ANALYSIS OF A DOUBLE FIN-TUBE FLAT CONDENSER-RADIATOR AND COMPARISON WITH A CENTRAL FIN-TUBE RADIATOR

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SUMMARY

An analytical study of a flat condenser-radiator with a double fin-tube geometry (closed sandwich) with variable tube side-wall thickness was performed for a Rankine space-power electric-generating system. The analysis of the double fin radiator included consideration of tube and header pressure drops, meteoroid protection for the tubes and headers, along with a detailed presentation of the heat rejection analysis and total weight characteristics. The double fin-tube radiator is compared to a conventional central fin-tube configuration on a heat rejection to weight basis for a four-panel radiator configuration.

Both fin and tube geometries are compared on the basis of the same power level, working fluid temperature, tube and header pressure drop, radiator material, and meteoroid protection criteria. A beryllium radiator for a 1-megawatt system and a columbium alloy radiator for a 500-kilowatt system, both at a radiating temperature of 1700° R, were chosen for the weight and geometry comparisons.

The conclusion reached indicates a substantial weight savings can be realized with the double fin-tube arrangement if the tube side-wall thickness can be reduced as a result of a possible meteoroid bumper effect of the enclosing fins. Weight reductions compared to the central fin-tube geometry of up to 32 to 39 percent were shown to be possible for the maximum reduction in side-wall thickness in the two examples considered. This result further substantiated the preliminary conclusions given in an earlier reference that compared the double and central fin-tube configurations neglecting the effects of headers, pressure drops, tube wall temperature drop, and powerplant thermodynamic cycle considerations. Thus an incentive is offered to further investigate the meteoroid bumper screen concept and its application in the double fin-tube geometry radiator.

INTRODUCTION

The generation of large amounts of electric power in space using the Rankine cycle concept requires that a large amount of waste heat be rejected
from the working fluid. This waste heat is the amount of energy that must be rejected from the working fluid vapor leaving the turbine in order to completely condense it. Since radiation is the only mode of heat transfer for rejecting this energy and since maximum temperatures are limited, the resulting radiator surface areas and weights are generally large. Previous studies on space radiators (refs. 1 to 5) have indicated that a central fin-tube arrangement is feasible since it reduces radiator weight by reducing the prime surface area vulnerable to critical damage by impacting meteoroids. Reference 6 presents the analysis and results of a central fin-tube radiator using flat plate fins of constant cross section.

A preliminary comparison of several fin-tube configurations was carried out in reference 7 that did not include cycle considerations, vapor and liquid headers, and tube and header pressure drops. The results of reference 7 indicated that a substantial weight savings could be obtained by using a double fin tube with reduced rectangular tube side-wall thickness instead of the central or open sandwich fin-tube arrangements. A reduced tube side-wall thickness can result from the assumption that the two fins of the double fin-tube configuration will act as a bumper screen to incident meteoroids, thus allowing some reduction in the thickness of the tube side wall. The double fin-tube configuration is attractive from a structural viewpoint since it provides a rigid structure that has a continuous smooth surface that could be the vehicle skin. These attractive features motivated the need for a more sophisticated analysis.

The purpose of this study is to analyze in a more comprehensive manner the heat rejection and weight characteristics of a double fin-tube configuration with variable rectangular tube side-wall thickness, and to more accurately identify the potential weight savings over radiators with a central fin-tube geometry. The vapor and liquid headers are also included in the geometry analyzed since they directly affect the meteoroid protection requirements of the radiator tubes and can be a significant portion of the total radiator weight. There is also a sizable amount of heat that can be rejected from the vapor header for large power systems (ref. 1). Working fluid pressure drops were also considered for the tubes and headers.

A rectangular cross-section fin was chosen for the analysis and comparison. A one-dimensional approach was taken in the development of the fin energy balance equation with the assumption that the base temperature of the fin is equal to the outer surface temperature of the tube armor. The radiator heat rejection and weight analysis was carried out for two typical Rankine power-plants: a 1-megawatt potassium cycle with a 1700° R beryllium armor and fin radiator, and a 500-kilowatt potassium cycle with a 1700° R columbium alloy radiator. This report presents the results of the heat rejection and weight analysis for the double fin-tube radiator along with a comparison of these results with those given in reference 6 for the central fin-tube configuration.

**SYMBOLS**

- A: surface area, sq ft
\( A_p \)  radiator planform area, sq ft
\( A_v \)  vulnerable area, sq ft
\( a \)  penetration correction factor
\( B \)  constant
\( C_p \)  specific heat, Btu/(lb)(^\circ F)
\( c_a \)  sonic velocity in armor material, \( \sqrt{\frac{E_a g}{\rho_a}} \), ft/sec
\( D \)  tube diameter, ft
\( E_a \)  Young's modulus of armor material, lb/sq ft
\( F \)  angle factor, fraction of the energy leaving a surface that is incident upon another surface
\( F_{VH} \)  vapor header occlusion factor
\( g \)  units conversion factor, 32.17 ft/sec^2
\( H \)  incident energy, Btu/(hr)(sq ft)
\( h \)  heat of condensation, Btu/lb
\( J \)  mechanical equivalent of heat, 778 (ft)(lb)/Btu
\( K \)  factor, \( \frac{(q_b + q_r)}{\epsilon \pi D_l T_b^4} \)
\( K_H \)  fluid turning loss factor from header to tubes
\( k \)  thermal conductivity, Btu/(hr)(ft)(^\circ R)
\( L \)  minimum half length of fin, equal to \( L^* - R_o \), ft
\( L^* \)  one-half the tube center to center distance, ft
\( l \)  actual half length of fin, ft
\( N \)  number of radiator tubes
\( N_c \)  conductance parameter, \( \sigma l^2 T_b^3/kt \)
\( P \)  cycle fluid pressure, lb/sq ft
\( P_e \)  powerplant output, kw
\( P(0) \)  probability of zero punctures
\( Q \)  heat rejection rate, Btu/hr
$Q_b$  
  tube radiant heat rejection rate, Btu/hr

$Q_f$  
  fin radiant heat rejection rate for a fin length 2l radiating from both sides, Btu/hr

$Q_{Rej}$  
  total radiator heat rejection rate, Btu/hr

$Q_{VH}$  
  vapor header entrance quality

$q_{VH}$  
  vapor header heat rejection rate, Btu/hr

$q$  
  heat rejection per unit length of tube, Btu/hr ft

$R$  
  radius, ft

$R_b$  
  tube side wall to tube centerline dimension, $R_b = R_o - \left(1 - \frac{S_o}{S_a}\right)S_a$, ft

$\phi$  
  fraction of flow area occupied by one phase

$Re$  
  Reynolds number

$T$  
  temperature, °R

$T_v$  
  vapor saturation temperature at tube inlet, °R

$t$  
  thickness of fin, ft

$u$  
  velocity of vapor, ft/sec

$V$  
  velocity of liquid, ft/sec

$\bar{v}$  
  average meteoroid velocity, ft/sec

$W$  
  weight, lb

$\dot{W}$  
  weight flow per tube, lb/sec

$\bar{W}$  
  panel width, ft

$X$  
  normalized distance coordinate, $x/l$

$X_{VH}$  
  fraction of total heat rejected by the vapor header

$X_{tf}$  
  fraction of total heat rejected by the tubes and fins

$x$  
  coordinate measuring distance along lower fin, ft

$Y$  
  normalized distance coordinate, $y/l$

$y$  
  coordinate measuring distance along upper fin, ft
\( Z \) radiator tube length, ft
\( \alpha, \beta \) constants in penetration formula
\( \delta \) tube wall thickness, ft
\( \epsilon \) surface hemispherical emissivity
\( \eta^* \) thermal effectiveness
\( \theta \) normalized temperature, \( T/T_b \)
\( \mu \) viscosity, lb/ft·sec
\( \rho \) density, lb/cu ft (unless otherwise specified)
\( \sigma \) Stefan-Boltzmann constant, \( 0.173 \times 10^{-8} \text{ Btu/(hr)(sq ft)(}^\circ \text{R}^4) \)
\( \tau \) mission exposure time, days
\( \phi, X \) two-phase-flow parameters

Subscripts:
\( a \) armor
\( b \) tube base surface
\( c \) tube liner
\( \text{cond} \) conduction
\( F \) friction
\( f \) fin
\( g \) vapor phase
\( i \) inside
\( \text{LH} \) liquid header
\( l \) liquid
\( m \) momentum
\( o \) outside
\( P \) particle
\( R \) total
rad  radiation
s   side wall
t   tube
tot total flow, liquid and vapor
VH vapor header
X  normalized distance coordinate, x/L
x  coordinate measuring distance along lower fin, ft
Y  normalized distance coordinate, y/L
y  coordinate measuring distance along upper fin, ft
0  conditions at tube inlet
1  base surface 1
2  base surface 2
3  radiator inlet conditions

RADIATOR CONFIGURATION AND THERMODYNAMIC CYCLE

The general radiator panel configuration considered for the analysis is shown in figure 1 where the vapor from the turbine exhaust is distributed to the finned tubes by a vapor header. The heat radiated from the vapor header and finned tubes causes the vapor to condense. The condensate is then sub-cooled and collected in the liquid header before being sent to the condensate pump. This scheme can be modified by dividing the radiator into a number of non-redundant segments, each of which could be treated as a separate entity.

The detailed cross-section drawing of the double fin-tube with variable tube side wall composed of tube liner inserted in an armor block, which provides meteoroid protection, and two rectangular fins is shown in figure 2. The tube liner, which is exposed to the working fluid, must be capable of withstanding possible corrosion. The liner thickness must also be compatible with current fabrication capabilities and structural requirements, and the presence of the liner may also substantially reduce the required armor thickness (ref. 8). The liner thickness is arbitrarily taken as $t = 0.040 D_1$ with a minimum wall thickness set at 0.020 inch. The liner thickness was increased as the inside diameter was increased, so as to provide necessary stiffening and strengthening of the radiator tubes.

For the double fin geometry of figure 2, the liner can be damaged by im-
pacting meteoroids in two general ways. The first is by any primary impacts occurring on the outer exposed surfaces of the tube block. These impacts are assumed to obey the conventional armor penetration and damage relations developed for tubes (refs. 8 to 10), with vulnerable area given by \( 4R_b zN \). Accordingly, the armor thickness \( \delta_a \) was determined using the criterion of reference 9. The armor thickness, which is a result of the optimization program (ref. 6), is applied in full on the upper and lower surface of the tube. A second damage source can arise from a spray of particles on the armor block side surface \((D_a - 2t)zN\) resulting from impacts on the fin surfaces. In view of the bumper action involved and the obliquity of the secondary impacts, however, a reduction will undoubtedly be allowed in the armor thickness required by the tube block side wall to resist the effects of these secondary impacts. Since no specific relations are at present available for the determination of this side-wall thickness, a parametric variation of the \( \delta_s/\delta_a \) is used to examine the effects of reduced side-wall thickness on radiator weight and geometry. The parameter \( \delta_s/\delta_a \) is defined as the fraction of the armor thickness retained on the enclosed side of the tube block.

The vapor header takes the form of a hollow paraboloid whose wall consists of 0.12-inch-thick liner, which was arbitrarily chosen, and whose meteoroid armor protection thickness is the same as that required by the tubes. The parabolic shape insures constant velocity in the header. For simplicity, the liquid header was designed with a constant diameter and a fluid velocity of 4 feet per second, so that a very low pressure drop would result. The liner for the liquid header follows the same schedule with the inside diameter as do the tube liners. However, a maximum liner thickness is set at 0.12 inch. The liquid header also has meteoroid armor.

The assumptions given herein for the double fin-tube radiator geometry, with the exception of the fin-tube configuration, also hold for the central fin tube that was analyzed in reference 7 and shown in figure 3.

The thermodynamic cycle used in this analysis is the Rankine cycle, which uses a working fluid that undergoes a change of phase. The working fluid is condensed in the radiator, which results in a near isothermal condition prevailing in the tubes and vapor header. In order to show sample results and compare the two fin-tube geometries on a heat rejection per unit weight and on a radiator geometry basis, two power levels were chosen. Potassium was chosen as the working fluid in the cycle for both power levels with a peak turbine inlet temperature of 2460° R and a radiator temperature of 1700° R. It was also specified that the radiator tubes would subcool the working fluid 100° R. Additional cycle requirements such as turbine and generator efficiencies were set at 0.75 and 0.90, respectively, with 10 percent of the generator output required for accessories and controls. The emittance of the radiator was taken to be 0.90, and the effective sink temperature for the radiator was assumed to be 0° R. The foregoing values plus the cycle temperatures, working fluid, and the power level chosen for the analysis and comparison enabled the determination of the total heat rejection rate and the mass flow rate of the working fluid for the radiator design inputs. Additional information required from the cycle analysis is the quality of the working fluid entering the vapor header. The analysis of the thermodynamic cycle used is given in detail in reference 5.
HEAT-TRANSFER ANALYSIS

Approach and Assumptions

The analysis considers the general case of two rectangular profile fins of length \( l \) attached to a tube enclosed in an armor block forming a double fin configuration as shown in figure 2. Energy input to the fin is comprised of heat conduction along the fin from the two tube side-wall surfaces. For any specific choice of \( L/R_0 \), the fin length \( l \) as shown in figure 2 will depend on the value of tube side-wall thickness.

The specific assumptions used in the development of the heat-transfer relations for the double fin-tube geometry as well as the central fin-tube geometry are

(1) Radiator emittance is constant with temperature.

(2) For the determination of radiator temperature variation the radiator surfaces act as blackbodies with incident and emitted radiation governed by Lambert’s cosine law.

(3) Hemispherical radiation to space from both outer surfaces of the radiator to a 0° R space sink temperature.

(4) The base surface temperature of the fin is assumed constant along the length of the tube and equal to the tube block outer surface temperature.

(5) Steady-state one-dimensional heat flow exists in the fins and tube block.

(6) Material properties are constant along the length of fin and tube block and are evaluated at the fin base temperature.

(7) The development of the fin and tube angle factors for the radiant interchange between fin and tube side walls is based on an infinite length of tube and fin, with fin thickness assumed negligible in the development.

(8) The inside tube wall temperature is circumferentially uniform and equal to the stagnation temperature of the fluid at the inlet of the header.

Tube Wall Temperature Drop

Since the tube block will have thick walls due to the armor required for meteoroid protection, a significant temperature drop will occur across the block. The assumed heat-transfer paths and the various fin and tube block temperatures are shown in figure 4. The inside tube wall temperature \( T^* \) is determined from reference 6 as

\[
T^* = T_5 \left(1 - \frac{1}{2} \frac{K_n u_0^2}{J gh}\right)
\]  

(1)
where \( T_3 \) is the radiator fluid stagnation temperature at the header inlet in degrees Rankine and \( u_0 \) is the tube inlet vapor velocity. The turning loss factor \( K_H \) in equation (1) was set at 1.15 (ref. 6).

The relation between the tube inside surface temperature and the tube block outer surface temperature is based on a simplified approach that assumed heat is transferred from the tube inner surface to the exposed surface of the tube block by one-dimensional conduction. This heat path was chosen since it represents the greatest flow of energy. The heat transmission by conduction is assumed to travel a distance \( \delta_a \) through a cross-sectional area \( 2R_b \) d\( Z \). The expression for the heat conduction is

\[
dq_{\text{cond}} = \frac{2kR_b}{\delta_a} (T^* - T_b) \, dZ
\]  

(2)

For simplicity it is assumed that the temperature drop \( T^* - T_b \) is governed primarily by the radiant heat transfer from the exposed surface of the block (neglecting the conduction to the fins). The expression for radiation from one surface element may be written as

\[
dq_{\text{rad}} = 2\varepsilon R_b T_b^4 \, dZ
\]

(3)

When equations (2) and (3) are combined for \( dq_{\text{cond}} = dq_{\text{rad}} \), the resultant equation for the approximate temperature drop through the tube block wall is given as

\[
\varepsilon \delta_a T_b^4 + k(T_b - T^*) = 0
\]

(4)

The temperature \( T_b \) obtained from equation (4) is then used in the development of the fin heat-transfer relations. The temperature of the tube side wall is also assumed to be equal to the temperature of the tube block exposed surface \( T_b \) regardless of the thickness of the tube side wall (all values of \( \delta_s/\delta_a \)).

Fin Temperature Profile

Formulation of equations. - Considering an element of the fin surface in figure 2 and employing the previous assumptions, the law of energy conservation for an element can be expressed as the energy balance between the net heat transfer due to conduction and radiation (ref. 11):

\[
dq_{\text{cond}} + dq_{\text{rad}} = 0
\]

(5)

The net internal heat conduction through the element of thickness \( t \) and length \( Z \) is expressed as

\[
dq_{\text{cond}} = \frac{d}{dx} \left[-kt Z \frac{dT}{dx}\right] dx
\]

(6)
The net radiant heat rejection from both sides of the element of the fin is composed of its emission minus the incident energy. This expression is given as

$$dQ_{\text{rad}} = \left(2\sigma T_x^4 - H_x\right)dA_x$$  \hspace{1cm} (7)

The incident energy term $H_x$ in equation (7) is composed of the incident energy from the two side walls to an element of area on the fin

$$\sigma T_b^4 \left(F_dA_{x-1} + F_dA_{x-2}\right)$$  \hspace{1cm} (8a)

and the energy leaving an opposing fin surface incident upon the other

$$\int_{y=0}^{2l} \sigma T_y^4 dF_{dA_{x-1}-dA_y}$$  \hspace{1cm} (8b)

Substitution of expressions (8a) and (8b) into equation (7) yields

$$dQ_{\text{rad}} = \left[2\sigma T_x^4 - \sigma T_b^4 \left(F_dA_{x-1} + F_dA_{x-2}\right) + \int_{y=0}^{2l} \sigma T_y^4 dF_{dA_{x-1}-dA_y}\right]dA_x$$  \hspace{1cm} (9)

Introducing equations (6) and (9) into equation (5) and the dimensionless variables

$$X = \frac{x}{l}$$

$$Y = \frac{y}{l}$$

$$\theta = \frac{T}{T_b}$$

$$N_c = \frac{\sigma \alpha b^2 z^2}{kt}$$

yield the expression

$$\frac{1}{N_c} \frac{d^2\theta}{dx^2} = \sigma^4(X) - \left(F_dA_{x-1} + F_dA_{x-2}\right) - \int_0^2 \sigma^4(Y) dF_{dA_x-dA_y}$$  \hspace{1cm} (10)

The angle factors in equation (10) are evaluated using a relation (ref. 12, eq. (31-58)) that applies to parallel surfaces of infinite length. For the configuration of figure 2 the angle factors in question are given as
\[ dF_{dA_x}dA_y = \frac{2(\frac{R_o}{t})^2}{(Y - X)^2 + 4(\frac{R_o}{t})^2} \, dY \]  
(11)

\[ F_{dA_x-1} = \frac{1}{2} \left\{ 1 - \frac{X}{\left[ X^2 + 4(\frac{R_o}{t})^2 \right]^{1/2}} \right\} \]  
(12)

\[ F_{dA_x-2} = \frac{1}{2} \left\{ 1 - \frac{2 - X}{\left[ (2 - X)^2 + 4(\frac{R_o}{t})^2 \right]^{1/2}} \right\} \]  
(13)

Introduction of equations (11), (12), and (13) and the symmetry of the temperature distribution about \( x = l \), that is, \( T(x) = T(2l - x) \), into equation (10) yield the expression

\[ \frac{1}{N_c} \frac{d^2\theta}{dx^2} = 2\theta^4(x) - 1 + \frac{1}{2} \left\{ \frac{X}{X^2 + 4(\frac{R_o}{t})^2}^{1/2} + \frac{2 - X}{(2 - X)^2 + 4(\frac{R_o}{t})^2}^{1/2} \right\} \]

\[ - 2(\frac{R_o}{t})^2 \int_0^1 \theta^4(x) dY \left\{ \frac{1}{(Y - X)^2 + 4(\frac{R_o}{t})^2}^{3/2} + \frac{1}{(2 - Y - X)^2 + 4(\frac{R_o}{t})^2}^{3/2} \right\} \]  
(14)

where the actual fin length \( l \) is given by the expression (fig. 2)

\[ l = L + \left( 1 - \frac{\delta}{\delta_a} \right) \delta_a \]  
(15)
Computational procedure. - It was necessary to use numerical techniques to solve equation (14) for the temperature profile of the fin of length \( l \). The numerical solutions were carried out on an IBM 7094 electronic digital computer.

The second order differential equation \( \frac{d^2 \theta}{dx^2} = B \theta^4 + f(x) \) was solved for the \( \theta \) profile in the interval \( X = 0 \) to \( X = 1 \). The boundary conditions are \( \theta = 1 \) at \( X = 0 \) and \( \frac{d\theta}{dx} = 0 \) at \( X = 1 \), where \( B \) is a constant and \( f(X) \) is a function of \( X \). Kalaba's method was used and is now described.

The term \( \theta^4 \) was approximated by a linear function of \( \theta \), namely by the first two terms of its Taylor series expansion. Central differences were used to obtain an expression for \( \frac{d^2 \theta}{dx^2} \). At each point of a mesh on the interval \( X = 0 \) to \( X = 1 \), the equation \( \frac{d^2 \theta}{dx^2} = B \theta^4 + f(X) \) was thus approximated as a linear function of \( \theta \). The set of resulting tridiagonal linear equations was reduced to two-diagonal form, and these were then solved for the \( \theta \) profile by backward substitution. An initial \( \theta \) profile guess was used, and each successive iteration yielded approximately one decimal place of accuracy. Solutions were obtained as a function of the input parameters \( N_c \) and \( l/R_0 \).

Temperature profile results. - Each solution of the fin energy equation, which is independent of power and temperature level or tube inside diameter, provided a temperature distribution along the fin. Results are plotted in figure 5 as a function of position \( X \) on the fin for several parametric values of \( N_c \) and \( l/R_0 \). It is seen that the temperature drop along the fin is very small when the conductance parameter \( N_c \) is small, which indicates a low thermal resistance of the fin. Additionally, there is little effect of \( l/R_0 \) when \( N_c \) is small, although a greater temperature drop occurs with increasing \( l/R_0 \) ratio. This holds regardless of the choice of the tube wall thickness ratio \( \delta_t/\delta_a \), since only the \( l/R_0 \) ratio is considered in the development of equation (14).

Radiator Effectiveness

Fin heat rejection. - After the temperature distribution and the slope of the temperature distribution curve at \( X = 0 \) have been determined, the net heat transferred by the fin can be calculated. The rate of heat loss from the fin outer surface at any point \( x \) on the fin from a differential area \( NZ \) \( dx \) is \( \sigma e Z N T_x^4 \) \( dx \). The overall rate of heat loss \( Q_T \) from the pair of fins that forms the double fin tube is

\[
Q_T = 2 \int_0^{2l} \sigma e Z N T_x^4 \ dx = 4\sigma Z N T_0^4 \epsilon \int_0^1 \theta^4(X) \ dx
\]

(16)

This equation is evaluated using the solutions of the fin energy equation (14). Comparison of this fin energy rejection to the total heat loss from both sides of an isothermal fin-tube section of length \( 2(l + R_0) \) can be expressed in dimensionless terms as
\[
\eta_f^* = \frac{Q_F}{4\varepsilon \sigma N \frac{L}{b}} \left( \frac{R_b}{1 + \frac{R_b}{L}} \right)^4 \int_0^1 \theta^4(x) \, dx \]

(17)

where

\[
\frac{R_b}{L} = \frac{R_o - \left( 1 - \frac{\delta_s}{\delta_a} \right) \delta_a}{L + \left( 1 - \frac{\delta_s}{\delta_a} \right) \delta_a}
\]

(18)

and \( \theta(x) \) is a function of \( x \) for specific values of \( \frac{\delta_s}{\delta_a} \), \( L/R_o \), and \( N_c \). Equation (17) can be defined as the thermal effectiveness of the double fin.

The results obtained from the solutions of equation (17) for fin effectiveness are presented for an example case in figure 6 as a function of \( L/R_o \) for parametric values of \( N_c \) and \( \delta_s/\delta_a \). In the aforementioned equations describing fin effectiveness (eqs. (17) and (18)), the tube wall thickness \( \delta_a \) must be known in order to obtain solutions. In order to obtain the actual value of \( \delta_a \), the tube inside diameter \( D_t \), tube liner thickness \( \delta_c \), power and temperature level, materials, meteoroid protection criteria, tube and header pressure drop, and definition of radiator tube vulnerable area must be specified. Thus, solutions for equation (17) require complete solutions for the entire radiator. Such radiator solutions will be described in detail in the section RADIATOR WEIGHT AND GEOMETRY.

Inspection of the curves in figure 6 reveals that for any fixed value of \( L/R_o \) and \( \delta_s/\delta_a \) the fin effectiveness decreases with increasing values of \( N_c \). The fin effectiveness is also seen to decrease with increasing \( \delta_s/\delta_a \) at constant \( N_c \). When \( L/R_o \) equals zero the value of fin effectiveness does not equal zero when the value of \( \delta_s/\delta_a \) is other than \( L \). This is due to an increase in the fin length caused by the reduction of tube side-wall thickness for any choice of \( L/R_o \) ratio. Thus, at \( L/R_o \) equal to zero, a fin remains of length \( l = \delta_a - \delta_s \).

Tube heat rejection. - The net heat loss from the external surface of the tube block is just its radiant emission since there is no incident energy from other parts of the system. This energy rejection when compared to the total heat loss from both surfaces of an isothermal fin-tube section of length \( 2(l + R_b) \) can be defined as the thermal effectiveness of the exposed surface of the tube. This expression is

\[
\eta_b^* = \frac{Q_b}{4\varepsilon \sigma N \frac{L}{b}} \left( \frac{R_b}{1 + \frac{R_b}{L}} \right)^4 \left( \frac{1}{1 + \frac{l}{R_b}} \right)
\]

(19)
where \( L/R_b \) is defined in terms of \( R_o, L, \delta_a, \) and \( \delta_s/\delta_a \) in equation (18). The heat radiation from the tube side-wall surfaces is already included as a contribution to the heat radiated from the fin, since in effect the radiant interchange acts in a manner similar to the heat conduction down the fin. This aspect is fully explained in reference 11. Solutions for equation (19) also require values of armor thickness \( \delta_a \) and thus are a result of the radiator calculation procedure for the double fin-tube block configuration.

Results of equation (19) for tube effectiveness are shown plotted for an example case in figure 7. Tube thermal effectiveness is seen to decrease as \( L/R_o \) is increased, which indicates a decrease in the relative importance of the tube portion as the fin length increases. Decreasing the tube wall ratio \( \delta_s/\delta_a \) also results in a reduced tube effectiveness, which is caused by a reduction in the tube block outer surface area and thus the energy it can reject to space. It is also seen from the figure that for an \( L/R_o \) ratio equal to zero the tube effectiveness does not equal 1 if the ratio \( \delta_s/\delta_a \) is less than 1. This is brought about because a fin of length \( \delta_a - \delta_s \) remains (see eq. (15)).

**Total fin-tube heat rejection.** For calculation purposes, it is desirable to formulate the total fin-tube thermal effectiveness that can be determined by summing the results of equations (17) and (19) for fin effectiveness and tube effectiveness, respectively. This expression is

\[
\eta^*_R = \frac{Q_f + Q_b}{4\sigma e N_z (1 + R_b/L)^2 R_b}\left[ 1 + \frac{L}{R_b} \int_0^1 \theta^4(x) dx \right]
\]  

(20)

Figure 8 shows a plot of total fin-tube effectiveness \( \eta^*_R \) against \( L/R_o \) ratio for several choices of conductance parameter \( N_c \) and tube block side-wall ratio \( \delta_s/\delta_a \) for an example case. Inspection of the curves shown in figure 8 reveals that for any fixed value of \( L/R_o \) and \( N_c \) the total fin-tube thermal effectiveness decreases as the \( \delta_s/\delta_a \) ratio decreases. This is reasonable since as the \( \delta_s/\delta_a \) ratio decreases the amount of isothermal base surface decreases. The fin-tube effectiveness is also reduced by increasing the conductance parameter or by increasing the \( L/R_o \) ratio.

Negligible variation in the magnitude of \( \eta^*_R \) was observed for the two power levels chosen for the comparison. Thus the curves given in figure 8 for the 500-kilowatt case also apply for the 1-megawatt power level case. It was also noticed that variations of the tube inside diameter had no appreciable effect on the value of \( \eta^*_R \) at a specific value of the \( L/R_o \) ratio, the \( \delta_s/\delta_a \) ratio, and the conductance parameter \( N_c \).

**PRESSURE DROP CONSIDERATIONS**

Another factor that is required to determine the geometry and weight of a radiator is the pressure drop determination in the radiator tubes and headers. This aspect of radiator design helps to determine the vapor header geometry and
the required tube diameter, the tube length, and the number of tubes. The equations presented are for a Rankine cycle condenser-radiator which assumed that vapor flow in the vapor header to be of the same quality as the turbine exhaust, two-phase flow in the radiator tubes and all-liquid flow in the liquid header. The development of the equations given in this section are given in reference 6.

Vapor header. - The determination of the pressure drop in the vapor header is simplified by assuming that only the gas phase affects the pressure drop. This pressure drop is expressed as a ratio of ΔP to the header inlet pressure P3 with the resulting ratio kept constant for comparative purposes. This equation, which is for turbulent flow, is

\[
\frac{\Delta P}{P_3} \text{VH} = \frac{0.00357 \rho g u_{VH}^2 \bar{W}}{2P_3 \text{Re}^0.2D_{VH}}
\]  

(21)

where \( u_{VH} \) is the uniform vapor velocity in the parabolic header (based on the turbine exhaust quality and neglecting the flow area occupied by the liquid) and \( \text{Re} \) is the vapor Reynolds number based on the vapor header maximum diameter \( D_{VH} \). The value of \( D_{VH} \) is obtained from the expression

\[
D_{VH} = \left(\frac{2\bar{W} \text{Qual}_3}{\rho g u_{VH}}\right)^{1/2}
\]  

(22)

and the total panel width \( \bar{W} \) in equation (21) is

\[
\bar{W} = \frac{NR_0}{2} \left(1 + \frac{L}{R_0}\right)
\]  

(23)

The term \( \text{Qual}_3 \) in equation (22) is the vapor quality at the entrance to the vapor header. The number of condensing tubes \( N \) in the previous equation is determined from the tube pressure drop analysis in conjunction with the optimization procedure used for this fin-tube geometry.

The amount of heat rejected to space from the parabolic vapor header is determined from the following expression:

\[
Q_{VH} = Q_{\text{rej}} X_{VH} = 2.09 \text{eoD}_{VH} F_{VH} \bar{W}^{1.4}
\]  

(24)

where the factor \( F_{VH} \) is defined as the vapor header occlusion factor for radiant emission to space and given a value of 0.85 for this analysis (ref. 13). The analysis and results of reference 13 (fig. 4) although primarily for tubes and fins are assumed valid for the occlusion of the header-panel arrangement shown in figure 1.

Radiator Tubes

The pressure drop in the radiator tubes where flowing vapor is condensing
was computed from a combination of several basic flow and energy equations that are given in detail in reference 6. The flow model (ref. 14) used assumed that at any given section perpendicular to the flow direction, the temperature and pressure in both the liquid and vapor are uniform and the same for both phases. This flow model also assumed turbulent flow with liquid and vapor velocities uniform in each phase at a given cross section, but that the two velocities were not necessarily equal. Pressure drops were computed for a series of incremental tube lengths, and the pressure drop for the whole tube was obtained by summing the incremental drops.

The total change in pressure for an entire radiator tube is comprised of a frictional and a momentum component. The friction pressure drop is described by the expression

\[ \frac{dP_f}{Re} = -\frac{1}{\eta} Re \left( 0.092 \frac{\mu^2}{\rho} dx \right) \]  

(25)

where \( Re = \frac{4W}{\pi D_1 \mu} \) and \( \eta \) is a function of \( x \). The differential form of the change in pressure due to a change in momentum is

\[ -\frac{dP_m}{\eta} = \frac{1}{\eta} \left[ d\left( \rho A \dot{V}^2 \right) + d\left( \rho A \dot{u}^2 \right) \right] \]  

(26)

A third relation is required that relates the increment of tube length and the increment of condensate formed:

\[ \frac{dW_g}{dx} = \frac{K_s \pi D_1^4}{3600} \frac{dP_f}{T} + C_{pg} \frac{d\dot{W}_g}{dx} + C_{pg} \frac{d\dot{W}_f}{dx} \]  

(27)

where for the double fin-tube radiator the definition of \( K \) is

\[ K = 4 \frac{R_b \pi D_1^4}{\eta \pi D_1} \left( 1 + \frac{L}{R_b} \right) \]  

(28)

Equations (25), (26), and (27) along with equation (1) are solved simultaneously for \( dW, dP_m, \) and \( dP_f \). The total change in pressure for each increment can then be found from the relation

\[ dP = dP_m + dP_f \]  

(29)

and the total pressure change for the entire radiator tube can be found by
summing the incremental changes. For comparative purposes, the total tube pressure drop is expressed as the ratio of pressure drop to the tube inlet pressure.

The pressure drop associated with turning the vapor from the vapor header into the radiator tubes and accelerating the flow can be calculated from the expression

\[ \Delta P_{\text{entrance}} = K_H \frac{1}{2} \rho_g \frac{w_0^2}{g} \]

(30)

where the tube entrance loss factor \( K_H \) is given the same value used in equation (1).

**Liquid Header**

The pressure drop in the liquid header is obtained by applying Fanning's equation with a friction factor for turbulent flow. This expression is

\[ \Delta P_{\text{LH}} = \frac{0.00102 \rho_g w_{\text{VH}}^2}{D_{\text{LH}} (Re_L)^{0.2}} \]

(31)

where \( Re \) is the Reynolds number corresponding to the maximum liquid velocity \( V_{\text{LH}} \) that, in these calculations, was taken as 4 feet per second. The liquid header diameter \( D_{\text{LH}} \), which is assumed constant, is determined by applying the continuity equation at the header exit to give the expression

\[ D_{\text{LH}} = \left( \frac{\dot{W}}{\pi \rho_{\text{LH}} V_{\text{LH}}} \right)^{1/2} \]

(32)

**RADIATOR WEIGHT AND GEOMETRY**

The heat-rejection analysis of the fin and the tube was nondimensionalized so that the results could be used for general design purposes. Using the previous results for analyzing the merits of the variable tube wall double fin-tube configuration in radiator designs is impractical without consideration of the total weight of the vapor and liquid headers as well as the tubes and fins. It is necessary, therefore, to consider the ratio of heat rejection per unit weight \( \frac{\dot{Q}_{\text{rej}}}{W} \) and the influencing effects of radiator geometry limitations on the maximum heat rejection per unit weight.

**Armor Thickness**

In order to determine both radiator weight and geometry, the effects of meteoroid penetration must be considered on both the tubes and headers. This will dictate the required armor protection thickness needed for the radiator.
The tube armor thickness $\delta_a$ is determined using the meteoroid protection criteria given in reference 9, which is based on a comprehensive appraisal of the available data and theories concerning the meteoroid penetration phenomenon. According to reference 9, the resultant equation for the armor thickness $\delta_a$ is given by the expression

$$\delta_a = 2a \left( \frac{\rho_P}{\rho_a} \right)^{1/2} \left( \frac{\bar{V}_P}{c_a} \right)^{2/3} \left( \frac{6.747 \times 10^{-5}}{\rho_P} \right)^{1/3} \left( \frac{\alpha \delta_a \tau}{-\ln P(0)} \right)^{1/3} \left( \frac{1}{\beta + 1} \right)^{1/3}\beta$$

where in the previous equation

$a = 1.75$

$\beta = 1.34$

$\rho_P = 0.44 \text{ g/cu cm}$

$\bar{V}_P = 98,400 \text{ ft/sec}$

$\alpha = 0.53 \times 10^{-10} \text{ g}^2/(\text{sq ft})(\text{day})$

Insertion of these constants into equation (33) along with utilizing Young's modulus in the definition of sonic velocity of the material yields the more compact form

$$\delta_a = \frac{1.48}{\rho_a^{1/6}} \left( \frac{\bar{V}_P}{c_a} \right)^{1/3} \left[ \frac{\alpha \delta_a \tau}{-\ln P(0)} \right]^{0.249}$$

(34)

The total exposed area to be protected by direct impacts $A_v$ is assumed to be the outer surface of the vapor header and the projected area of the tube block. The liquid header contribution is assumed to be negligible since its surface area is small compared to that of the vapor header. Thus

$$A_v = A_t + A_{VH}$$

(35)

The radiator tube projected area is given by the expression

$$A_t = 4R_b W_z = \frac{Q_{rej} X_{tf}}{\alpha c T_b^4 \left( 1 + \frac{l}{R_b} \right) \eta^*_{R}}$$

(36)

where $l/R_b$ is obtained from equation (18) and $\eta^*_{R}$ from equation (20). The fraction of the total radiator heat that is rejected by the tubes and fins $X_{tf}$ is defined by the expression

$$X_{tf} = \frac{Q_t + Q_f}{Q_{rej}} = 1 - X_{VH}$$

(37)
The vulnerable area of the vapor header is assumed to be its full surface area and is given as

$$A_{VH} = \frac{Q_{rej}}{\sigma e F_{VH} T_{v}^4}$$  \hspace{1cm} (38)

where $F_{VH}$ is the occlusion factor for the effect of the radiator panel on the heat rejection of the vapor header surface.

Combining equations (35) to (38) yields

$$A_V = \frac{Q_{rej}}{\sigma e} \left[ \frac{1 - X_{VH}}{1 + \left( \frac{L}{R_b} \right) \eta_{VH} T_{v}^4 b} \right] + \frac{X_{VH}}{F_{VH} T_{v}^4}$$  \hspace{1cm} (39)

**Weight Ratio**

The total heat rejection per unit weight of a fin-tube radiator can be expressed as

$$\frac{Q_{rej}}{W} = \frac{Q_{rej}}{W}$$  \hspace{1cm} (40)

where for radiation from both sides of the radiator and from the vapor header

$$\frac{Q_{rej}}{NZ} = \frac{Q_{rej}}{W} = \frac{Q_{rej}}{W} \left[ 4R_O T_{v}^4 \eta_{R} \left( 1 + \frac{L}{R_O} \right) + 2.095 \frac{D_{VH} F_{VH} T_{v}^4}{NZ} \right]$$  \hspace{1cm} (41)

Insertion of equation (23) into equation (41) yields the following form of the total radiator heat rejection per unit length of tube:

$$\frac{Q_{rej}}{NZ} = \frac{Q_{rej}}{W} \left( 1 + \frac{L}{R_O} \right) \left( 4 \eta_{R} T_{v}^4 b + 1.045 \frac{D_{VH} F_{VH} T_{v}^4}{Z} \right)$$  \hspace{1cm} (42)

where the outside tube diameter (block width) $D_O$ can be expressed in terms of the liner thickness $\delta_c$, the inside tube diameter $D_i$, and the armor thickness $\delta_a$ and can be given as

$$D_O = D_i + 2\delta_c + 2\delta_a$$  \hspace{1cm} (43)

The total weight of the radiator is comprised of the individual weights of
the vapor header, the liquid header, and the tube-fin panel. The weight of the liquid inventory in the subcooler portion of the radiator tubes is neglected since the subcooler length is small. The vapor header weight is given by (ref. 6)

\[ W_{\text{VH}} = \frac{4W}{3} \pi \left[ (D_{\text{VH}} + \delta_{\text{VH}}) \delta_{\text{VH}} \rho_c + (D_{\text{VH}} + 2\delta_{\text{VH}} + \delta_a) \delta_a \rho_a \right] \]  

(44)

and that of the liquid header and condensate by

\[ W_{\text{LH}} = 4\pi W \left[ \frac{D_{\text{LH}}^2}{4} + \rho_c \delta_{\text{LH}} (D_{\text{LH}} + \delta_{\text{LH}}) + \rho_a \delta_a (D_{\text{LH}} + \delta_a + 2\delta_{\text{LH}}) \right] \]  

(45)

The fin and tube panel weight can be calculated using the expression

\[ W_{\text{f}} = 4l \rho \pi NZ + \rho_c \pi \frac{NZ}{4} \left[ (D_i + 2\delta_c)^2 - D_i^2 \right] + NZ \rho_a \left[ 4R_b - \frac{\pi}{4} (D_i + 2\delta_c)^2 \right] \]  

(46)

When \( t = \sigma T_b^3 / kN_c \), \( D_0 \), from equation (43), \( l \) from equation (15), and \( R_b = R_i + \delta_c + (\delta_b / \delta_a) \delta_a \) are introduced into equation (46), the equation for the tube and fin panel weight becomes

\[ W_{\text{f}} = \frac{4\rho \pi NZ \sigma T_b^3}{kN_c} \left[ \frac{L}{R_b} + \left( 1 - \frac{\delta_b}{\delta_a} \right) \frac{\delta_b}{R_b} \right]^3 (R_i + \delta_c + \delta_a)^3 + \rho_c \pi NZ (D_i + \delta_c) \]

\[ \left\{ 2(D_i + 2\delta_c + 2\delta_a) \left[ R_i + \delta_c + \left( \frac{\delta_b}{\delta_a} \right) \delta_a \right] - \pi (R_i + \delta_c)^2 \right\} \]  

(47)

The total radiator weight is then obtained by summing the results of equations (44), (45), and (47). The denominator of equation (40) can be found by dividing the total radiator weight by the total tube length \( NZ \). This result along with the results of equation (42) when inserted into equation (40) yield the radiator heat rejection per unit weight. The peak value of \( Q_{\text{re,j}} / W \) can then be obtained by plotting the results of equation (40) as a function of \( L / R_b \) for specific values of conductance parameter \( N_c \) and tube side-wall ratio \( \delta_b / \delta_a \).

Panel Geometry

In addition to the important aspect of minimizing weight for practical radiator designs, it is also of interest in most cases to investigate the geometry of the radiator as it might affect the integrating of the vehicle and radiator. Planform area, aspect ratio, and fin thickness are three facets of the geometry of the radiator panel that must be determined in order to satisfy radiator-space vehicle integration and structural and fabricational require-
ments of the fin-tube configuration.

Radiator planform area $A_p$ is obtained from the equation

$$A_p = 2NZ(L + R_0) = \frac{Q \tau e_j X_t t_f}{2\sigma \epsilon \bar{T}_b \eta_R^x}$$

(48)

It is seen from this equation that planform area will vary inversely with overall fin-tube thermal effectiveness $\eta_R^x$ for a specific choice of power and temperature level. The planform area will generally increase with increasing $L/R_0$ because $\eta_R^x$ decreases as the $L/R_0$ ratio is increased (fig. 8).

Another interesting factor with respect to the geometry of the radiator is the magnitude of the fin thickness. Radiator applications might require that the fin have structural or fabricational qualities that could result in non-optimum weights and dimensions. Fin thickness $t$ for the closed sandwich configuration can be calculated from the expression

$$t = \frac{\sigma \bar{T}_b^3}{kN_c^2}$$

(49)

The panel aspect ratio, which is defined as the ratio of panel width $\bar{W}$ to tube length $Z$, is obtained by using equation (23) for $\bar{W}$ and the results of the pressure drop calculations for $Z$.

Method of Solution

Simultaneous solution of equations (4), (20), (21), (29), (33), and (39) requires inputs of inside tube diameter $D_i$, tube and vapor header pressure drop ratios, power level, temperature level, and the properties of the materials and cycle fluid. This results in values of the parameters $\delta_a$, $T_b$, $X_{VH}$, $N$, and $u_0$ for a selected variation of $L/R_0$, conductance parameter $N_c$, and the tube side-wall ratio $\delta_s/\delta_a$.

Important results required for weight and geometry calculations will include the number of tubes $N$, tube length $Z$, tube outside radius $R_0$, panel width $\bar{W}$, and the inside diameter of the vapor header $D_{VH}$. An electronic digital computer was used to obtain the desired results that required an iterative type solution.

RESULTS AND DISCUSSION

Calculation Inputs

Calculations making use of the resultant equations developed in the analysis for the double fin-tube geometry require inputs such as inside tube diameter, radiator vapor inlet temperature, cycle power level and conditions, mate-
rials of construction, meteoroid protection criteria, tube block side-wall thickness ratio, and pressure drop in the tubes and header in order to completely specify a radiator solution. For this reason two specific cases are used in order to show the effects of some of the previously mentioned variables on heat rejection per unit weight and on radiator geometry.

The first case considered is a 1-megawatt electrical output powerplant with the radiator at 1700°C R. The tube armor and the fin were assumed to be made of beryllium, and the tube liner was assumed to be a columbium alloy. The second case is a 500-kilowatt system with the radiator at 1700°C R. For this case, the tube armor, tube liner, and the fin were all taken to be columbium - 1-percent zirconium alloy. These two cases both used a peak cycle temperature of 2460°C R and potassium as the cycle fluid. Inside tube diameters of 3/8, 1/2, 5/8, 3/4, and 1 inch were chosen with tube lengths increased to allow 100°C R of subcooling. A 500-day mission time and a probability of no puncture P(0) of 0.995 were chosen for the calculation of meteoroid protection thickness. Pressure drop ratios for the two cases were set at ΔP/P = 0.02 for the vapor header and ΔP/P = 0.05 for the radiator tubes. The emittance of the surface coating on the fins, tubes, and headers was taken to be 0.90. Radiator material properties were assumed constant with temperature and evaluated at 1700°C R. The all columbium alloy radiator had a density of 530 pounds per cubic foot, thermal conductivity of 34 Btu per hour per foot per °F, and a modulus of elasticity of 0.202x10^10 pounds per square foot. The beryllium radiator used a density of 115 pounds per cubic foot, a thermal conductivity of 51.5 Btu per hour per foot per °F, and a modulus of elasticity of 0.397x10^10 pounds per square foot. The beryllium radiator used a columbium liner with the material properties previously mentioned.

Calculations using the given inputs and specifications were also made for the central fin-tube geometry that used the method described in reference 6.

Radiator Weight

Radiator heat rejection per unit weight Q_{rej}/W was plotted for each value of inside tube diameter D_i and tube block side-wall ratio δ_s/δ_a chosen for the comparison over a range of the parameters N_c and L/R_o. Results showing the variation in heat rejection rate per unit weight as a function of L/R_o for several values of conductance parameter N_c are shown plotted for two sample cases in figure 9. The figure shows results for the tube block side-wall thickness to tube armor thickness ratio δ_s/δ_a equal to 0.5 with tube inside diameters of 3/8 and 5/8 inch chosen for the 500-kilowatt and 1-megawatt cases, respectively. Each constant N_c curve is seen to peak at a specific value of L/R_o with the value of the L/R_o at peak Q_{rej}/W increasing as N_c is increased.

Plotting the maxima for each N_c curve of figure 9 and the additional results for the δ_s/δ_a = 0 and 1.0 cases yield a performance map (fig. 10) that plots peak heat rejection per unit weight against L/R_o. It is seen from figure 10 that decreasing the armor block side-wall thickness results in a sub-
stantial increase in the value of $Q_{rej}/W$. According to figure 10, the maximum $Q_{rej}/W$ occurs at an $N_{c}$ approximately equal to 0.75 for the 500-kilowatt system at a tube inside diameter of 3/8 inch. The results for the 1-megawatt system indicate this maximum also occurs near an $N_{c} = 0.75$ for a near optimum tube inside diameter of 5/8 inch.

Plotting the maxima of figures 10(a) and (b) on figures 11(a) and (b), respectively, and the results for additional inside tube diameters yield the maximum value of $Q_{rej}/W$ and its corresponding inside tube diameter for the three values of $\delta_{g}/\delta_{a}$ chosen for the comparison. The curves given in figure 11(a) for the 500-kilowatt case indicate that maximum $Q_{rej}/W$ for any choice of $\delta_{g}/\delta_{a}$ occurs at a tube inside diameter between 3/8 and 1/2 inch with the $Q_{rej}/W$ curve relatively flat in this region. Maximum $Q_{rej}/W$ occurred at a diameter between 1/2 and 5/8 inch for the 1-megawatt system as shown in figure 11(b).

The weight results of the double fin-tube configurations are compared to the central fin-tube results in figures 11(a) and (b) in order to show the desirability of the double fin-tube with reduced tube side-wall thickness. It is observed from figure 11 that the central fin-tube configuration has a larger heat rejection per unit weight than the double fin-tube with $\delta_{g}/\delta_{a} = 1.0$. For the 500-kilowatt case, the double fin tube with $\delta_{g}/\delta_{a} = 0.5$ affords an 11-percent weight advantage at maximum $Q_{rej}/W$ over the central fin tube, and a maximum upper limit of 39 percent when $\delta_{g}/\delta_{a} = 0$. The radiator weight per kilowatt of electrical power for the 500-kilowatt case using a columbium radiator is 10.2, 15.0, and 19.0 pounds per kilowatt for the $\delta_{g}/\delta_{a} = 0$, 0.5, and 1.0 cases, respectively.

The percentage weight savings are reduced for the 1-megawatt power level with the beryllium radiator and are 8 percent for the $\delta_{g}/\delta_{a} = 0.5$ case and 32 percent for the $\delta_{g}/\delta_{a} = 0$ case. For this power level, the specific weight was 3.50, 4.66, and 5.73 pounds per kilowatt for the $\delta_{g}/\delta_{a} = 0$, 0.5, and 1.0 cases, respectively. The beryllium radiator has a smaller percent increase in $Q_{rej}/W$ than the columbium radiator because there is less relative weight in the beryllium armor.

The conclusions reached in figure 19 of reference 7, which compared the double fin-tube and central fin-tube configurations without taking into account the effects of header heat rejection and weight as well as pressure drops in the radiator tubes, showed a similar percentage increase in $Q_{rej}/W$ by reducing $\delta_{g}/\delta_{a}$. This data, which was for a 1-megawatt, 1700° R, beryllium radiator, indicated a 6-percent weight advantage for the double fin tube at a $\delta_{g}/\delta_{a} = 0.5$ and a 26 percent advantage at $\delta_{g}/\delta_{a} = 0$ over the central fin-tube configuration.

The conductance parameter obtained at maximum heat rejection per unit weight is plotted as a function of inside tube diameter for a $\delta_{g}/\delta_{a} = 0.5$ for both geometries in figure 12. Results obtained for $\delta_{g}/\delta_{a}$ other than 0.5 showed very little variation in the optimum value of $N_{c}$. Both the 500-
kilowatt and 1-megawatt examples are shown with the double fin-tube results yielding the higher value of \( \eta_c \) for a specific choice of inside tube diameter. It is also noted that the values of \( \eta_c \) obtained for the 500-kilowatt columbium radiator are somewhat larger than those obtained for the 1-megawatt beryllium radiator. Reference 7 indicated the double fin-tube configuration reached maximum \( Q_{rej}/W \) at much larger values of \( \eta_c \) than those obtained in this report.

**Radiator Geometry**

**Planform area.** - The planform area of the double fin-tube geometry is shown plotted in figure 13 for the two power levels chosen for this investigation. This figure illustrates the calculated variations of planform area with \( L/R_0 \) ratio, conductance parameter \( \eta_c \), and tube side-wall thickness to armor thickness ratio \( \delta_s/\delta_a \) for the two power cycles for peak \( Q_{rej}/W \) conditions. The two inside tube diameters used for the results shown in figure 13 correspond to near minimum weight conditions. Reducing the tube side-wall thickness ratio for peak \( L/R_0 \) ratios results in only a small variation in planform area for a constant value of \( \eta_c \), but results in a sizable decrease in \( L/R_0 \). However, it should be kept in mind that the actual fin length \( L \) is not decreasing that rapidly since the fin length is being increased by the amount \( \delta_s - \delta_a \) as \( \delta_s/\delta_a \) approaches zero. The decrease in \( L/R_0 \) (which is a measure of the tube center-to-center distance) as \( \delta_s/\delta_a \) decreases causes a reduction in tube block effectiveness (due to reduced \( Q_b \)) and an increase in fin effectiveness (fig. 6) that result in only a small variation in total fin-tube effectiveness for a specific choice of \( \eta_c \). This accounts for the negligible variation of planform area with varying \( \delta_s/\delta_a \). Similar results are obtained for the other inside tube diameters investigated.

Comparison of the planform area results of the double fin-tube geometry with those of the central fin-tube geometry indicates good agreement as shown by figure 14. The planform area obtained for the central fin tube is less than that of the double fin tube for internal tube diameters greater than 1/2 inch for the 500-kilowatt columbium radiator. At the 3/8-inch diameter, which corresponds to near minimum weight, the two fin-tube geometries agreed to within 4 percent. For the 1-megawatt beryllium system, the double fin tube has the larger planform area at tube diameters greater than 5/8 inch. At near optimum weight corresponding to a 5/8-inch diameter, the central fin-tube geometry offers only a small savings in planform area. The planform area obtained for the double fin-tube geometry in reference 7 for the 1-megawatt radiator is much greater than that obtained from the comparison calculations of this report. This is brought about because the conductance parameter at maximum \( Q_{rej}/W \) for the case of reference 7 was larger than the values obtained in this report. Figure 13 verifies that increasing conductance parameter \( \eta_c \) results in increased planform area.

**Fin thickness.** - The effect of reducing the tube side-wall ratio \( \delta_s/\delta_a \) on fin thickness is shown for the two examples in figure 15 for peak \( Q_{rej}/W \) conditions at near optimum inside tube diameter. In both cases, reducing
δ_s/δ_a results in a sizable decrease in fin thickness at constant N_c. Accompanying this decrease in fin thickness is a reduction in L/R_0 and thus tube spacing. The magnitude of the fin thickness obtained for the two sample cases at optimum values of N_c (fin thickness t is greater than 0.020 in. for the columbium radiator and greater than 0.035 in. for the beryllium radiator) resulted in values of fin thickness that are reasonable and should satisfy structural and fabricational requirements.

The comparison of the fin thickness obtained for the double fin-tube configuration indicates an increasing total fin thickness with increasing values of tube side wall to armor ratio δ_s/δ_a throughout the range of D_1 investigated for both power levels (fig. 16). Curves are given for fin thickness at maximum Q_rej/W for both geometries and both system power levels with central and double fin-tube δ_s/δ_a = 0.5 cases agreeing closely for the 1 megawatt radiator.

Panel aspect ratio. - Radiator panel aspect ratio, which is defined as the ratio of panel width W to the tube length Z for the four panel radiator of figure 1, is shown plotted in figure 17 for the two double fin-tube example cases. The aspect ratio of a panel is seen to decrease as the tube block side-wall ratio δ_s/δ_a decreases at a constant N_c. The decrease in aspect ratio with reduced δ_s/δ_a is primarily a result of the reduced L/R_0 since reductions in δ_s/δ_a had little effect on planform area as shown by figure 13. These results can be indicated by inspection of equation (43), which describes planform area and length, and equation (23), which describes the total panel width W. The reduction in aspect ratio at constant planform area is explained by the calculations which showed that as δ_s/δ_a gets smaller the number of tubes and the tube length increase as the L/R_0 ratio decreases. This was accompanied by a large reduction in the working fluid inlet tube velocity.

The panel aspect ratio showed practically no difference between the central fin-tube and double fin-tube configurations as indicated in figures 16(a) and (b) for the 500-kilowatt and 1-megawatt systems, respectively.

Number of tubes. - The number of radiator tubes required was found to increase substantially as the tube block side-wall ratio decreased. The 500-kilowatt columbium radiator with a tube inside diameter of 3/8 inch had 298, 337, and 430 tubes for δ_s/δ_a ratios of 1.0, 0.5, and 0, respectively, at maximum heat rejection per unit weight. The 1-megawatt beryllium radiator using a tube inside diameter of 5/8 inch had 223, 250, and 302 tubes for δ_s/δ_a ratios of 1.0, 0.5, and 0, respectively, at maximum Q_rej/W.

For simplicity and reliability of fabrication it is desirable to reduce the number of tubes. A sizable reduction in the number of tubes without much reduction in Q_rej/W can be had by just increasing the size of the tube inside diameter. For the 500-kilowatt columbium radiator with tube block side-wall ratio set at 0.5, a 3/8-inch inside tube diameter required 337 tubes whereas a 1/2-inch inside diameter results in 206 tubes. This reduction in the number of tubes results in only a 2-percent decrease in Q_rej/W. The 1-megawatt beryl-
lum radiator with $\delta_s/\delta_a$ set at 0.5 required 250 tubes for a 5/8-inch inside tube diameter whereas a 3/4-inch inside diameter required just 182 tubes. This reduction resulted in only a 3-percent decrease in $Q_{rej}/W$. Similar reductions in the number of tubes can be obtained at values of $\delta_s/\delta_a$ equal to 0 and 1.0 with small reductions in $Q_{rej}/W$.

Comparison of the number of tubes for the central and double fin-tube radiators at maximum $Q_{rej}/W$ for the 500-kilowatt case indicated the central fin-tube geometry required 308 tubes compared to 293 tubes for the double fin-tube $\delta_s/\delta_a = 1.0$ radiator at an inside tube diameter of 3/8 inch. This trend also held for the 1-megawatt case for which the central fin-tube required 226 tubes compared to 223 for the double fin-tube $\delta_s/\delta_a = 1.0$ radiator.

Interpolation of results. - If the values of the ratio of $\delta_s/\delta_a$ other than the three values chosen for this investigation (0, 0.5, and 1) are designated by forthcoming meteoroid bumper results, additional points may be obtained from curves fitted through the calculated points to obtain intermediate values of panel aspect ratio $W/Z$, fin thickness $t$, radiator planform area $A_p$, and radiator heat rejection per unit weight $Q_{rej}/W$.

SUMMARY OF RESULTS

An analysis of the double fin-tube configuration with variable tube block side-wall thickness and a comparison with a comparable central fin-tube radiator for the sample cases of a 1-megawatt beryllium radiator and a 500-kilowatt columbium radiator for a 500-day mission time and probability of no puncture $F(0)$ of 0.935 showed that

1. A substantial increase in radiator heat rejection per unit weight is possible for the double fin-tube geometry if the tube block side-wall thickness can be reduced to less than 75 percent of that required for normal armor thickness.

2. The double fin-tube configuration with ratio of tube block side-wall thickness to normal armor thickness of 0.5 offers an 11 and 8 percent weight savings over the central fin-tube geometry for the 500-kilowatt and 1-megawatt cases, respectively. This corresponds to 15.0 pounds per kilowatt for the 500-kilowatt case and 4.68 pounds per kilowatt for the 1-megawatt case considered herein compared to 16.7 and 4.83 pounds per kilowatt for the central fin-tube 500-kilowatt and 1-megawatt cases, respectively.

3. The percent weight savings is increased to 39 and 32 percent for the 500-kilowatt and 1-megawatt cases, respectively, if the ratio of tube block side-wall thickness to normal armor thickness is set at 0. In this case, the corresponding radiator specific weights would be 10.2 and 3.50 pounds per kilowatt for the double fin-tube geometry and 16.7 and 4.83 pounds per kilowatt for the central fin-tube geometry.

4. The double fin-tube configuration reached maximum heat rejection per
unit weight for the 1-megawatt system at a conductance parameter of 0.75 whereas maximum heat rejection per unit weight for the central fin-tube geometry occurred at 0.63. These values for the 500-kilowatt case were 0.63 and 0.75, respectively, for the double and central fin-tube. However, the conductance parameter associated with maximum heat rejected per unit weight decreases with decreasing inside tube diameter for both the double and central fin-tube radiators.

5. The double fin-tube radiator reaches maximum heat rejected per unit weight for a tube inside diameter in the range from 3/8 to 1/2 inch for the 500-kilowatt case, and from 1/2 to 5/8 inch for the 1-megawatt case.

6. The physical dimensions of the double fin-tube radiator (planform area, panel aspect ratio, number of tubes, inside tube diameter, and fin thickness) can be varied over a fairly wide range without seriously decreasing the radiator heat rejection per unit weight.

7. The fin thickness obtained for maximum heat rejection per unit weight for the double fin-tube geometry is of reasonable fabricational and structural magnitude (greater than 0.02 in. for the 500 kw columbium radiator and greater than 0.035 in. for the 1 Mw beryllium radiator).

8. The values of planform area and panel aspect ratio obtained for the double and central fin-tube geometries were in close agreement regardless of the choice of tube block side-wall ratio near the minimum weight condition. Comparison of the total fin thickness for the two geometries agreed well at a tube block side-wall thickness ratio of 0.5, but showed considerable variations at side-wall ratios of 1.0, and 0.

9. Reduction of the tube block side-wall thickness ratio substantially increases the number of radiator tubes at maximum heat rejection per unit weight. However, the number of tubes can be reduced to the values corresponding to the central fin-tube geometry by increasing the value of $D_1$ above that for maximum heat rejection per unit weight. The weight penalty involved is only several percent.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, August 12, 1964

REFERENCES


Figure 1. - Schematic of four-panel radiator configuration.

Figure 2. - Double fin-tube geometry with variable side-wall thickness.
Figure 3. - Central fin-tube geometry.

Figure 4. - Assumed radiator heat-transfer paths.
Figure 5 - pin temperature profiles; closed sandwich fin tube.
Figure 8. Pin effectiveness; double fin-tube geometry. Powerplant output, 500 kilowatts; temperature at radiator inlet, 1700°F; inside tube diameter, 5/4 inch; columbium radiator.
(a) Powerplant output, 500 kilowatts; inside tube diameter, 3/8 inch; columbium radiator.

(b) Powerplant output, 1 megawatt; inside tube diameter, 5/8 inch; beryllium radiator.

Figure 9. - Heat rejection per unit weight characteristics; double fin-tube geometry. Temperature at radiator inlet, 1700°F; ratio of tube side-wall to armor thickness, 0.5.
(a) Powerplant output, 500 kilowatts; inside tube diameter, 3/8 inch; columbiunum radiator.

(b) Powerplant output, 1 megawatt; inside tube diameter, 3/8 inch; beryllium radiator.

Figure 10. - Comparison of peak heat rejection characteristics of double fin-tube geometry with variable tube block side wall. Temperature at radiator inlet, 1700°F.
(a) Powerplant output, 500 kilowatts; columbium radiator.

(b) Powerplant output, 1 megawatt; beryllium radiator.

Figure 11. - Maximum heat rejection per unit weight. Temperature at radiator inlet, 1700° R.
Figure 12. - Conductance parameter at maximum heat rejection per unit weight.
Figure 13. - Comparison of radiator planform area at peak heat rejection per unit weight; double fin-tube geometry. Temperature at radiator inlet, 1700° R.

(a) Powerplant output, 500 kilowatts; inside tube diameter, 3/8 inch; columbium radiator.

(b) Powerplant output, 1 megawatt; inside tube diameter, 5/8 inch; beryllium radiator.
(a) Powerplant output, 500 kilowatts; columbium radiator.

(b) Powerplant output, 1 megawatt; beryllium radiator.

Figure 14. - Planform area comparison at maximum heat rejection per unit weight for double and central fin-tube radiators. Temperature at radiator inlet, 1700° R.
(a) Powerplant output, 500 kilowatts; inside tube diameter, 3/8 inch; columbia radiators.

(b) Powerplant output, 1 megawatt; inside tube diameter, 6/8 inch; beryllium radiator.

Figure 15. - Radiator fin thickness at peak heat rejection per unit weight; double fin-tube geometry. Temperature at radiator inlet, $1700^\circ F$. 
Figure 16. - Total fin thickness at maximum heat reduction per unit weight for double and central fin-tube radiators. Temperature at radiator inlet, 1700ø F.
Figure 17. - Radiator panel aspect ratio at peak heat rejection per unit weight; double fin-tube geometry; four panels. Temperature at radiator inlet, 3700° K.
Figure 18. - Panel aspect ratio comparison at maximum heat rejection per unit weight for double and central fin-tube radiators. Temperature at radiator inlet, 1700°F.
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An analytical study of a flat condenser-radiator with a double fin-tube geometry was performed for a Rankine space-power electric-generating system. The analysis of the double fin-tube configuration included consideration of tube and header pressure drops, meteoroid protection for the tubes and headers, and reduced tube side-wall thickness due to a possible bumper screen effect of the fins. The double fin-tube radiator is compared to a conventional central fin-tube configuration on a heat rejection to weight basis. The conclusion reached indicates a substantial weight savings can be realized when using the double fin-tube configuration with reduced tube side-wall thickness compared to the central fin-tube arrangement. |

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