Guidance Methods for Accurate In–Flight Alignment of Navy Theatre Wide Missiles

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OUTLINE

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• ADOP - In-Flight Alignment Metric

• Second Stage Guidance Methods

• In-Flight Alignment Analysis

• Summary
Background
**Stage 1**
- IMU initialized at launch
- Radar acquisition

**Stage 2**
- Command guidance to a specified injection velocity vector
- Trajectory shaping to maximize kinematic performance or improve IFA
- GPS acquisition

**Stage 3**
- 1 pulse or 2 pulse TSRM
- Burnout reference guidance during pulses to produce a ballistic collision course to minimize ZEM
- Nosecone ejection after final pulse or between pulses

**Stage 4**
- KW IR acquisition of target
- KW divert used to steer out 3rd stage ZEM
Successful Intercept Requirements

Pointing Requirement

- At kinetic warhead (KW) separation the target must be within the seeker field of regard (FOR)

Divert Requirement

- The zero effort miss (ZEM) must be within the kinetic warhead divert capability
In-flight Alignment Required to Achieve Pointing Error Allocations

- The missile IMU alignment with respect to the ship defined navigation (ECEF) coordinate frame may have a large unknown error at launch (up to 26 mrad)

- This error dominates the error budget and degrades performance

- The in-flight alignment (IFA) process calibrates the IMU alignment with respect to the navigation coordinate frame during flight

- An integrated GPS/IMU missile navigation system was first used on Standard Missile to perform this in-flight alignment as part of the Terrier LEAP experiment

In order To Meet The Pointing Error Allocation The Missile Initial Attitude Error Must Be Reduced Inflight
How Does Inflight Alignment Work?

• Background Facts
  – The major alignment error component to be calibrated is the IMU alignment with respect to the navigation frame (≤ 26 mrad)
  – When accelerations are transformed with an IMU alignment error to the navigation frame an acceleration error develops

• The Aiding Process
  – Acceleration errors, when integrated, result in velocity errors which result, in turn, in position errors
  – Navigation errors are observable by comparing inertial navigation estimates of the position and velocity to measurements from outside sources:
    • Radar measurements (position)
    • GPS measurements (position & velocity)
  – Errors are corrected via an on-board Kalman Filter
In-Flight Alignment Metric

“ADOP”
Attitude Dilution Of Precision (ADOP) was developed as a trajectory induced observability metric of in-flight alignment.

There are two fundamental ingredients in the ADOP metric:

- The missile acceleration time profile
- The GPS and radar measurement noise error time profiles

An interpretation of the ADOP metric:

- Missile total (RSS) attitude alignment error with respect to the navigation coordinate frame ($3\sigma$ value expressed in milli-radians)
- A value less than 5 mrad is considered good performance and a value greater than 5 mrad is considered degraded performance
ADOP Attributes

- Trajectory induced observability metric for in-flight alignment
- A simplified error model that is economical to run
- Provides lower bound on attitude errors for benchmarking in-flight alignment performance
- Can be used to generate observability maps over the tactical battlespace
- Shows difficult regions of the battlespace for in-flight alignment

ADOP Observability Contour Map Spanning the Battlespace

<table>
<thead>
<tr>
<th>ADOP Values (mrad)</th>
<th>Ground Range</th>
<th>Altitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>&gt; 20</td>
<td>Poor IFA</td>
<td></td>
</tr>
<tr>
<td>20 - 15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15 - 10</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 - 5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 - 0</td>
<td>Better IFA</td>
<td></td>
</tr>
</tbody>
</table>
Second Stage Guidance Methods
• **Cross Product Guidance (CPG)**
  - Guides to a specified injection velocity vector
  - Approximates an optimal kinematic trajectory

• **Delayed Cross Product Guidance (DCPG)**
  - Similar to CPG, guides to a specified injection velocity vector
  - Guidance initiation is delayed to improve IFA

• **Modified Cross Product Guidance (MCPG)**
  - Similar to CPG, guides to a specified injection velocity vector
  - Guidance initiation is delayed
  - Adds a shaping term to improve IFA
Guidance Law Definitions

\[ \vec{A}_C = -K_1 \vec{V} \sin \hat{\delta} + K_2(t)\hat{\delta} \]

- **Cross Product Term:**
  - CPG, DCPG, & MCPG
  - Nulls heading error and forces convergence to injection velocity vector
  - \( K_1 \) gain is scheduled with \( \gamma_{INJ} \) to minimize angle-of-attack

- **Shaping Term:**
  - MCPG only
  - Applies short-lived acceleration in direction opposite to cross product term to induce observability
  - \( K_2 \) gain is scheduled with \( \gamma_{INJ} \) to maximize effect in regions of poor IFA

\[ \vec{u} = \hat{\vec{v}} \times (\hat{\vec{v}} \times \hat{\vec{v}}_{INJ}) \]
\[ |\vec{u}| = \sin \delta \]
\[ \hat{\delta} = \frac{\vec{u}}{|\vec{u}|} \]

\( \vec{A}_C \) = commanded acceleration vector
\( \vec{V} \) = velocity magnitude
\( \hat{\vec{v}} \) = current velocity unit vector
\( \hat{\vec{v}}_{INJ} \) = commanded injection velocity unit vector
\( \hat{\delta} \) = cross product unit vector
\( d \) = angle between \( \hat{\vec{v}} \) and \( \hat{\vec{v}}_{INJ} \)
\( K_1 \) = cross product term gain
\( K_2 \) = shaping term gain
• Second stage guidance is closed-loop

• At lower injection angles, accelerations are limited early in second stage to meet the aero-thermal constraint

• Used to generate a fan of trajectories for varying injection angles and flight times to span the kinematic battlespace

• ADOP measured at various flight times along each trajectory to create observability maps
IFA Analysis
• IFA performance measured by ADOP observability maps covering the battlespace

• ADOP maps generated for each guidance law:
  • CPG
  • DCPG
  • MCPG

• ADOP maps examined for two types of aiding:
  • Radar only
  • Radar & GPS

• ADOP maps examined at two trajectory events:
  • $2^{nd}/3^{rd}$ stage separation
  • $3^{rd}/4^{th}$ stage separation
Example ADOP Histories

- ADOP time histories show improvement in IFA performance at 3\textsuperscript{rd}/4\textsuperscript{th} stage separation over 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation.
- IFA performance improvement at the later flight time results from:
  - Additional time for aiding from outside sensors
  - Additional accelerations from the 3\textsuperscript{rd} stage
• IFA improves from 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation to 3\textsuperscript{rd}/4\textsuperscript{th} stage separation for both aiding methods
• IFA improves for radar & GPS aiding over radar only aiding
• IFA requirement satisfied over majority of the battlespace for the radar & GPS aiding case at the 3\textsuperscript{rd}/4\textsuperscript{th} stage separation point
- IFA improves from 2nd/3rd stage separation to 3rd/4th stage separation for both aiding methods
- IFA improves for radar & GPS aiding over radar only aiding
- IFA requirement satisfied over majority of the battlespace for the radar & GPS aiding case at the 3rd/4th stage separation point
• IFA improves from 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation to 3\textsuperscript{rd}/4\textsuperscript{th} stage separation for both aiding methods
• IFA improves for radar & GPS aiding over radar only aiding
• For radar & GPS aiding, IFA requirement satisfied over most of the battlespace at 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation and satisfied over the entire battlespace at 3\textsuperscript{rd}/4\textsuperscript{th} stage separation
Maps of Burnout Velocity

- Battlespace is slightly reduced in ground range with DCPG and further reduced in altitude with MCPG
- Burnout velocities are slightly decreased for DCPG and further reduced for MCPG in the regions of largest trajectory shaping

<table>
<thead>
<tr>
<th>Normalized Velocities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 - 0.6</td>
</tr>
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</table>
Summary
Summary

- IFA is necessary to meet the KW seeker pointing requirement
- ADOP is the trajectory induced IFA observability metric
- IFA performance has been analyzed for three different second stage guidance laws
- The addition of GPS aiding significantly improves IFA
- The longer aiding period for 3\textsuperscript{rd}/4\textsuperscript{th} stage separation improves IFA over 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation
- Both DCPG and MCPG provide improved IFA performance over CPG
- Using MCPG and with radar & GPS aiding, the IFA requirement is satisfied over the majority of the battlespace at 2\textsuperscript{nd}/3\textsuperscript{rd} stage separation and over the entire battlespace at 3\textsuperscript{rd}/4\textsuperscript{th} stage separation
- Both burnout velocity and the overall battlespace are slightly reduced for DCPG and MCPG
Backup Slides
Example Pointing Error Allocation

**Assumptions:**
- Pr target within radius = 0.9974
- \( V_C = 4068 \text{ m/s} \)
- \( T_{GO} = 24 \text{ s} \)
- \( R_{SHIP/TARGET} = 261.3 \text{ km} \)
- Angle Error = Range Error / \( (V_C \times T_{GO}) \)

**KW Field of Regard Radius**
- 11.0 mr [15.6 mr]
  \( \times 3.45 / 3 \)

**Boresight Error Requirement**
- 9.6 mr \( 3\sigma \) [13.6 mr]

**KW Attitude Error**
- 4.2 mr

**Third Stage Attitude**
- 7.1 mr

**KW ACS Control**
- 1.5 mr

**KW Seeker to KW IMU**
- 2.5 mr

**KW IMU to 3rd IMU Transfer Alignment**
- 3 mr

**Uncorrelated Relative Target/ Missile State Errors From Ship**
- 5.0 mr [10.8 mr]

**GPS used to achieve allocation**
- Ship Radar to Missile Stage 3 IMU Inflight Alignment
  - 7.1 mr \( (A_{21}) \)

**GPS used to achieve allocation**
- Msl Stage 3 IMU to True SBEF (ECEF)
  - 5.0 mr \( (A_{32}) \)

**Filter Position Errors**
- 450 m, 4.6 mr

**Filter Velocity Errors**
- 5 m/s \( \times 5 \text{ s} = 25 \text{ m}, 0.26 \text{ mr} \)

**Time Tag Errors**
- 20 ms (small)

**Target-to-Missile Track Bias**
- 131 m, 1.3 mr [950 m, 9.7 mr]

**Notes:**
- Bold numbers are allocated values
- Shaded boxes indicate where GPS measurements are used to achieve allocations
- Brackets are target/missile track on different radar faces
GPS And Radar Measurement Aiding For Missile Navigation

\[ \{ \rho_i, i = 1,2,3,4 \} = \text{GPS Pseudo Range Measurements} \]

\[ \{ \Delta \rho_i, i = 1,2,3,4 \} = \text{GPS Delta Pseudo Range Measurements} \]

\[ \rho_1 \]

\[ \rho_2 \]

\[ \rho_3 \]

\[ \rho_4 \]

\[ \mathbf{L}_2 = \text{Linear Position Change From The Initial SBEF Reference} \]

\[ \mathbf{P}_{M/S} = \text{Position Of The Missile Relative To The Ship From AEGIS WCS Radar Track Processing} \]

\[ \mathbf{L}_1 = \text{Ship Based Earth Fixed Reference Vector} \]

\[ \mathbf{L}_1 \text{ REFERENCE} \]

\[ \mathbf{P}_{M/S} \text{ REFERENCE} \]

\[ \text{EARTH CENTER} \]

Note:
\[ L_1 \text{ and } L_2 \text{ are taken from the AEGIS Ship Navigation System} \]

AEGIS Derived Missile Position In The ECEF Frame Requires Both Radar Measurements And Ship Navigation System Data
• 3 Position Errors
• 3 Velocity Errors
• 3 Missile Attitude Errors
• 3 Gyro Drifts
• 3 Accelerometer Biases
• 2 GPS Receiver Clock Errors (Bias & Drift)
• 3 SPY Radar Face Misalignments
• 3 Ship Initial Position Biases
Note: The radar track of the missile is assumed to be constrained to SPY face 0.
ADOP Calibrated Against Detailed Navigation Simulation

- ADOP Alignment Error Comparisons with Detailed 6-DOF Navigation Simulation:

<table>
<thead>
<tr>
<th>Trajectory Case</th>
<th>ADOP Alignment Error @ KW Ejection (mrad)</th>
<th>Radar Only</th>
<th>Radar &amp; GPS</th>
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</thead>
<tbody>
<tr>
<td></td>
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<td>ADOP</td>
<td>NAVSIM</td>
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<tr>
<td>2</td>
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<tr>
<td>11</td>
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<td>16.3</td>
<td>18.2</td>
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