally, the interpolation formulas minimize errors due to both kinematic model mismatch and recorded data precision, while being computationally practical. This is achieved by assuming a constant jerk rate kinematic model (i.e., a fouth order polynomial) during the interval, but making full use of position, velocity, and acceleration state data recorded at the bounding times (i.e., six data values). Since only five coefficients must be solved for a fourth order polynomial and six data values are available, smoothing is performed to mitigate round-off error in the data values.

Assuming a constant jerk rate kinematic model is justified from both theoretical and experimental perspectives. Constant jerk (i.e., constant acceleration rate) during an interval is compatible with linear change of air vehicle body axes during that interval, i.e., with linear change in air vehicle body attitude, there will be linear change in the angle of attack of aerodynamic lift surfaces and a corresponding approximately linear change in the rate of lift force, giving rise to a linear change in acceleration. If the time interval between recorded truth data is small enough, changes in body axes can indeed appear to be linear within the interval, indicating that a constant jerk (i.e., third order polynomial) kinematic model would be adequate over that interval. However, as the time interval grows larger, the body axis change tends to look more parabolic with time than linear, meaning that a constant jerk rate (i.e., fourth order polynomial) kinematic model is more appropriate. In experimental tests of the ability to reproduce closely the position and velocity states from a full 6-degree-of-freedom aerodynamic object model by interpolating with position, velocity, and acceleration data, it was found that a constant jerk rate kinematic model did support significantly larger time intervals before accuracy tolerances were exceeded. This could be taken a step further to argue that a fifth order polynomial kinematic model would permit even larger time intervals. However, solving for the coefficients of a fifth order polynomial requires all the position, velocity, and acceleration data values at the bounding times, leaving no option for smoothing to mitigate round-off error, as is possible using a fourth order polynomial. Tolerance against round-off error is considered a useful characteristic of the interpolation scheme, so a constant jerk rate (fourth order polynomial) kinematic model is a good compromise.

Position Interpolation from Scripted Truth Data

Since constant jerk rate is assumed during the interval, the equation for position is a fourth order polynomial in time, having five parameters (coefficients). However, there are a total of six truth states available from the boundary conditions: position, velocity, and acceleration at T_{last} and position, velocity, and acceleration at T_{next}. Thus, many different constant-jerk rate solutions are feasible and they can be blended to mitigate substantially the effects of round-off error in the recorded truth data at the boundaries. The criteria employed for selecting the position interpolation scheme that is presented here were to (1) minimize the worst-case error and then (2) minimize the square root of the average error variance over the entire interval. After examining many different combinations, it was found that the weighted sum of a solution formed by using the boundary conditions {P_{last}, V_{last}, A_{last}, P_{next}, V_{next}} and one formed by using the boundary conditions {P_{last}, V_{last}, P_{next}, V_{next}, A_{next}}, referred to as the left-side and right-side solutions, respectively, had better (possibly the best feasible) potential for minimizing errors. This is the scheme described below.

For symmetry, define T_{rel} as the time relative to the midpoint time of the interval,

$$T_{rel} = T - \frac{T_{next} + T_{last}}{2} \qquad \text{(C-1)}$$

and, for normalizing, define ΔT as duration from the midpoint of the interval to either boundary,

$$\Delta T = \frac{T_{last} + T_{next}}{2} - T_{last} = \frac{T_{next} - T_{last}}{2} \qquad \text{(C-2)}$$

The left-side interpolation formula for position, i.e., the one from the boundary conditions {P_{last}, V_{last}, A_{last}, P_{next}, V_{next}}, is given by

$$
\begin{aligned}
P_{left-side}(T_{rel}) = \frac{1}{16}\Big\{ & \left[11P_{last} + 5P_{next} + 8V_{last}\Delta T - 2V_{next}\Delta T + 2A_{last}\Delta T^2\right] \\
& -4\left[3P_{last} - 3P_{next} + V_{last}\Delta T + V_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right) \\
& -2\left[3P_{last} - 3P_{next} + 6V_{last}\Delta T + 2A_{last}\Delta T^2\right]\left(\frac{T_{rel}}{\Delta T}\right)^2 \\
& +4\left[P_{last} - P_{next} + V_{last}\Delta T + V_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right)^3 \\
& +\left[3P_{last} - 3P_{next} + 4V_{last}\Delta T + 2V_{next}\Delta T + 2A_{last}\Delta T^2\right]\left(\frac{T_{rel}}{\Delta T}\right)^4 \Big\}
\end{aligned}
\tag{C-3}
$$

The right-side interpolation formula for position, i.e., the one from the boundary conditions {P_{last}, V_{last}, P_{next}, V_{next}, A_{last}}, is given by

$$
\begin{aligned}
P_{right-side}(T_{rel}) = \frac{1}{16}\Big\{ & \left[5P_{last} + 11P_{next} + 2V_{last}\Delta T - 8V_{next}\Delta T + 2A_{next}\Delta T^2\right] \\
& -4\left[3P_{last} - 3P_{next} + V_{last}\Delta T + V_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right) \\
& +2\left[3P_{last} - 3P_{next} + 6V_{next}\Delta T - 2A_{next}\Delta T^2\right]\left(\frac{T_{rel}}{\Delta T}\right)^2 \\
& +4\left[P_{last} - P_{next} + V_{last}\Delta T + V_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right)^3 \\
& -\left[3P_{last} - 3P_{next} + 2V_{last}\Delta T + 4V_{next}\Delta T - 2A_{next}\Delta T^2\right]\left(\frac{T_{rel}}{\Delta T}\right)^4 \Big\}
\end{aligned}
\tag{C-4}
$$

The weighted blending of the left- and right-side solutions is then just

$$
P(T_{rel}) = w_{pos}(T_{rel})P_{left-side}(T_{rel}) + [1 - w_{pos}(T_{rel})]P_{right-side}(T_{rel}) \tag{C-5}
$$

for a weighting function $w_{pos}(T_{rel})$. It can be shown that the simple weight $w_{pos}(T_{rel})$ = 1/2 will give the smallest maximum error, but is not optimum with respect to minimizing the average variance over the entire time interval. The average variance can be improved by specially chosen higher order polynomial functions of T_{rel}, but these result in larger maximum error magnitude. Given this dilema and the general desire to keep the interpolation formula simple, a weight of $w_{pos}(T_{rel})$ = 1/2 is recommended.

Velocity Interpolation from Scripted Truth Data

Since constant jerk rate is assumed during the interval, the equation for velocity is a third order polynomial in time, having four parameters (coefficients). In theory, there are a total of six truth states available from the boundary conditions; however, use of position states in the velocity interpolation formulas introduces an extra ΔT in the denominators of terms containing P_{last} or P_{next}, thereby amplifying the effects of their round-off errors. Consequently, unless position states were to be recorded with higher precision in the truth files, the useful set of boundary conditions for velocity interpolation is just four states: velocity and acceleration at T_{last} and velocity and acceleration at T_{next}. Since all four boundary condition are needed as polynomial coefficients, only one feasible solution remains $\{V_{last}, A_{last}, V_{next}, A_{next}\}$.

Using the same T_{rel} and ΔT definitions as for position interpolation, the interpolation formula for velocity is given by

$$
\begin{aligned}
V(T_{rel}) = \left(\frac{1}{4}\right)\Bigg\{ & \left[2V_{last} + 2V_{next} + A_{last}\Delta T - A_{next}\Delta T\right] \\
& - \left[3V_{last} - 3V_{next} + A_{last}\Delta T + A_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right) \\
& - \left[A_{last}\Delta T - A_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right)^2 \\
& + \left[V_{last} - V_{next} + A_{last}\Delta T + A_{next}\Delta T\right]\left(\frac{T_{rel}}{\Delta T}\right)^3 \Bigg\}
\end{aligned}
\tag{C-7}
$$

Interpolation of Empirical Truth Trajectory Data

The determination of the interpolated ground truth position and velocity states for the tracks-to-truth assignment and for performance scoring is much more difficult for empirical data as compared to interpolation of scripted truth data. Interpolation using higher order state derivatives may not be applicable when using empirical data, since the data employed in the truth determination may have large errors that make the use of such an interpolation method unreliable. Moreover, the empirical data may only provide position states. Conventional filtering and least-squares techniques can be employed to estimate the position and velocity states at arbitrary times, but these techniques fall short if very accurate trajectory state reconstruction is needed. For these reasons, smoothing is recommended for estimating (interpolating) ground truth position and velocity from empirical data.

Estimates of the air vehicle true position and velocity states can be obtained by sequentially filtering the position measurements and calling the outputs the ground truth. This can be accomplished using a single kinematic model or more accurately using a multiple model estimation technique, such as the Interacting Multiple Model (IMM) algorithm (Bar-Shalom, Cang, and Blom, 1989; Blair and Watson, 1997; Blom and Bar-Shalom, 1988; Tugnial, 1982). The time at which the state must be known (the interpolation time) will not, in general, coincide with a measurement time. Thus, prediction is required, which will introduce even more errors into the state estimate of ground truth.

Fixed-Interval Smoothing

Fixed-Interval Smoothing (FIS) is a more reliable reconstruction technique when compared to conventional forward-time filtering because it uses the past, present, and future position measurements from the data set to estimate the target state (Blair and Watson, 1997; Helmick, *et. al.*, 1993, 1995, and 1996). Single model and multiple model FIS algorithms have been employed successfully to reconstruct the ground truth trajectories for live experiments (Blair and Watson, 1997, and Watson, 2001). The FIS is accomplished as follows. First, an estimate of the state is computed in the forward-time direction with all the measurements. Next, an estimate of the state is computed in the backward-time direction with all the data. The last step is to combine optimally the forward-time and backward-time estimates to achieve a more accurate state estimate than the individual directions alone. The estimation error at the onset and completion of maneuvers should be much smaller or vastly improved using this method by reducing the error associated with the lag of each filter.

To accomplish the smoothing process, the forward-time and backward-time Kalman filter estimates and associated error covariances at each position measurement time should be stored. (For a multiple model filter, the state estimates, error covariances, and mode probabilities for each model must be stored. There is a significant increase in required storage when using a multiple model approach.) The use of forward and backward filtering is done once and the stored filter estimates and error covariance are then used for all subsequent ground truth state interpolation.

The smoothed estimate must be computed at a particular interpolation time. This requires that the forward-time and backward-time estimates be predicted and properly fused using a minimum variance method. The smoothed estimate at the interpolation time will be more accurate than the conventional filtered states predicted to that time, since future data are employed in the smoothing process. The smoothed state is accomplished according to the following three steps:
Time alignment of the filtered data;
Fusing of the state estimates;
Smoothing measurement update.

Step 1: Time Alignment of Filtered Data

For interpolation time t_k, the forward-time data employed in the smoothing process are selected to be the Kalman filter estimate that is closest to but not exceeding t_k. The backward-time data employed in the smoothing process is selected to be the Kalman filter estimate that is closest to but not preceding t_k. The state prediction process of the Kalman filter is then applied to the forward-time and backward-time filtered estimates to make them coincident with t_k. The following data must be computed from the stored filtered data for a single model track filter:

$$X^f_{k|k_f} = \text{forward - time state estimate predicted to } t_k$$
$$X^b_{k|k_b} = \text{backward - time state estimate predicted to } t_k$$
$$P^f_{k|k_f} = \text{forward - time error covariance predicted to } t_k$$
$$P^b_{k|k_b} = \text{backward - time error covariance predicted to } t_k$$

with k_f and k_b being the times of the most recent forward-time and backward-time estimates, respectively.

Step 2: Fusing of State Estimates

The forward-time and backward-time state estimates and error covariances are fused using a minimum variance approach given by

$$\begin{aligned} X_{k|N} &= P_{k|N}\left(\left(P^f_{k|k_f}\right)^{-1} X^f_{k|k_f} + \left(P^b_{k|k_b}\right)^{-1} X^b_{k|k_b}\right) \\ P_{k|N} &= \left(\left(P^f_{k|k_f}\right)^{-1} + \left(P^b_{k|k_b}\right)^{-1}\right)^{-1} \end{aligned} \tag{C-8}$$

Step 3: Smoothing Measurement Update

The smoothed state estimate and error covariance at t_k, $X^s_{k|N}$ and $P^s_{k|N}$, are computed with the fused values from Step 2 and the measurement at t_k if one exists. There are two cases to consider since a measurement may not be available at t_k.

Case 1: If there is a measurement at t_k, the measurement update is accomplished with the Kalman filtering equations given by

$$\begin{aligned} X^s_{k|N} &= X_{k|N} + K_{k|N}\,\tilde{Z}_{k|N} \\ P^s_{k|N} &= \left[I - K_{k|N} H_k\right] P_{k|N} \end{aligned} \tag{C-9a}$$

where

$$
\begin{aligned}
\tilde{Z}^{s}_{k|N} &= Z_k - H_k\, X_{k|N} \\
K_{k|N} &= P_{k|N}\, H_k^T\, S_{k|N}^{-1} \qquad \text{(C-9b)} \\
S_{k|N} &= H_k\, P_{k|N}\, H_k^T + R_k
\end{aligned}
$$

with

Z_k = measurement at t_k
R_k = covariance of Z_k
H_k = gradient of Z_k with respect to X_k

Case 2: If there is no measurement at t_k, the smoothed state estimate and error covariance are given by

$$
\begin{aligned}
X^{s}_{k|N} &= X_{k|N} \\
P^{s}_{k|N} &= P_{k|N}
\end{aligned} \qquad \text{(C-10)}
$$

Simulation results comparing the FIS with the forward-time and backward-time filters are presented for a single constant velocity model. The simulated target shown in Figure C-1 is employed for this example. The target has an initial range of 83 km, speed of 457 m/s, and altitude of 3.0 km. The air vehicle flies straight and level for the first 30 seconds. A 4-g turn is then performed through a 45 degree course change. Straight and level, non-accelerating flight is continued for the next 30 seconds. A second 4-g turn through a 90 degree course change is performed while the aircraft decelerates to a speed of 274 m/s. Straight and level flight is maintained for the remainder of the flight after the course change is completed. The root-square-error (RSE) of position and velocity computed between the estimates and true states at each measurement time is presented in Figure C-2. The error of the smoother is significantly less than the forward filtering or the backward filtering over the entire length of the track and it is especially noticeable during target maneuvers. The uncertainty in the estimates, represented by the Filter Generated Standard Deviation (FGSD), is also much smaller for the smoother. The FGSD is a measure of how well the filter perceives its own performance. The single model FIS employed in the example illustrates the improved performance that can be obtained for trajectory reconstruction using smoothing when compared to a conventional filtering approach. Even though the error covariance is small, the single model FIS has the same drawback as the conventional Kalman filter by not responding to the target maneuvers. The single model filter does not provide consistent estimates since its error covariance does not increase to reflect the degradation in performance during maneuvers. A multiple model FIS can provide an error covariance that is nearly consistent while at the same time providing more accurate estimates (Blair and Watson, 1997).

For implementation, the forward-time and backward-time estimates will need to be computed and stored prior to the evaluation process in order execute in a timely manner. However, the accuracy of the estimates may be so good that an interpolation scheme could be employed over portions of the reconstructed trajectory using the smoothed estimates obtained for each measurement time. The use of an interpolation scheme would reduce the amount of stored data needed to compute the trajectory state at the interpolation time.

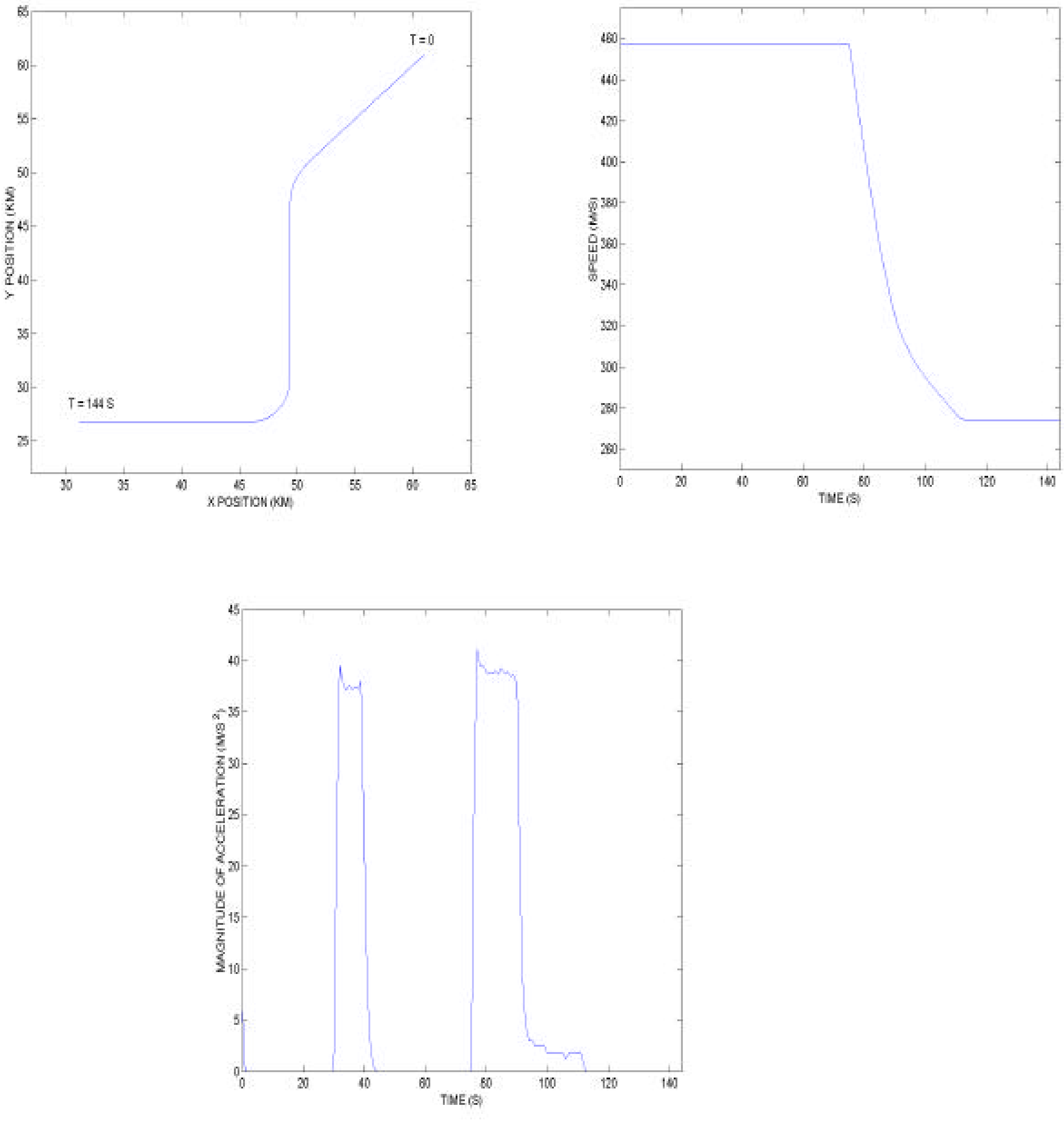


Figure C-1. Target profile.

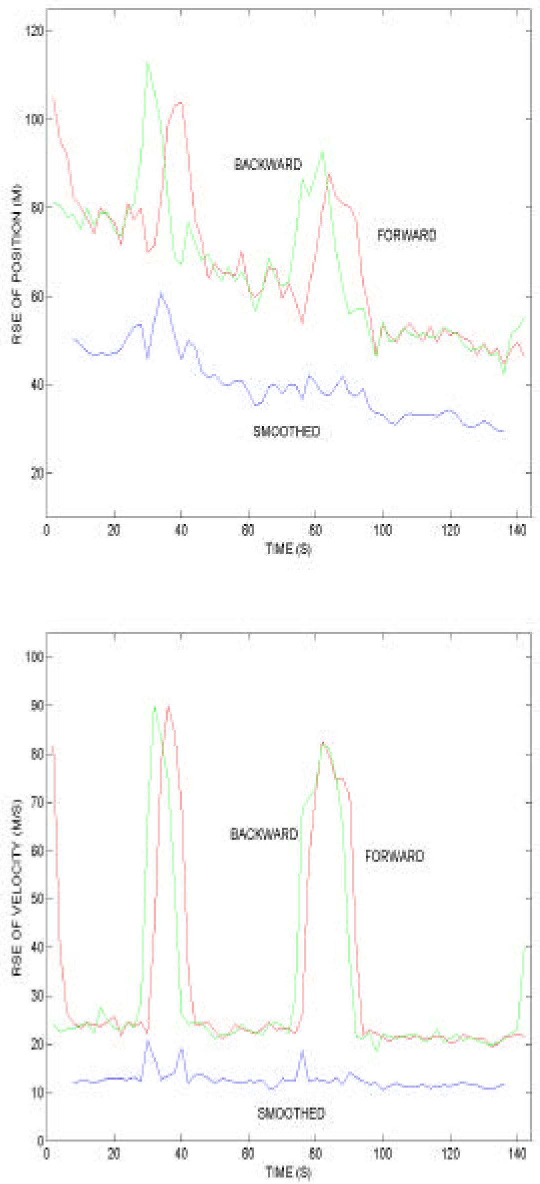
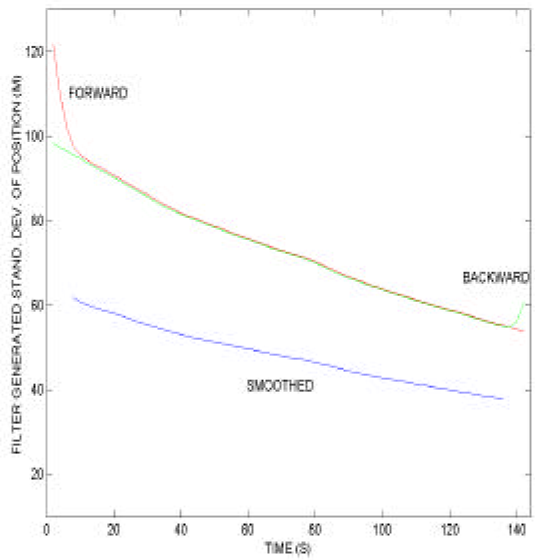


Figure C-2.   Performance comparison.



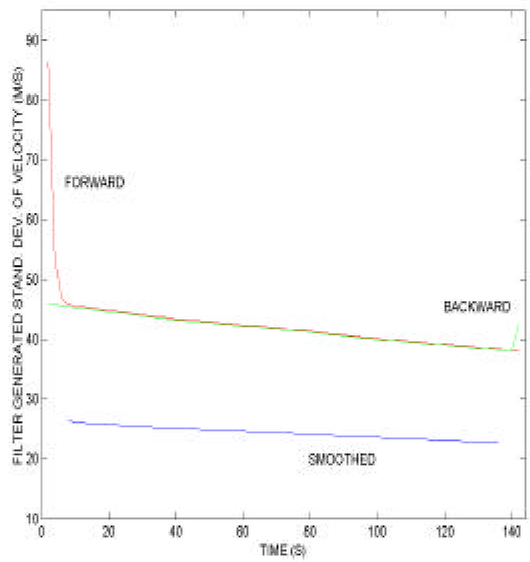


Figure C-2. Performance Comparison. (continued)

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