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Practical Calculations for the Newtonian Secondary Mirror

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INTRODUCTION

A Newtonian reflecting telescope can be designed to perform more efficiently than any other type of optical system, if one is careful to follow the laws of nature. One must have optics made from precision Pyrex or a similar material and figured to a high quality. The mounting hardware must be well planned out and properly constructed using highest quality materials.

The Newtonian can be used for visual or photographic work, for "deep sky" or "planetary" observing, or a combination of all these. They are easy to layout and to construct using simple household tools. The design mathematics is simple and can be easily accomplished by hand calculator. The Newtonian reflector can be easily modified for other types of observing such a photometry, photography, CCD imaging, micrometer work, and more.

Newtonian telescopes are less expensive than Refractors or Cassegrains and when properly designed can equal the high contrast images of a Refractor without the associated chromatic aberration. Approaching a chromatic aberration free system is the modern APO refractor. A Schmidt-Cassegrain can cost as much as three times more than a Newtonian, a Refractor as much as ten times more. Of course, the "roll-your-own" idea usually reduces this cost even further. Remember this can vary greatly according to how large the aperture is. The larger the aperture the higher the cost. Don't forget you do have to account for the cost of the tools used and your time in the total.

Since a reflecting telescope does not suffer for chromatic aberration we don't have to worry about focusing each color while observing or photographing with filters as we would in a single or double lens refractor. This is a problem especially associated with photography. Since the introduction of the relatively low cost Apochromatic refractors (APO) in the past few years these problems no longer hamper the astrophotographer as much. However, the cost of large APO's for those requiring large apertures is prohibitive to most of us who require instruments above 12 or so inches.

Remember, even with the highest quality optics a Newtonian can be rendered nearly useless by tube currents, misaligned components, mirror stain, and a secondary mirror too large for the application of the instrument. The size of the Newtonian secondary and how it effects image contrast will be the main topic discussed here.

Some fundamentals in Newtonian design will be discussed along with methods of optimizing existing telescopes if one chooses not to build from the ground up. However, when selecting the aperture and focal length of a new telescope we must first decide what the instrument is going to be used for.
A LITTLE ABOUT APERTURE AND FOCAL LENGTH

It is no secret among telescope makers (TM) that the larger the aperture the more light the instrument gathers and the higher the resolution will be. Resolution is determined solely by the aperture and is often confused with image quality or a loss in contrast.

The choice for those primarily interested in "deep sky" observing is usually a fast focal ratio, or Richest Field, Newtonian (f/4 to f/5). Planetary work requires a large image scale usually provided by the standard (f/7 to f/8) or long focus Newtonian (f/10 to f/12). The focal ratio (f/#) is the focal length divided by the aperture. Also, don't believe those who warn you to stay away from faster focal ratio telescopes because of some rumors they may have heard. Faster mirrors are more difficult to figure, yes; however, no one says they are impossible. In fact, there are several opticians around this country who can figure excellent f/3's and f/4's. Remember that a Classical Cassegrainian is composed of f/4 or f/5 primacy with a magnifying secondary to increase the effective focal length to the system.

We often meet two types of people who do not like to figure fast mirrors. First are those who can't, and second, those making mirrors for planetary observers who need a longer focal length to increase the image scale. However, fast or short focal ratio reflecting telescopes usually requires a larger secondary and produces a smaller coma free field than the slower or longer focal ratio instruments. This will be discussed later. As a point of interest, the light gathering power of a mirror depends its square area and is calculated simply by:

$$A = \pi r^2,$$

where $A$ is the area of the mirror, $\pi$ is 3.14159265, $r$ is the radius of the mirror.

An example of the loss in light gathering caused by a 3-inch secondary on an 8-inch primary mirror, e.g., $r_{\text{sec}} = 1.5$

and $r_{\text{prim}} = 4$:

$$A_{\text{sec}} = \pi (1.5)^2 = 7.1 \text{ square inches},$$

and,

$$A_{\text{prim}} = \pi (4)^2 = 50.2 \text{ square inches}$$

Dividing the square area of the secondary by the square area of the primary yields the percent of loss in light gathering power:

$$\text{loss} = \frac{A_{\text{sec}}}{A_{\text{prim}}} = \frac{7.1}{50.2} = 0.14 \text{ or } 14\%$$

Before going any further an understanding of several aspects of the human eye needs to be addressed. After all, in the final analysis the human eye is the ultimate test instrument for judging telescope image quality. It takes a long period of time while observing with a full range of astronomical objects in various atmospheric conditions and weather to fully test a particular telescope. Changes in design and component placement are bound to occur, so, to
minimize this we should consider the individual's physical constraints—such as age, how much they are willing to invest, their astronomical interest, and conditions of the eyes.

THE OBSERVER'S EYE

It is commonly known the pupils of our eyes may open as much as 7mm, even 8mm, after we have been in complete darkness for twenty or thirty minutes. While this may be true when we are young, remember; as we grow older our pupils do not open as wide and generally by the age of 45 a person's eyes may only open to around 5mm. So, we have the first design constraint to work with—the maximum opening or aperture of our eye pupil. You are not wasting light if this is larger than your pupil opening is, just a beginning design constraint. Also, eye fatigue or observing bright objects in the telescope will cause a smaller pupil opening.

When looking through the telescope eyepiece we actually see the magnified image of the primary mirror and the focal plane is located a short distance in front of the field lens when the image is in focus. It's like looking at a small disk of light with a micrometer. The image is projected onto our eye a short distance from the eyepiece "eye lens" and this point is called the exit pupil (EP) (See Figure 1). To determine the lowest effective magnification for our instrument divide the aperture by the exit pupil, which in essence is the observer's fully opened eye pupil. This can be found by:

\[
M = \frac{D}{EP},
\]

where \( M \) is the magnification, \( D \) is the aperture, and \( EP \) is exit pupil.

By dividing the telescope focal length by this magnification gives us the longest focal length eyepiece you will need. With the lowest effective magnification that produces a 5mm or 7mm EP you can select the linear image size of the focal plane. Most often though the size and types of eyepieces generally dictate this on hand, so make an inventory of you eyepieces.

For example, consider a person with a collection of eyepieces with field lenses of 0.75 inches (19mm) and an assumed dark-adapted pupil diameter of 8mm. His or her telescope may have an aperture of 10 inches (254 mm) with a focal length (FL) of 60 inches (1524mm). The lowest effective magnification will be:

\[
M = \frac{254}{8} \text{ or } 31.75x
\]

In this example a 48mm eyepiece would be the longest focal length eyepiece you would need \( (1524/31.75 = 48) \).
Figure 1. The position of the exit pupil (EP) relative to the eyepiece lenses and the focal plane.

Generally speaking, most eyepieces have field stops or diaphragms a short distance in front of the field lens in the barrel. Some don't, so, the field stop will then be the diameter of the field lens. Usually, field lenses in 1.25-inch barrel eyepieces are 0.75 to 1.0 inch in diameter. In two-inch barrel eyepieces the field lens is usually 1.50 to 1.75 inches (considering the field lens cell and barrel thickness).

As a rule, for most casual planetary and deep sky observers a linear image size between 0.5 to 0.75 inches will do fine. The critical planetary observer may want a small image, say, and 0.25 to 0.375 inches for a small angular field. The critical deep sky observer with two-inch eyepieces may want a 1.0 to 1.5-inch image for a wide angular field. It makes no sense to use eyepieces that produces a larger exit pupil than your eye can accommodate and this should be one of the limiting factor in determining the secondary size.

Another consideration with Newtonians is how much of the linear image is free from coma, so, we figure the coma free field (CFF) in inches by:

\[ \text{CFF} = 0.000433 \left( \frac{F}{R} \right)^3 \]

where \( F/R \) is the focal ratio.

In the above example the focal ratio of the 10" is FL/D or 60/10 = 6. So, the coma free field would be:

\[ \text{CFF} = 0.000433 \times 6^3 = 0.094 \text{ inch} \]

To calculate the angular field (I) in degrees of our telescope use the following:

\[ I = \text{arctan} \left( \frac{\text{ID}}{\text{FL}} \right) \]

where \( \text{ID} \) is the field stop or field lens I.D. and \( \text{FL} \) is the focal length of your primary mirror.

http://www.m2c3.com/alpocs/tdl2000/telescopemath06202000/field1.htm

6/15/01
The Effects of the Newtonian

The angular field of the 10" f/6 in the example above with a 0.5-inch linear image will be:

\[ I = \arctan (0.5/60) = \arctan (0.008) = 0.477 \text{ degrees} \]

or 28.3 minutes of for the coma free field:

\[ I = \arctan (0.094/60) = \arctan (0.0016) = 0.09 \text{ degrees or 5.4 minutes of arc} \]

Remember the rule about contrast -- we must keep the size of the obstruction or secondary small for good image contrast. But, what is contrast?

**IMAGE CONTRAST**

Since Newtonains require a secondary mirror to reflect the primary image to the side of the tube it has to be positioned somewhere in the optical path. This causes an obstruction in the optical path and reduces some of the light gathering power of the primary. More important this obstruction adversely affects image contrast. If we scatter stray light throughout the image it makes the dark areas of the object brighter and the bright areas darker, therefore, a loss in image contrast. What really happens is the obstruction of the secondary tends to remove light energy from the Airy disc and distributes it among the dark and bright rings in the diffraction disc of a stellar image or many points in an extend image.

Image contrast, as perceived by our eye, is the difference in brightness or intensity between various parts of the telescopic image, i.e., and a star against the background sky. A simple formula for calculating contrast is as follows:

\[ c = (b2 - b1) / b2, \]

where \( b1 \) and \( b2 \) are the intensities levels or brightness measured in candle power/meter squared (cd/m²) of two areas of the object and \( c \) is the contrast.

For example, Jupiter has a surface brightness of around 600 cd/m² for light areas. If we compare a dark belt of 300 cd/m², then the contrast between these areas would be:

\[ c = (600 - 300)/600 = 0.5 \text{ or 50%} \]

If we scatter light from the bright area, say 50 cd/m² and add it to the dark belt then the contrast between the two becomes:

\[ c = (550 - 350)/550 \text{ or 0.36 or 36%} \]

A relatively small amount of scatter may cause a significant decrease in image contrast. The Earth's daylight sky brightness has been measured at about 8000 cd/m² [Chapman et al, 1980].

An image of a star formed by a perfect lens or mirror (perfect paraboloid in a Newtonian) is

http://www.m2c3.com/alpocs/tdl2000/telescopemath06202000/field1.htm
seen as a spot of light surrounded by several bright rings and dark spaces separating the rings. We theorize that 84% of the light falls in the central spot, or "Airy Disc," 7.1% in the first bright ring, 2.8% in the second, and 1.5% and 1.1% in the third and forth rings [Hurlburt, 1963, and Johnson, 1964, and Stoltzmann, 1983].

When something blocks off some in the telescope entrance light will flood or scatter into the rings and other aberrations will scatter light to the outer edge of the ring system. This all lowers the image contrast. A poorly figured mirror will also produce an image of a star with a dull "Airy disc," a weak 1st ring, and very bright and broad second ring and so on, which decreases image contrast! (See Figure 2).

From the graphs and tables published in the referenced articles on Newtonian improvements in the Journal of the Association of Lunar and Planetary Observers (J.A.L.P.O.) a general equation can be arrived approximating the "contrast factor" value for your system (See Table I):

\[
CF = 5.25 - 5.1x - 34.1x^2 + 51.1x^3
\]

where x is the obstruction ratio or secondary/primary diameters.

Table I. Values for obstruction and contrast factor. The left-hand column is obstructions in percentage of secondary to primary mirror in linear diameters. The values in the right-hand column can be found in the reference states in the text.

<table>
<thead>
<tr>
<th>Obstruction</th>
<th>CF</th>
</tr>
</thead>
<tbody>
<tr>
<td>0%</td>
<td>5.25</td>
</tr>
<tr>
<td>10%</td>
<td>4.46</td>
</tr>
<tr>
<td>20%</td>
<td>3.28</td>
</tr>
<tr>
<td>30%</td>
<td>2.03</td>
</tr>
<tr>
<td>40%</td>
<td>1.02</td>
</tr>
<tr>
<td>50%</td>
<td>0.55</td>
</tr>
</tbody>
</table>

An image of a planet or extended deep sky object may appear sharp and bright, but, barley show any surface details in a telescope with 35% obstruction. If the obstruction was reduced then this same telescope will show very fine surface details and give refractor quality high contrast images. If the obstruction is reduced below around 15% one may be hard pressed to see much difference from the on. This can be accomplished without perceptible vignetting.
of the image.

SECONDARY TO FOCAL POINT

The Newtonian secondary mirror, or diagonal, is usually centered within the telescope at a point near the opposite end of the tube from the primary mirror. The distance from the secondary mirror to the focal point (C) can be found by dividing the outside diameter of the tube (T) by two and adding the length of the fully racked-in focuser (H). This distance becomes important when selecting the final image size and size of the secondary mirror. The size of the secondary mirror is a key aspect when attempting to optimize our telescope for maximum efficiency. Some telescope makers add a half-inch (See Figure 3).

Figure 3. Cross section drawing of placement of secondary mirror centered in the optical path of a typical reflecting telescope. C is the distance from the secondary mirror to the focal point, T is the outside diameter of the tube, and H is the fully racked-in focuser.
Figure 4. Drawing of placement of secondary mirror centered in the optical path of a typical reflecting telescope. $C$ is the distance from the secondary mirror to the focal point.

The final choice for the tube diameter is predicated on the aperture and how much extra space is required for airflow. You will want at least 0.75 to 1 inch clearance between the primary and the inside of the tube because heat waves ("tube currents") from its walls will enter the optical path causing similar effects as bad seeing. So, we have one parameter for selecting $C$ -- tube inside diameter should be the aperture plus 1.5 or 2 inches.

Next we will now choose our focuser. In recent times several manufacturers have opted for shorter and more compact focusers because the longer ones require a larger secondary mirror to form a decent size image at the focus. Now we can purchase a rack and pinion focuser with a low profile of about 1.4 to 1.6 inches or some helicals as low as 0.75 to 1 inch. This distance is added to the half diameter of the telescope tube to determine the important secondary to eyepiece distance. The shorter this distance the smaller the secondary. Another thing, the depth of focus for fast Newtonians, $f/4$ to $f/5.5$, is usually very shallow and focusers with 3 or 4 inch travel is way too much for visual observing or even placing most cameras for photographing.

**SECONDARY SIZE DETERMINED BY IMAGE SIZE**

To determine the absolute minimum size the Newtonian secondary mirror can be and remain in the geometric cone of light formed by the primary mirror to the focal point let's start with this equation:

$$\text{minimum secondary} = \frac{DC}{F},$$

where $C$ = the distance from the secondary to the focal plane, $D$ = the primary diameter, and $F$ = the focal length.

However, this calculation only gives the minimum size and in order to fully illuminate the image of the primary at the focal plane we must increase the size of the secondary by a small amount. This can be found by the following:

$$100\% \text{ illuminated sec.} = \frac{C(D - i)}{F + i},$$

where $i$ is the linear image size at the focal plane.

If we use the 10" $f/6$ in the above examples with a 12" O.D. tube and 1.6" focuser then the minimum secondary will be:

$$\text{minimum secondary} = \frac{(10 \times 8.1)}{60} = 1.35 \text{ inches}$$

where $C$ = half tube diameter plus focuser height plus a half inch fudge

$$C = \frac{12}{2} + 1.6 + 0.5 = 8.1 \text{ inches}$$
For a fully illuminated image of 0.5-inch, the secondary would be:

\[
\text{fully illuminated secondary mirror} = 8.1(10 - 0.5)/60 + 0.5 = 1.78 \text{ inches}
\]

In other words, the focal plane is not just a point, it is a circle or disc that represents the image of the primary mirror. But, how large should this illuminated image be and is it necessary to illuminate the entire field?

REDUCED SECONDARY MIRROR SIZE

Some telescope makers point out that a loss of a half magnitude at the edge of the image field is barely noticeable to the visual observer. This is only one photographic \( f/ \)-stop and they say this is acceptable for photometric purposes. An article illustrating a reasonable falloff in the illuminated field of no more than 0.5 magnitude was published in the March 1977 Sky and Telescope [Peters et al., 1977] and gave a set of complex equations as follows:

\[
M = 2.5 \log(1/I),
\]

where "I" is found from:

\[
I = \left[ \arccos A - x \sqrt{1 - A^2} + r^2 \arccos B \right] / \pi
\]

hence:

\[
r = cF/D, \quad x = 2b(F - I)/D, \quad A = (x^2 + 1 - r^2)/2x, \quad B = (x^2 + r^2 - 1)/2xr
\]

where \( M \) = magnitude, \( D \) = primary diameter., \( F \) = primary focal length, \( I \) = secondary to focal point, \( I \) = image diameter (linear \( c \) = secondary minor axis diameter, \( b \) = distance from center of secondary to edge of field, and \( \pi = 3.14159265 \)

NOTE: angles expressed in radians

For example, a 6-inch \( f/4 \) with a 1-inch linear image, 3.5-inch high focuser, and 7-inch tube diameter, should have a 2.46-inch secondary (41% obstruction with a 0.94:1 CF). Following the equations above, the secondary could be reduced to 1.75" (29.2% obstruction and 2.13:1 CF) before a 0.5 magnitude loss would occur at the edge of the field. In fact, you will not reach even 0.35 magnitude loss with this setup. Quite a difference in obstruction and you would most likely notice an increase of more than twice the contrast before noticing the loss in magnitude at the edge of the field. Besides, you lose 17% of the light gathering power with the larger secondary and only 8.5% with the smaller one. In practice, however, the difference in contrast is barely perceptible below 15% obstruction.

To prove the above theory, let's step through the equations:

\[
r = cF/D
\]
\[ r = \frac{1.75 \times 24}{7.25 \times 6} = 0.966 \]

\[ x = 2b(F - l)/D, \ b = \frac{1}{2} \text{ linear image, so } 1/2 = 0.5'' \]

\[ x = \frac{(2 \times 0.5) \times (24 - 7.25)}{6.25 \times 6} = 0.385 \]

\[ A = \frac{x^2 + 1 - r^2}{2x} \]

\[ A = \frac{0.385^2 + 1 - 0.966^2}{2 \times 0.385} = 0.281 \text{ (arccos } A = 1.29 \text{ radian)} \]

\[ B = \frac{x^2 + r^2 - 1}{2xr} \]

\[ B = \frac{0.385^2 + 0.966^2 - 1}{2 \times 0.385 \times 0.966} = 0.108 \text{ (arccos } B = 1.46 \text{ radian)} \]

\[ I = \frac{\text{arccos } A - x \sqrt{1 - A^2} + r^2 \text{ arccos } B}{\pi} \]

\[ I = \frac{\text{arccos } 0.281 - 0.385 \sqrt{1 - 0.281^2} + 0.966^2 \text{ arccos } 0.108}{3.14} \]

\[ = \frac{1.29 - 0.37 + 1.46 \times 0.613}{3.14} \]

\[ = \frac{0.92 + 1.36}{3.14} = 0.726 \]

\[ M = 2.5 \log(1/I) \]

\[ M = 2.5 \log(1/0.726) = 2.5(0.139) = 0.349 \]

The sizes of secondary mirrors in the above examples are calculated values and one should note that most manufacturers produce common sizes and you will have to pick a size nearest or slightly above the calculated size if it is close to the minimum.

**DISCUSSION**

The size of the secondary mirror in a reflector causes lots of arguments among amateur telescope makers (TM) and much of it comes from books that may confuse people or in effect establish hard and fast rules that are misunderstood. Sometimes, these books only define the extreme limits of the telescope optics and omit practical limits. Equations are published that are usually correct and one only has to find the necessary variables to insert in equation to come up with a reasonable design.

One variable is the illuminated field or finally image diameter at the focuser a short distance past the focuser. How much of a field do we need to produce an image of a planet or a deep sky object of comparable size? This variable is one thing the designer has control over, so this may be a start in this discussion. Usually the illuminated field is larger than the particular object we focus in the field. A telescope used for wide field or low magnification observing then they will of course require a larger secondary mirror than a telescope that is used for or higher magnification narrow field observing. The field is the important factor.
here that we can use to determine the smallest linear image we require for our observing needs and therefore the size of the secondary mirror.

Generally speaking, most eyepieces have field stops or diaphragms a short distance in front of the field lens in the barrel. A Typical field lens diameter is around 0.75 to 1.0 inches (19 to 25.4mm) in diameter for 1.25-inch barrel eyepieces. The field lens of shorter focal length eyepieces can range from 0.9 to 0.6 inches (5 to 15 mm). As a rule, at least for most casual planetary and deep sky observers, a linear image size of 0.5 inches (12.7mm) will do fine to sufficiently illuminate an image field. The critical planetary observer may want a smaller image, say, 0.25 to 0.375 inches (6.3 to 9.5mm) for a small angular field.

The critical deep sky observer with two-inch eyepieces may want a 1.0 to 1.5-inch (25.4 - 38mm) image for a wide angular field. It makes no sense to use eyepieces that produces a larger exit pupil than your eye can accommodate and this should be one of the limiting factor in determining the secondary size. Observers younger than around 45 years of age may be able to open their iris to 7 or 8 millimeters. However, as age catches up with this diameter of he eye’s iris may not open wider than 5 or 6 mm.

Looking through the telescope eyepiece we see the magnified image of the focal plane located a short distance usually in front of the field lens. The image is projected onto our eye a short distance from the eyepiece "eye lens" and this point is called the exit pupil (EP). As a rule, we need only to use eyepieces that would yield an illuminated field of no more than the angular size of the planet that presents the largest subtended angle, plus a little room for the object to move about. Since Venus and Jupiter at closest approach are around 50 seconds of arc we may want to work for there. Jupiter observers may wish to see the four moons in the field so that would constitute about 30 minutes of arc.

REFERENCES


http://www.m2c3.com/alpocs/tdl2000/telescopemath06202000/field1.htm 6/15/01

FURTHER READING

A brief list of important papers on Newtonian telescopes. Some will be hard to find, but, they are out there. Clubs and societies should have a good library that includes reprints of this stuff. If not, start one.


http://www.m2c3.com/alpocs/tdl2000/telescopemath06202000/field1.htm 6/15/01