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Algorithmics Of Motion
A Multi-University Research Initiative

Final Report

2000

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Abstract

The main goal of our research program is to develop the mathematical concepts and algorithms for performing a wide range of operations on motions of physical objects, including: capture, representation, synthesis, segmentation, editing, optimization, simulation, and execution. In addition, we have focused on the support tools that are critical for understanding and analyzing physical and synthetic agents interacting in a dynamic environment. These include the underlying data structures, the architecture, and the analysis and synthesis tools. The term Algorithmics of Motion refers to the overarching framework, the mathematical concepts, the algorithms and the tools for this work. This document presents an executive summary of our work over the last five years from October 1995 to September 2000.

1 Introduction

Algorithmics of Motion is a multi-university research initiative between two universities: The University of Pennsylvania and Stanford University. The main goal of the MURI is to bring together a group of distinguished researchers with complementary expertise to develop the framework, mathematical concepts, algorithms, data structures, architecture, and analysis tools for performing a wide range of operations on motions of physical objects including: capture, representation, synthesis, segmentation, editing, optimization, simulation, and execution.

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- Ruzena Bajcsy (University of Pennsylvania)
- Kostas Daniilidis (University of Pennsylvania)
- Leonidas Guibas (Stanford University)
- Vijay Kumar (University of Pennsylvania)
- Jean-Claude Latombe (Stanford University)
- Max Mintz (University of Pennsylvania)
- Rajeev Motwani (Stanford University)
- Jim Ostrowski (University of Pennsylvania)
- Camillo Taylor (University of Pennsylvania)
- Carlo Tomasi (Stanford University)

2 Our Goals and Vision

The main goals of our research program are the framework, mathematical concepts, and algorithms, for performing a wide range of operations on motions of physical objects, including: capture, representation, synthesis, segmentation, optimization, simulation, and execution. An important aspect of this research is the suite of support tools that are critical for understanding and analyzing physical and synthetic agents interacting in a dynamic environment.

The world is governed by fundamental laws of physics that describe the relationships between the position and motion of objects and the forces that act on them. In order to understand such physical agents, it is necessary to have the ability to model, analyze and control such machines as cruise missiles, computer-controlled tanks, and autonomous and semi-autonomous agents at the micro, meso, and macro scale. An important aspect of the functioning of these machines is their interaction with a dynamically changing environment. As computers and computer networks become ubiquitous, we now have the ability to create synthetic human agents that can interact with virtual models of physical objects (including machines) in a virtual world. However, even here, it is important that the motion and interaction of the synthetic agents conform to the laws of physics and the dynamic constraints that are representative of the real world. Finally as computers and computer controlled machines are designed to augment a human user's capabilities, the relative motions and forces involved in the interaction between the human agent with other physical agents becomes critical. Our research is driven by the fundamental problems that arise in capturing and representing motion, synthesizing and planning motion, and executing or controlling a planned motion.

While the objective of this research initiative is to solve fundamental research problems, our thrust areas and the problems are driven by many important applications.

Military Applications:
- Autonomous or semi-autonomous vehicles traversing a hard, uneven terrain, and transporting heavy equipment;
- Support for marines landing on hostile beaches and mine laden ports; and
- Support for special operation forces in urban warfare, monitoring illegal activities, scouting buildings, clandestine access to buildings, and reconnaissance.

Civilian Applications:
- Manufacturing applications: designing collision-free paths for assembly, reverse engineering of complex three-dimensional products, and design of one-of-a-kind products (assistive devices for people with disabilities and aging people);
- Medical applications: synthesis of new drugs, computer controlled surgery, and tele-immersive technologies for tele-surgery.
3 Technical Objectives

Our technical objectives and research agenda are addressed under the important thrust areas that we have identified. These thrust areas can be divided into the three different foci of the initiative.

3.1 Motion Capture

Motion capture entails sensing and measuring attributes of the external world, analyzing the measurements, and representing this information efficiently in a form that can be used for motion synthesis and execution.

The key thrust areas under motion capture are:

- Recovery of 3-dimensional environments;
- Image motion analysis; and
- Vision-based reconstruction for mobile robots and telerobotics.

3.2 Motion Synthesis and Planning

Motion synthesis refers to the techniques used to automatically generate motion plans (e.g., collision-free paths based on geometric information, or sensor-based motion strategies) from spatial, geometric, and temporal data. Plans may be generated from scratch or by modifying or editing pre-existing plans to optimize over a set of meaningful criteria.

The key thrust areas are:

- Advanced motion planning;
- Sensor-based motion planning; and
- Sensor fusion for tracking and interception of moving targets.

3.3 Motion Execution

Motion execution is the controlled generation of the synthesized motion in a graphically simulated or in a physical world. Motion execution in the real world requires a tight coupling with motion capture, modeling and representation of motion, synthesis, and planning. Motion execution in the graphical world entails representation of motions, synthesis, planning, and rendering.

The key thrust areas are:

- Collision detection using kinetic data structures;
- Control and coordination of multiple robots; and
- Control of advanced locomotion systems.

The key accomplishments and the plans for future research in each thrust area are briefly described in the following sections. The effort in the thrust areas is summarized in Table 1.
4 Technical Approach

Our technical approach is anchored in a problem solving loop. As stated in our technical objectives, we are interested in a class of problems that involve agents interacting with dynamically changing environments. Hence, the problem solving loop consists of:

1. Using mathematical tools to create conceptual models that we believe best describes the problem or the sub-problem. These models can be equations and/or algorithms.
2. Use these models to develop computer simulations to test the validity of our problem solving.
3. If the problem addresses an interaction of physical agent and/or physical environment, we build a physical model to verify our initial assumptions and conceptual models.
4. Therefore we conduct experiments for validation and benchmarking.
5. If these experiments agree with our expectations/problem specifications then we are done, otherwise we refine the conceptual model and repeat the steps 2-4.

This methodology is a common theme in all the research presentations.

Table 1 The Key Thrust Areas, Principal Researchers, and Leveraged Funding and Collaborations.

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5 Synergistic Efforts

5.1 Collaboration Between Researchers at the University of Pennsylvania and Stanford University

Our interaction has consisted of formal visits by the P.I.s on a regular basis. For example, Professors Latombe, Guibas and Tomasi have visited Penn, while Professors Bajcsy, Daniilidis, Kumar, Metaxas, and Taylor have visited Stanford. In addition, the principal post-doctoral students have exchanged visits. Messrs, Lavalle and Gonzales from Stanford have visited Penn, while Desai and Goktas have visited Stanford to discuss MURI collaborations.

In addition to the formal planned visits, we have a number of meetings at such premier conferences as ICRA, CVPR, ISRR, and IROS. As the conference proceedings publications list shows, our MURI is well represented at such conferences.

We list below examples of collaboration between MURI researchers.

- The research conducted by Bajcsy and Latombe on the construction of 3-D models of robot’s environments has resulted in an NSF proposal to develop jointly a software system that will enable a robot to navigate, make sensing operations, and build a 3-D model of its environment. The focus of this proposal is on dealing with uncertainty (errors in robot localization, uncertainty in sensing). NSF has awarded us a one-year grant (approximately $120,000).

- Kumar and Latombe have collaborated on motion planning algorithms. Latombe’s methods are used to generate an initial plan that satisfies geometric constraints while Kumar’s methods are used to refine the plan to satisfy kinematic and dynamic constraints while optimizing a cost function.

- Lavalle, Kumar, and Mintz have had a fruitful collaboration on game-theoretic methods for motion planning in the presence of uncertainty.

- Daniilidis, Tomasi, and Taylor have had discussions on image analysis and geometric problems in computer vision.

- Guibas and Kumar have started discussions on the complementary properties of kinetic data structures and control graphs, and their applications to controlling multiple autonomous agents.

- Daniilidis, Tomasi, and Taylor were together with Sastry, Kosecka, and Ma the instructors of the ICRA Tutorial on Structure from Motion, San Francisco, April 28, 2000.

5.2 Collaboration with Other Multi-University Research Initiatives

In addition to collaboration within the MURI, we have active collaborations with other MURIs funded by DoD. We provide some examples of such collaborations below.

- Leonidas Guibas and Pankaj Agarwal (Brown/Duke/Johns Hopkins MURI) have been collaborating closely for several years now. In particular they have worked extensively on static and kinetic binary space partitions and their applications in graphics. A student of Agarwal, T. Murali, is now at Stanford as a post-doc under Guibas and Latombe. Guibas and Agarwal have also collaborated on several problems involving moving points or graphs with changing edge weights.
• Bajcsy, Daniilidis, Kumar, and Ostrowski have an ongoing interaction with S. Sastry, J. Kosecka, R. Alur and others (Berkley/Stanford/Cornell MURI) on studies of hybrid systems and vision based control.

• Kumar has started discussion with Robert Howe (Stanford/Harvard/Johns Hopkins MURI funded by ONR) biomimetic robots. A student of his, Jaydev Desai, worked as a post-doctoral researcher at Harvard. Kumar spent his sabbatical (Fall 1999) at Johns Hopkins interacting with the MURI group there.

6 Significant Accomplishments

6.1 Motion Capture

6.1.1 Recovery of 3D environments

We (Daniilidis et al) have developed an algorithm for the recovery of 3D-scenes with a moving camera using minimal calibration. We developed an algorithm for estimating camera parameters and registering several stereo views based only on a few landmarks. As a member of the National Tele-immersion Initiative we set as our goal to make feasible an immersive sense of remote presence, so compelling that it instantaneously creates the illusion of being near to people and objects which is physically miles away. The key issues here are the real-time scene acquisition and the efficient representation of scene descriptions. We have developed algorithms for view-independent scene reconstruction and motion-predicted stereo matching (ISAR 2000, ECCV 2000, ICPR 2000). During an unprecedented session on May 9, 2000, we demonstrated a three-party tele-immersion system with real-time on-line 3D reconstructions of the participating persons transmitted from Philadelphia and Armonk over Internet2, and projected in a spatially augmented display designed in Chapel Hill.

Figure 1 The National Tele-immersion Initiative. A viewer in Chapel Hill, North Carolina, is able to interact with collaborators in Philadelphia (left) and Armonk, New York (right) by viewing three-dimensional reconstructions of their offices (U Penn).
6.1.2 Omnidirectional Geometry

Being aware of the significance of representation in the visual algorithms performance we espoused the opinion with a few other researchers that it would be worth realizing such representations in the sensor level rather than during processing. Our work (Daniilidis and co-workers) in the image formation process was strongly motivated by biological findings. Artificial visual systems face difficulties in tasks like navigating on uneven terrain or detecting other movements. Paradoxically, these are tasks which biological systems like insects, with very simple brains, can very easily accomplish. The difference is that there are hundreds of evolved biological eye designs but only one artificial camera: the CCD-chip with a lens. As a prominent and useful example, we chose to study the basic questions underlying omnidirectional sensing and its realization with cataoptric designs - combinations of reflective and refractive components. Our motivation for a unifying representation of these new sensors lead (ICCV-99, ECCV-2000) to the discovery of a novel geometric theory that described reflections of the 3D world on mirrors followed by projections on the omnidirectional plane. This model unifies all second order reflective surfaces and includes the traditional CCD camera as a degenerate case. It sheds light on the constraints for self-calibration of a camera and facilitates new projective reconstructions based on the concepts of oriented projective geometry. Moreover, in collaboration with Hicks (JIVM 2000) we established robust spatiotemporal signatures for mapping and localization using angular measurements picked up in omnidirectional images.

Work on omnidirectional vision had direct impact on our contribution to the Tactical Mobile Robotics program. During the October 2000 TMR-Demo we showed how the Urbie robot can climb several flights of stairs using omnidirectional vision sensors. Our ongoing work includes a novel sensor design being able to deliver registered and fused information from many directions and in three bands of the spectrum (far IR, near IR, visible).

6.1.3 Camera Motion Estimation

Based on the analysis techniques described above, Tomasi has developed new video processing methods for the computation of three-dimensional properties of the scene and of the camera motion. He proved previous methods to provide heavily biased estimates, and proposed a general and efficient technique for overcoming bias in the presence of outliers. The new method estimates the direction of camera motion from a pair of consecutive video frames with an accuracy of better than one degree, compared to about fifteen-twenty degrees of the best competing methods under equal circumstances. These advances enable the use of camera-driven vehicles and three-dimensional scene reconstruction in realistic scenarios, for applications in automatic surveillance, site inspection, and aerial navigation and survey.

Figure 9: Computation of camera motion. In spite of the very obvious outliers (circled, left) in the image motion measurements, the direction of camera motion (right) is computed with less than one degree of error (red true, blue computed).
6.1.4 Detection of Stereo and Motion Discontinuities

In order to process video, it is important to segment foreground from background, and more generally different objects from each other. The motion field and the disparities from stereo image pairs can both provide important clues for this segmentation. We (Tomasi) have developed motion and disparity based segmentation methods based on energy minimization techniques. Our algorithms alternate between segmenting the image into a number of nonoverlapping regions and finding the affine parameters describing the displacement function of each region. Experiments on real images show the algorithm's ability to find accurate segmentations and displacement maps, as well as discontinuities and creases, from a wide variety of stereo and motion imagery. The results can be used in tracking systems that focus on one of many people in a scene, or in splitting foreground from background for algorithms that use the background to determine camera motion and process the foreground for different purposes.

![Image](image_url)

Figure 10: Depth and motion discontinuities. Even with shallow depths (top left), depth discontinuities (top right) are crisp and accurate. Features moving by small and complex motions (bottom left) are accurately segmented (bottom right) into foreground and background.

6.1.5 The Sensitivity of Structure from Motion

In spite of decades of research, it is still an open question how well camera motion and scene geometry can be recovered from standard video. Tomasi has been conducting a systematic effort to answer this question in a quantitative manner. He has developed a mathematical technique that allows computing the probability density of parameter estimation errors from the probability density of image noise. He has also extended Cramer-Rao bounds to account for constraints, making this powerful predictive tool applicable to computer vision problems. These techniques have made it possible to make statements like the following: "video from a standard television camera with a 20mm lens in front of a scene between 2 and 3 meters away can lead to camera motion estimates that are accurate within 1.2 degrees with image noise of one pixel standard deviation." Bias and probabilistic skew of algorithms can be calculated as well. This type of analysis makes the performance of vision algorithms predictable, and therefore usable in a wide array of applications.
6.1.6 Object Recognition

Taylor has developed a novel approach to the problem of object recognition which exploits a new method for determining whether two sets of image points could be projections of the same constellation of 3D points under scaled orthography. This method shares some of the advantages of image based recognition schemes in that all computations are based solely on image measurements and no information about the underlying 3D structure is assumed. However, the proposed scheme has a significant advantage over such methods in that it is able to correctly recognize images of objects taken from viewpoints other than those in the database. Importantly, the method avoids the combinatoric issues encountered in other feature based recognition schemes by recasting the correspondence problem as a sorting problem.

![Image](image.png)

Figure 2 Three-dimensional mapping for supervised autonomy. A robot with an omnidirectional camera explores the interior of an abandoned building in Fort Sam Houston (left). Omnidirectional images (center) are used to automatically reconstruct the interior of the building (right). (U Penn).

6.1.7 Accurate measurement of material reflectance properties

In order to design robust vision systems, one must understand the Role of Light/illumination and its interaction with surfaces. Towards that goal, Bajcsy and Angelopoulou have designed a laboratory with a spectrograph that enables us to measure Bidirectional Reflectance Distribution Function (BRDF) of surfaces in the spectral domain with resolution of 0.7nm. As far as we know, this is the highest resolution available currently in the vision community and we plan to make it widely available.

This facility enables us to measure the spectral response on different types of light sources and transmittance of various color filters. We developed a new theory of invariance of incident illumination, and performed initial tests to verify this theory, which in turn will help us to separate the effect of illumination from surface reflectance and scene geometry.

6.1.8 Template Matching and Neuroanatomy

To characterize the natural anatomic variability of the human brain so that its confounding effect can be minimized in the identification of brain structures as revealed on high-resolution medical images, a computational framework has evolved in which individual anatomies are modeled as spatially warped versions of a canonical representation of neuroanatomy, known as a brain atlas or template. The assumption is that at a certain spatial scale normal instances of the same anatomy share a common topological scheme and that differences between individuals appear only as variations in the local shape details.

To realize this framework, Gee and Bajcsy have invented the method of elastic matching for determining the spatial mapping between a three-dimensional (3-D) image pair in which one image volume is modeled as an elastic continuum that is deformed to match the appearance of the second volume. Remarkably, in addition to deformable representations of anatomy, the elastic registration technique enables many useful applications. An MRI (magnetic resonance imaging) or X-ray CT (computed tomography) study can be interpreted using knowledge—such as anatomic labels and structural boundaries—transferred from an atlas once the mapping is computed that spatially aligns the atlas with the neuroanatomy depicted in the study. The transformed labels automatically localize their corresponding structures in the image study.
Visualization is achieved by deforming, according to the mapping, the structural boundaries contained in the atlas and then displaying the result. Quantitative measurements of any structure in the study can be directly inferred from the morphometric information associated with its warped atlas version. If functional scans have also been acquired, the physiologic information can be conveniently related through the same mapping to a standard anatomic space defined by the atlas, after the subject's pose in these scans is made the same as in the structural images.

A major thrust of current research has been to construct probabilistic representations that share the practical benefits of the deformable modeling paradigm but additionally attempt to capture the statistics governing biological variability. The work on establishing covariances on the substructures of heterogeneous anatomy entails development of new techniques in applied statistics and has important consequences for the modeling of very high-dimensional structures.

6.2 Motion Synthesis and Planning

6.2.1 Randomized Motion Planning

The complexity of motion planning is known to grow exponentially in the number of degrees of freedom. Latombe and his co-workers have developed and refined successive planning algorithms based on a random sampling scheme of the configuration space of the moving objects (robots or any kind of physical agents). We have implemented and experimented with these algorithms on many examples and applications. Along the way we have also developed important techniques needed to implement random-sampling planners (e.g., collision checking, sampling strategies, space indexing). We have formally proven that under generally satisfied assumptions our random-sampling approach finds a motion path with high probability. In practice, our planners have shown to be quite fast.

Our work on randomized planning has led to what is now commonly considered as the only available practical approach to robot path planning with many degrees of freedom. Many other research institutions, both in the US and in Europe, have students and researchers working on improving and/or extending our random-sampling approach. For example, at MIT, Prof. Feron uses our approach to generate motion strategies for autonomous helicopters; at Texas A&M, Prof. Amato uses this approach to predict protein folding motions. Ourselves have applied our planning methods to a variety of problems, including:

- Planning of radiosurgical procedures of brain tumors for a new system (the Cyberknife) -- this system is now in clinical use at Stanford Medical Center and is marketed by Accuray, Inc.
- Animation of autonomous digital actors -- a company, The Motion Factory, was formed to market motion planning software for digital actors.
- Prediction of ligand-protein binding motions in order to select drug candidates from molecule databases -- this work was initially funded by a grant from Pfizer and is now continued in cooperation with the Biochemistry Department as part of the Stanford's Bio-X initiative.

We recently obtained a 5-year grant from General Motors to adapt our motion planning methods to manufacturing problems.
Figure 3 The Cyberknife System for Computer Assisted Surgery (left) and the use of randomized motion planning algorithms for surgical planning (middle, right). (Stanford).

Figure 4 Biomolecular modeling, planning and simulation algorithms from projects leveraged from MURI funding (Stanford).

6.2.2 Motion Planning with Visibility Constraints

In visibility-based planning, Latombe and Guibas have developed entirely new motion planning methods to allow an agent to move in a physical space (without collision) in order to achieve sensing goals (for instance, map a new environment, find a hiding target, track an evading target). Our methods typically search an information space built on top of the agent's configuration space. They combine two types of computational geometry techniques: collision checking and visibility analysis. Two major methods developed are a next-best-view planner to efficiently build the map of a new environment and an environment-sweep planner to find and locate moving targets.

Probabilistic road-map (PRM) planners have shown great promise in attacking motion planning problems with many degrees of freedom that were previously infeasible. Yet when such a planner fails to find a path, it is not clear that no path exists, or that the planner simply did not sample adequately or intelligently the free part of the configuration space. We propose to attack the motion planning problem from the other end, focusing on disconnection proofs, or proofs showing that there exists no solution to the posed motion planning problem. Just as PRM planners avoid generating a complete description of the configuration space, our disconnection provers search for certain special classes of proofs that are compact and easy to find when the motion planning problem is 'obviously impossible,' avoiding complex geometric and combinatorial calculations. We have demonstrated such a prover in action for a simple, yet still realistic, motion planning problem. Failure of the prover suggests key milestones, or configurations of the robot that can then be passed on and used by a PRM planner. Thus by hitting the motion planning problem from both ends, we can resolve the existence of a path, except in truly delicate border-line situations.
6.2.3 Sensor Based Motion Planning

Kumar and Mintz have developed a framework for motion planning that incorporates set-valued estimates of uncertainty at different levels of granularity. These algorithms yield the best case performance under the worst possible uncertainty, without any assumptions on the underlying distribution.

We have developed algorithms for:

- *open loop motion planning* - the robot(s) plan the optimal path from a start position to a goal position based on the information available at the start position;
- *recursive open loop motion planning* - the robot(s) plan the optimal path from a start position to a goal position based on the information available at the start position, but this path is refined as additional information becomes available;
- *closed loop motion planning* - the robot(s) chose the optimal feedback policy (a control law that relates real time sensory information to actuator inputs) from a start position to a goal position based on the information available at the start position; and
- *recursive closed loop motion planning* - the robot(s) chose the optimal feedback policy (a control law that relates real time sensory information to actuator inputs) from a start position to a goal position and this path is refined as additional information becomes available.

Our main contributions have been threefold. First, we have established saddle point properties for open loop motion plans in a wide class of problems thus guaranteeing global optimality. Second, we have developed algorithms for developing piecewise linear closed loop motion plans. We have shown that at least two linear feedback laws are necessary to circumvent a single obstacle and have established an upper bound on the number of switches required in the general case. Finally, we have algorithms that generate min max solutions for all four classes of strategies.

6.2.4 Sensor Fusion for Tracking and Interception of Moving Targets

Mintz and his students have solved a set of important problems that are fundamental to sensor fusion for control and planning of autonomous robots.

- We have solved the set-valued (optimal fixed-geometry confidence set) problem for an extremely wide range of sampling distributions. The class of distributions includes: unimodal, multimodal probability density functions, which may be symmetric or non-symmetric, and possessing heavy-tailed behavior. The optimal solutions to these decision problems are either continuous monotone piecewise linear rules with alternating sections of unit and zero slope, or non-monotone rules which can be very accurately approximated as piecewise linear rules with sections where the slope is greater than unity or is zero.

- The multivariate versions of the confidence set problem have been solved when the component noise distributions are independent. If the noise components exhibit dependencies, highly accurate approximate solutions are described which are appropriate in all but the most extreme cases. For example, in the case of a multivariate normal distribution, if the correlation coefficients are less than 0.9 in absolute value, then we obtain results that are within 3 percent of optimal.

- For the dynamic version of the confidence set problem, we have obtained optimal results which impose virtually no limits on the dynamic models, and only impose an independence assumption on the additive sensor noise.

- We have applied the theory to a practical industrial application using multiresolution sensors and verified that extremely good correspondence between the theoretical (predicted) performance and the empirically obtained (actual) performance was achieved.
All of these results make use of game-theoretic solutions and robust statistical inference techniques which we have developed over the past several years.

![Figure 5](image)

Figure 5 Sensor fusion for localization and control while climbing stairs. The ISR robot climbing stairs (left). Different estimates of the heading angle provided by the acoustic and vision sensors (center) and their confidence measures (right). (U Penn).

## 6.3 Motion Execution and Control

### 6.3.1 Control and Coordination of Multiple Robots

Kumar and Ostrowski have developed a framework for the control of a team of robots navigating in a terrain with obstacles, while maintaining a desired formation and changing formations when required. This problem is formulated by modeling the team as a triple consisting of a group element that describes the gross motion of the team, a set of shape variables that describe the relative positions of robots, and a control graph that describes the behaviors of the robots in the formation. This model captures the different levels of coordination required for such tasks. For example, at the lowest level, it is necessary for each robot to control its motion, to avoid collisions with its neighbors, and to move along a desired trajectory. This is captured by the shape variables for the formation. At an immediately supervisory level it is necessary to outline a strategy for maintaining a formation. In a system of two robots, this might be as simple as a leader-follower strategy where a leader plans and follows a preferred trajectory while the follower maintains a specified relative position and orientation with respect to the leader. The control laws may be specially designed for manipulation where the two robots grasp and carry a payload cooperatively. When the number of robots is more than two, there might be more than one leader-follower pairs or a more complex structure of interaction. This discrete level of coordination is captured by the control graph. In contrast to the control laws for individual robots where the state variables and inputs change continuously, there are only a finite number of distinct formations and the changes in formations represent discrete changes in the organization or the control graph for the team. Finally, at the highest level, it is necessary to plan the trajectory for the entire team of robots based on the available terrain information. This essentially boils down to finding the optimal trajectory, $g(t) \in SE(2)$.

We have used mathematical tools from graph theory, differential geometry and control theory to solve what is essentially a problem in hybrid systems. We have characterized the different possible formations for a team of $n$ robots by enumerating the possible control graphs. This includes a proof for the upper bound on the total number of possible control graphs and an enumeration of the equivalence classes of mixed graphs that are relevant to this problem. The gross motion is obtained using tools from Lie theory that allow us to exploit the Lie structure of the Euclidean group. Finally, we developed a discrete coordination strategy for changing formations in terms of a sequence of elementary transitions in the control graph that satisfy the constraints in the problem.
The framework and the algorithms are relevant to any scenario in which it is necessary to coordinate a large number of networked embedded systems. For example, in military applications, vehicles are often required to maintain a close formation, and in autonomous vehicle highway systems, there is a need to control platoons of cars in formation while allowing the cars to perform such maneuvers as lane changes, merges, and avoiding obstacles on the road. In the DARPA Tactical Mobile Robotics program, considerable emphasis is being placed on the coordination of autonomous robots in search and rescue missions, collaborative scouting and reconnaissance, and in three-dimensional mapping. Our ideas have direct application in these settings.

6.3.2 Visual Servoing

Taylor and Ostrowski have worked on the problem of controlling the spatial position and orientation of a robotic platform based on the image data obtained from a video camera mounted on that platform. More specifically, they proposed control laws that generate translational and angular velocities that would cause the robot to achieve and maintain a fixed position and orientation with respect to a set of feature points in the scene.

The proposed control schemes make use of well established techniques for computing estimates for the relative orientation of two camera positions from a set of feature correspondences. An important advantage of these control schemes is that it is possible to demonstrate analytically that they are globally convergent even in the presence of large calibration errors in both the intrinsic parameters of the camera and in the extrinsic parameters which relate the frame of reference of the camera to the body frame of the robot platform which is being controlled. Furthermore no a priori knowledge about the structure of the scene is assumed.

The research carried out under the aegis of MURI funding has opened up a number of further research opportunities and collaborations. Taylor’s research on real-time vision based guidance systems carried out under this grant has been incorporated into a project funded by Delphi Automotive to develop real time vision based lane tracking systems for advanced collision warning systems. The research on Vision Based human computer interfaces has led to new ideas on how to develop flexible interface systems that can be customized for disabled individuals. This work will be pursued under a recently awarded NSF grant. Our efforts on vision based control of teams of mobile robots has led to our participation in the DARPA funded MARS program and has sparked a new effort on collaborative robotics in the laboratory.

6.3.3 Robust Control of Remote Systems in the Presence of Communication Delays

Bajcsy and her coworkers have developed a robust motion control algorithm for the appropriate treatment of long latency times. Delays are the most crucial problem in controlling robot vehicles over the network.
Our algorithm is based on $\mu$-synthesis based control and outperforms the known algorithms of the literature. This work was conducted in close collaboration with Latombe and his students at Stanford.

6.3.4 Control of Advanced Locomotion Systems

Ostrowski and his coworkers study of motion planning and control for dynamic mobile robots. Examples from this class that have been studied include a blimp robot, an eel-like robot, and multiple walking (Sony's legged robot) and wheeled robots. The blimp robot provides an excellent example of an underactuated mechanical system, and has involved the study of very fundamental issues of trajectory tracking (e.g., how do you parallel park a blimp?). There have also been important contributions made to the area of dynamic visual servoing, where the dynamics of the robot are embedded in the sensor space (here the vision plane). This allows one to properly control a dynamic system, such as planes, blimps, helicopters, etc., using visual feedback. The eel robot has served to study biomimetic principles applied to the design of amphibious (land-based and underwater) robots. We have developed a methodology for developing control laws for such dynamic robots, and a hierarchical architecture that is used to solve the full motion planning problem of building controllers to move to any desired location. The strategy has been to rely on a decomposition of the system into multiple layers of abstraction, each of which simplifies the control and motion planning problem. Finally, we have been performing work on the coordination and control of multiple autonomous robots. This has been performed in parallel on two separate platforms, with the goal of understanding the trade-offs and benefits of developing modular code that can be applied to both platforms. The wheeled robots have been used to study formation control as well as issues in multi-robot grasping. The legged robots have primarily been used to study formation control and decision-making strategies for cooperative robots competing in robotic soccer through our involvement in the international RoboCup competitions.

Our work has been at the forefront of several areas of modern research. In visual servoing, we have proposed two new paradigms--that of dynamic visual servoing, where the dynamics of the system are directly incorporated into the vision-based control; and visual motion planning, where motion plans are developed in the sensor space (visual plane) rather than with respect to some inertial frame. These tools have important implications for use in autonomous robot exploration and tracking, using dynamic mobile robots such as planes, blimps, spacecraft, or helicopters. Through the support of this MURI, our work has led to two new projects targeted towards extended duration aerial vehicles. The first is a project with Carnegie Mellon University towards developing "infinite" (very long) duration blimps, used for surveillance, environmental monitoring, etc. The second is a project we are starting up with Global Aerospace to develop a control methodology for a global network of balloons. At Penn, there is also a newly established project to build a 30 ft. outdoor blimp to be flown autonomously. Lastly, we are in the planning stages right now to kick off an autonomous aerial vehicle project that seeks to develop a sensor fusion approach (using vision as a central sensor) towards formation flight of airplanes.

![Figure 7 Examples of novel locomotion systems: A modular, robotic eel that is able to swim underwater (left); and an indoor, vision-guided blimp (right). (U Penn).](image)

Our work on the eel has provided new insights into underwater and amphibious locomotion, with applications to covert operations and surf-zone counter-mine and de-mining efforts. The hierarchy developed while working on the eel is applicable to a wide range of dynamic locomotion systems. The general methodology of working with a modular robot has also led to an NSF CAREER award targeted towards the study of modular locomotion systems. The final area of work on multiple mobile robots has led to funding through the DARPA MARS initiative, and sponsorship by Sony Corporation to compete in the Sony legged league of the RoboCup competition.
6.3.5 Collision Detection using Kinetic Data Structures

Computer systems commonly cache the values of variables to gain efficiency. In applications where the goal is to track attributes of a continuously moving or deforming physical system over time, caching relations between variables works better than caching individual values. The reason is that, as the system evolves, such relationships are more stable than the values of individual variables. Kinetic data structures (KDSs) are a novel formal framework for designing and analyzing sets of assertions to cache about the environment, so that these assertion sets are at once relatively stable and tailored to facilitate or trivialize the computation of the attribute of interest. Formally, a KDS is a mathematical proof animated through time, proving the validity of a certain computation for the attribute of interest. KDSs have rigorous associated measures of performance and their design shares many qualities with that of classical data structures.

The KDS framework has led to many new and promising algorithms in graphics and robotics applications. Specific accomplishments include new algorithms for tracking:

- **Extent Attributes**: convex hull, width, diameter, etc. among moving points
- **Proximity Attributes**: closest pair, Voronoi/Delaunay diagrams, etc. among moving points, polygons, or balls; local environment of moving 'agents'
- **Space Partitions**: BSPs for occlusion culling among moving triangles
- **Optimization Problems**: MSTs for moving points or graphs with variable edge weights; linear programming
- **Visibility**: visibility with moving observers and obstacles
- **Collision Detection**: for rigid and deformable objects
- **Routes and Locations**: of mobile stations in ad-hoc networks

A key feature of all these algorithms is that they sample parts of the system under motion only as needed to maintain the desired attribute(s). These leads to algorithms that are efficient, robust, and often easily scalable, as the computations involved are local. KDSs are a completely new research area that came to be primarily because of this MURI funding.

![Figure 8](image)

Figure 8 -- A and B represent ongoing work on protein folding with the goal of simulating the effect of the solvent molecules (water) by a set of potentials and avoiding their explicit modeling in the simulation (Stanford).

7 Interactions with Private Industry

As a group, the MURI researchers have close collaborations and sponsorship from several industrial organizations. A list of industrial partners that are involved in MURI related activities is provided below.
Figure 9 Results from leveraged projects. A wheelchair augmented with robotic legs for superior traction and for overcoming obstacles (left); An inexpensive feeder being used by an individual with a C-6 lesion (center); and (c) A robot arm for children with cerebral palsy with a “game boy” like interface (U Penn).

We provide below several examples of our collaborations with industry.

- Initial demonstrations developed under the MURI research has enabled Tomasi to request and obtain funding from Intel in Santa Clara, CA, for the development of a PC-based system for American Sign Language recognition. This funding comes under a three-year agreement, and involves both student support and equipment.

- Additionally, initial demonstrations developed under the MURI research has enabled Guibas, Latombe, Motwani, and Tomasi to request and obtain an equipment gift from Intel in Santa Clara, CA, for a project entitled "Motion Representations and Algorithms". This gift consists of 5 Pentium II-based PC equipped with graphics cards.

- Researchers at the Intel Microprocessor Research Labs are using Kinetic Data Structure ideas to speed up interactive computer animations, such as games. An Intel researcher is implementing collision detection algorithms based on work recently completed by Guibas and his students. Intel is supporting this research of Guibas by providing the stipend for a student, some summer salary support, and equipment. A renewal of this grant is pending.

- Research developed under MURI has attracted a joint three-year grant funded by Sony, Intel, and Interval. This multi-million dollar grant is managed by Prof. Pat Hanrahan (Stanford), and aims at developing immersive television applications. Tomasi is responsible for motion and shape acquisition
through computer vision techniques. The grant funds research by five faculty members and ten
graduate students at Stanford.

- The research conducted by Latombe and Motwani on randomized path planning has enabled us to
  establish a collaboration with General Motors and Deneb Robotics. General Motors supports our
  research on finding the quasi-optimal location of a robot through SIMA (the Stanford Industrial
  Manufacturing Association). Deneb Robotics has given us a free license of their robot simulation
  software IGRIP.

- A cooperation is in progress between Tomasi and the General Reality Company (GRC) in San Jose,
  CA, a developer of hardware and software for 3D computer graphics and 3D interactive devices.
  Research started under MURI is being further developed at GRC to measure camera motion and to
  reconstruct the shape of 3D scenes and objects for virtual reality applications.

- Ostrowski’s work on control of nonholonomic locomotion systems has led to a project with Global
  Aerospace to develop a control methodology for a global network of balloons.

- Researchers at Microsoft Research are interested in using Guibas’ Kinetic Data Structures for
  maintaining visibility among moving objects.

- The research conducted by Latombe and Motwani on randomized path planning was adapted to
  develop software aimed at checking whether given ligands (small flexible molecules) are likely to bind
  against given protein cavities. This work was done under a broader research contract with Pfizer
  Pharmaceuticals covering other aspects of computational drug design.

- Taylor has worked with Honda Motors at Honda’s Tochigi Research Center in Japan as part of the
  Berkeley activity on Vision-Based Control of Autonomous Vehicles. Honda Motors have active
  collaborations with Ostrowski, Kumar and Taylor.

- General Motors is funding Bajcsy, Kumar, Taylor, and Ostrowski’s work on problems related to
  sensing and control for automated vehicle highway systems.

- Tokheim Corporation sponsored research on vision based control of robots for automated fueling of
  passenger automobiles. We jointly developed an experimental prototype consisting of a robot, a set of
  cameras, and a suite of algorithms for control and sensing.

- Kumar has a close collaboration with Applied Resource Corporation on the development of a virtual
  prototyping environment to design robotic aids for people with disabilities. He is also working on a
  new collaboration on a semi-automated, ultra-mobile wheelchair system for people confined to
  wheelchairs.

- Our work on path optimization and finding the location of a robot’s base has been done in interaction
  with GM Research Labs in Warren (MI). We gave one presentation at GM Research Labs.

- The application of PRM to Ligand Binding was presented to several visitors from pharmaceutical
  companies including Merck and Rhones-Poulenc.

- Bajcsy and Daniilidis are involved in the National Tele-immersion Initiative sponsored by Advanced
  Network and Services, Inc., NYC. The GRASP lab demonstrated the world-wide first 3D-
  Teleconferencing and is actively collaborating with the above mentioned company, Brown University,
  and the University of North Carolina.

- Daniilidis applied principles studied in the Algorithmics of Motion in a collaboration project with
  Siemens Research Corporation, Princeton, NJ. In this project, industrial plants are reconstructed
  from multiple views.
- The Microprocessor Research Laboratory (MRL) of Intel is interested in Kinetic Data Structures work and researchers from MRL are currently involved in implementation efforts mentioned above.

- We have recently attracted the donation of several experimental robots from Sony Corporation, and will be working on algorithms for vision-based motion planning of small, legged robots.

- General Motors is funding Latombe to adapt randomized motion planning to industrial manufacturing problems, in particular the programming of spot-welding robots.

- Work developed under this MURI grant has led to a cooperation between Tomasi and ST Microelectronics for the automatic motion segmentation in video. The aim of this grant is to develop a chip to be used in television set-top boxes for various video enhancement purposes.

- Tomasi has established a collaboration with Honda for the development of image motion analysis algorithms to be used for Honda's P3 humanoid robot.

8 Honors, Awards, and Accomplishments

8.1 Our Students

One of the very important tasks we perform is training, educating, and mentoring graduate students who then go on to important positions in academic, government, and industrial institutions. We listed below examples of such students and their achievements.

- Stan Birchfield (Stanford Ph. D.), has joined Quindi, a startup company in Palo Alto developing image processing techniques based on motion analysis.

- Luca Bogoni (Penn Ph.D.) is working at Sarnoff SRI research Corp. on DARPA sponsored contracts.

- Jaydev Desai (Penn, Ph.D.) worked with Professor Howe (Harvard University) as a post-doctoral fellow on a MURI on bio-mimetic robots and is currently an Assistant Professor at Drexel University.

- Hany Farid (Penn, Ph.D.) is currently an Assistant Professor (NSF Career Award winner) at Dartmouth University.

- David Hsu (Stanford, Ph.D.) has joined Compaq Research Lab in Cambridge, MA.

- Lydia Kavaraki (Stanford Ph.D.) is an Assistant Professor (NSF Career Award winner) at Rice University. She was recently awarded a Sloan Fellowship.

- Jana Kosceka (Penn Ph.D.) worked with S. Sastry on researched sponsored by DoD, including a MURI grant. She is currently an Assistant Professor at George Mason University.

- James Kuffner (Stanford, Ph.D.) is working at the University of Tokyo under Prof. Inoue to apply random planning techniques to humanoid robots.

- Venkat Krovi (Penn, Ph.D.) accepted an offer from McGill University, one of the top two or three universities in Canada. He won a prestigious student paper competition in 1997, sponsored by AMR.

- Steve LaValle (Post-doc, Stanford) is currently a faculty member (NSF Career Award winner) at Iowa State University.
Robert Mandelbaum (Penn, Ph.D.) is working at Sarnoff SRI Research Corp. on DARPA sponsored contracts.

Peng Song’s (Penn) paper with Kumar was one of three finalists for the best paper award at the International Conference in Robotics and Automation.

Tom Sugar (Penn, Ph.D.) is now an Assistant Professor at Arizona State University.

Milos Zefran (Penn, Ph.D.) is now an Assistant Professor at the University of Chicago.

Hong Zhang (Penn, Ph.D.) is currently an Assistant Professor at Rowan College in Rowan, NJ.

John Zhang (Stanford Ph. D.) is currently working at SUN Microsystems.

Tong Zhang (Stanford Ph.D.) is currently a researcher at the IBM T. J. Watson Research Center.

8.2 Our Faculty

Leonidas J. Guibas was elected as ACM Fellow.

James Ostrowski has been appointed as an Associate Editor of the IEEE Control Systems Society Conference Editorial Board. He received a 1999 NSF CAREER award for his work on “Hybrid Locomotion Systems for Varying Terrains and Environments”, which has direct relationship to the MURI’s aim of “Algorithmics of Motion”, in developing algorithms for controlling new types of locomotion systems operating in complex (“non-laboratory”) environments.

Camillo Taylor received a NSF CAREER Award in 1999 for his work on three-dimensional reconstruction from two-dimensional Images and vision-based control.

James Ostrowski has been re-appointed as an assistant professor. This process of reappointment is the first review after a three year period. The second three year review results in tenure.

Ruzena Bajcsy was named the Assistant Director of the Computer and Information Science and Engineering (CISE) directorate at the National Science Foundation after a nationwide search.

Ruzena Bajcsy was elected to the National Academy of Engineering in 1998.

Kostas Daniilidis chaired the IEEE Workshop on Omnidirectional Vision, Hilton Head island, June 12, 2000.
9 Publications in 1995-2000


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This paper was one of five nominations for the best paper award for the IEEE Transactions on Robotics and Automation in 1996.


\(^4\)This paper won the best student paper award at the 5th National Conference on Applied Mechanisms and Robotics, 1997.

\(^5\)This paper won the best student paper award at the 4th National Conference on Applied Mechanisms and Robotics, 1995.


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6 This paper was one of the three finalists for the Best Paper Award.

163. Taylor, C.J., "Recognizing Point Constellations under Scaled Orthography" in submission to CVPR 99.


166. Taylor, C.J., “VideoPlus: A Method for Capturing the Structure and Appearance of Immersive Environments” Second Workshop on 3D Structure from Multiple Images of Large-scale Environments

167. Taylor, C.J., "Reconstruction of Articulated Objects from Point Correspondences in a Single Image" : Computer Vision and Image Understanding, *Accepted for publication.*


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This paper has been nominated for the best paper award at the 1988 IEEE International Conference on Robotics and Automation.