THESIS

OPTIMIZATION OF PROCUREMENT SCHEDULING FOR MAJOR DEFENSE ACQUISITION PROGRAMS

by

Donald E. Humpert

September 2000

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Optimization of Procurement Scheduling for Major Defense Acquisition Programs

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As the Defense Acquisition Executive (DAE), the Under Secretary of Defense for Acquisitions, Technology and Logistics has full responsibility for supervising the performance of the DoD Acquisition System. A challenge to the DAE is in determining the most efficient allocation of funding in procuring of over eighty Major Defense Acquisition Programs. This thesis develops six different cost functions based on the Unit Theory learning curve model for estimating the cost of each of these MDAP systems. The most suitable of these adds an annual overhead component to the cost modeled by the learning effect. This function is implemented in an integer-linear optimization model, the Procurement Scheduling Optimization Model (PSOM). PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints, finding the optimal quantity of each system to be procured each year of the planning horizon. This thesis models the cost of seventeen of the MDAP systems and optimally schedules them over an eighteen year planning horizon. PSOM can easily be expanded to include all eighty-plus MDAP systems. PSOM is a tool available to acquisition planners and decision makers to assist in optimally allocating procurement funding.
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OPTIMIZATION OF PROCUREMENT SCHEDULING FOR MAJOR DEFENSE ACQUISITION PROGRAMS

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ABSTRACT

As the Defense Acquisition Executive (DAE), the Under Secretary of Defense for Acquisitions, Technology and Logistics has full responsibility for supervising the performance of the DoD Acquisition System. A challenge to the DAE is in determining the most efficient allocation of funding in procuring of over eighty Major Defense Acquisition Programs. This thesis develops six different cost functions based on the Unit Theory learning curve model for estimating the cost of each of these MDAP systems. The most suitable of these adds an annual overhead component to the cost modeled by the learning effect. This function is implemented in an integer-linear optimization model, the Procurement Scheduling Optimization Model (PSOM). PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints, finding the optimal quantity of each system to be procured each year of the planning horizon. This thesis models the cost of seventeen of the MDAP systems and optimally schedules them over an eighteen year planning horizon. PSOM can easily be expanded to include all eighty-plus MDAP systems. PSOM is a tool available to acquisition planners and decision makers to assist in optimally allocating procurement funding.
DISCLAIMER

The reader is cautioned that the computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the planner.
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<table>
<thead>
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<th>Acronym</th>
<th>Description</th>
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<tbody>
<tr>
<td>AAAV</td>
<td>Advanced Amphibious Assault Vehicle</td>
</tr>
<tr>
<td>AFIT</td>
<td>Air Force Institute of Technology</td>
</tr>
<tr>
<td>ACAT 1</td>
<td>Acquisition Category 1</td>
</tr>
<tr>
<td>AUC</td>
<td>Average Unit Cost</td>
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<tr>
<td>BIFV</td>
<td>Bradley Infantry Fighting Vehicle</td>
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<tr>
<td>DAE</td>
<td>Defense Acquisition Executive</td>
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<tr>
<td>DPP</td>
<td>Defense Program Projection</td>
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<tr>
<td>EELV</td>
<td>Evolved Expendable Launch Vehicle</td>
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<tr>
<td>FRP</td>
<td>Full Rate Production</td>
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<tr>
<td>FYDP</td>
<td>Future Years Defense Program</td>
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<tr>
<td>GAMS</td>
<td>General Algebraic Modeling System</td>
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<tr>
<td>IDA</td>
<td>Institute for Defense Analysis</td>
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<tr>
<td>JSOW</td>
<td>Joint Stand-Off Weapon</td>
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<tr>
<td>LRIP</td>
<td>Low Rate Initial Production</td>
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<td>LMP</td>
<td>Algebraic Lot Midpoint</td>
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<td>LURP</td>
<td>Linearized-Unit-Rate-Penalty</td>
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<td>OUSD(AT&amp;L)</td>
<td>Office of the Under Secretary of Defense (Acquisition, Technology and Logistics)</td>
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<tr>
<td>PSOM</td>
<td>Procurement Scheduling Optimization Model</td>
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<tr>
<td>QDR</td>
<td>Quadrennial Defense Review</td>
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<td>SAR</td>
<td>Selected Acquisition Reports</td>
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<td>USD(AT&amp;L)</td>
<td>Under Secretary of Defense (Acquisition, Technology and Logistics)</td>
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EXECUTIVE SUMMARY

As the Defense Acquisition Executive, the Under Secretary of Defense for Acquisitions, Technology and Logistics (AT&L) has full responsibility for supervising the performance of the DoD Acquisition System. A phenomenon that the Office of the Under Secretary of Defense (AT&L) has recognized is that the total estimated cost of any given defense system increases as the rate of procurement for that system decreases. In a world of unlimited resources, all systems could be acquired at the maximum rate possible at the lowest possible cost; unfortunately, we live with the reality of a limited annual budget. The decision maker’s dilemma is in choosing how to “best” schedule the acquisition of these systems, subject to budget limitations and other requirements. This thesis provides the decision maker with a tool to address this problem.

The most critical requirement of such a tool is that it accurately reflect the estimated cost of the systems that are scheduled with it. This thesis uses Selected Acquisition Report (SAR) data for seventeen Major Defense Acquisition Programs to develop six different cost estimating relationships and evaluate their suitability for this purpose. The “Base Model” is simply an expression of basic learning curve theory: each system has a “beginning” price per unit which decreases as more units of the system are produced. The five other cost estimating relationships are excursions from this model. The “Multiplicative-Rate Model,” “IDA Rate-Penalty Model,” and “Linearized-Unit-Rate-Penalty Model” all include the rate of production as a predictor of unit cost. The “Rate-Change Model” assumes that changes in rate of production from one period to the next contribute to the cost of each system. The “Base + Overhead Model” adds a fixed cost component to the basic learning curve cost for each lot of the system procured. The primary measure of effectiveness (MOE) for comparing each cost estimating relationship is how well their predicted lot costs agree with the SAR data lot costs. By this MOE, the Base + Overhead Model is the best cost estimating relationship of the six.

Implementing each cost function in a spreadsheet tool provides additional insight into its suitability. The cost functions are used to estimate the annual and total procurement costs for the seventeen systems for which we have data. The difference between the modeled total cost and the “true” total cost (calculated from SAR data)
serves as a second MOE. By this MOE, the Base + Overhead Model is again superior. Although the spreadsheet tool can be used to “manually” adjust the procurement plan for each system in search of the least expensive schedule, this is prohibitively tedious. This implies the requirement for an optimizing planning tool.

The Base + Overhead cost function is implemented in an integer-linear program, the Procurement Scheduling Optimization Model (PSOM). PSOM is implemented in the General Algebraic Modeling System (GAMS), and schedules the quantity of each MDAP system to be procured per year over an 18-year planning horizon. PSOM allows the planner to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest full rate production (FRP) start periods; and low rate initial production (LRIP) costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints. Data input to the model requires a working knowledge of GAMS.

PSOM can be used to construct a chart of the efficiency frontier, a plot of the minimum total cost of all systems at varying budget limits. This is built by repeatedly solving PSOM in a loop, with the budget decreasing after each iteration, from an amount in which the constraint is slack to the point at which the model becomes infeasible. The efficiency frontier for the seventeen systems modeled in this thesis is presented below.
Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. The decision maker can use the information in many ways. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate the effect of changing the annual budget limit; the additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.

PSOM can be easily expanded to include all 80+ MDAP systems. Expansion and use of PSOM or a similar optimization model is recommended for use by acquisition planners.
ACKNOWLEDGMENT

I would like to thank Professor Alan Washburn for his patient guidance through this effort, and for his mentoring throughout my two years at the Naval Postgraduate School. He never hesitated in allowing me his time, always accompanied by valuable advice. Lieutenant Commander Tim Anderson also deserves special thanks. His willingness in sharing his knowledge and enthusiasm towards this topic were truly inspiring.

I would also like to thank: my sponsor, Dr. Nancy Spruill, Principal Deputy Under Secretary of Defense (Acquisition, Technology and Logistics), for her generosity in funding this effort; Philip Rodgers, Deputy Director, Resource Analysis at OUSD(AT&L) for remembering his alma matter in making this project available; Dr. Stephen Balut, Director of the Cost Analysis and Research Division at IDA for his support in providing me with six weeks of priceless “real-world” experience; and Dr. Chuck Weber, the “Master Production Schedule” project leader at IDA for integrating me into his team and providing me with the data I needed for this endeavor.

Finally, I thank my wife Judi, and our three amazing children Brad, Kim and Angela. Their patience throughout this entire postgraduate experience has been truly remarkable. Of all the insights I’ve gained from my time in Monterey, the most valuable is in realizing once again the importance of family.
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I. INTRODUCTION

In his keynote address to Advanced Program Managers Course 99-02, Dr. Jacques Gansler, Under Secretary of Defense (Acquisition, Technology and Logistics) listed three "vital priorities" for the DoD acquisition community over the next few years (Gansler, 1999):

- To equip the early 21st century warfighter with the right equipment to assure our security and withstand any potential threat.
- To accelerate, broaden, and institutionalize our acquisition reform efforts in order to optimize our limited resources in providing those weapons.
  (emphasis added)
- To modernize our logistics systems -- so as to cut costs, infrastructure, and cycle time in support of our 21st century forces.

The intent of the second priority—"in order to optimize our limited resources"—provides the basis for this thesis.

A. THE CHALLENGE

A phenomenon that the Office of the Under Secretary of Defense (Acquisition, Technology and Logistics) (OUSD(AT&L)) has recognized is that the total estimated cost of any given system being procured increases as the rate of procurement for that system decreases. As a limited budget forces fewer of each system to be procured per year, the systems must be produced at a lesser rate, over a longer period. Either the rate or length of production, then, can be considered a contributor to the total procurement cost of a system. If not for the limitation of an annual budget and manufacturing constraints, the least costly strategy would be to procure all units of a system in one year—clearly an impractical real-world solution. Nonetheless, even considering annual budget limits and plant capacities, it is a relatively simple task to determine the optimal procurement schedule for a single system. Conceptually, it should also be possible to determine the optimal procurement schedule for all systems in combination. This "master" schedule is much more difficult to find, requiring suitable cost functions to be developed for each system and modeling their interactions. The 2001 Quadrennial Defense Review (QDR) offers an opportunity to take advantage of scheduling efficiencies if they can be
discovered. Figure (1) illustrates hypothetically how rearranging a procurement schedule could reduce overall cost.

![Current and Optimal Procurement Schedules](image)

**Figure 1. Example of Optimized Schedule**
The top chart shows the current procurement schedule for eight MDAP systems. A cost function that incorporates the effects of learning and production rates was used to model the cost of each system. The bottom chart shows the same total quantity of each system, procured over a more efficient schedule. The total cost of all systems in the current schedule is 38,541 CY00SM; the total cost of all systems in the optimal schedule is 37,464 CY00SM. Over a billion dollars is saved.

During each Quadrennial Defense Review (QDR), the DoD reviews its strategy for acquisition of new weapons systems and equipment. Each service and DoD entity argues in support of its priority interests. As the Defense Acquisition Executive, USD(AT&L) has full responsibility for supervising the performance of the DoD Acquisition System. A task of OUSD(AT&L) during the QDR will be to provide an economic framework for these decisions. A minimum cost, optimized procurement
schedule would be an ideal baseline from which the cost of deviations could be readily assessed. Therefore, an OUSD(AT&L) goal is to have developed an optimal Master Production Schedule for the DoD Major Defense Acquisition Programs (MDAP) for use in the 2001 QDR. This schedule will provide the least costly strategy to purchase MDAP systems for the next eighteen years.

In April 1998, OUSD(AT&L) tasked the Institute for Defense Analysis (IDA) to explore the use of optimization technology for long range defense acquisition planning (Weber, 1999). IDA was asked to focus on the development of such a Master Production Schedule for Acquisition Category (ACAT 1) systems that would meet the following requirements:

- Incorporate the approximately 80 Acquisition Category 1 (ACAT 1) systems in development or planned for the future.
- Cover the 18-year Defense Program Projection (DPP) planning horizon.
- Serve as an aggregate-level planning tool. The model would be developed at the system level, not being concerned with the detailed modeling of systems' components and sub-components.
- Reflect long-term planning issues, avoiding short-term scheduling issues such as assembly line balancing, potential strikes, etc.
- Reflect existing peacetime conditions to the extent reflected by the DPP (not concerned with wartime attrition).
- Use procurement cost functions for systems that reflect as realistically as possible the various factors affecting their values.

Fulfilling the last of these requirements represents one of the most challenging aspects of this effort. Since the cost function for each system is required in both the objective function and budgetary constraints of an optimization model, the model will be sensitive to the correctness of these cost estimating relationships.

The optimization model will seek to minimize the total cost of all systems over all years, subject to constraints. The most conspicuous of these constraints is the annual budget. Other constraints include: the requirement to meet the quantity demanded for each system by certain years; limitations on minimum and maximum production rates; and the desire to maintain production stability by forbidding breaks in production. The
model will also allow the user to specify startup and shutdown years, and allow the input of Low Rate Initial Production (LRIP) allowances. Figure (2) illustrates a typical acquisition schedule for a single hypothetical system.

![Procurement Schedule for a Hypothetical System]

**Figure 2. Hypothetical Procurement Schedule for a System**
A hypothetical acquisition schedule for a system starting full rate production (FRP) during period six. Quantities produced during low rate initial production (LRIP) are given as data. There are no production breaks once started, and LRIP immediately precedes FRP.

**B. PURPOSE**

This thesis develops the cost functions, spreadsheet scheduling tools, and a scalable optimization model, for a subset of the MDAP systems that IDA has been tasked with including in their model. The systems in the subset, although not selected at random, were selected to represent the whole population of MDAP systems. The following seventeen systems were selected:

1. Advanced Amphibious Assault Vehicle (AAAV)
2. Abrams Tank upgrade
3. Bradley Infantry Fighting Vehicle (BIFV) upgrade
4. C-17 Aircraft
5. CH-47 F Helicopter Upgrade
6. Crusader Self Propelled Howitzer with Resupply Vehicle
7. DDG 51 Guided Missile Destroyer
8. Evolved Expendable Launch Vehicle (EELV) Space Launch Vehicle
9. F/A-18 E/F Aircraft
10. Joint Standoff Weapon (JSOW)
11. Minuteman III Intercontinental Ballistic Missile
12. NAVSTAR Global Positioning Satellite
13. MV-22 Osprey Tilt-rotor Aircraft
14. SSN 774 Virginia Class Submarine
15. Standard Missile 2
16. T-45 TS Aircraft
17. Trident II Ballistic Missile

The following additional objectives are accomplished:

- Modeling six cost functions, to include comparisons between the models regarding their respective strengths and weaknesses.

- Formulation of a mixed integer program in the GAMS algebraic modeling language using the most appropriate cost function, with the capability to be easily expanded by IDA or OUSD(AT&L) to optimize all eighty ACAT I systems.

C. THESIS ORGANIZATION

Chapter II presents the six cost functions. The characteristics of each, the assumptions underlying them, and the data analysis which determines the respective system parameters for each function is given. The use of spreadsheet scheduling tools that use the cost functions developed is explored in Chapter III. The optimization model assumptions are listed, the formulation is offered, its implementation is described and its output is presented in Chapter IV. Chapter V details analysis of the optimization model output and conclusions.
II. COST FUNCTIONS

Arguably the most crucial step in creating a procurement scheduling tool is to derive a sound cost estimating relationship for each system being modeled. The cost function must as accurately as possible fit available data, while its behavior must agree with our intuition as to what is "correct."

A. DATA

The data required to estimate the cost functions of each system consist of the lot quantities and lot costs of each system, from beginning to end of procurement. The data for all systems but the Crusader and AAAV was obtained from their respective Selected Acquisition Reports (SAR). The SAR data was the most readily available, since IDA maintains copies of them.

A second source of this data is the Procurement Annex of the Future Years Defense Program (FYDP), which is a detailed five-year projection of procurements. This data mirrors that which is reported in the SARs. Since both the Crusader and AAAV are currently still in the RDT&E phase, however, neither their SARs nor the FYDP contained their appropriation data.

A third source of the required data is the DPP, which is less detailed but spans eighteen years. The DPP contains the most accurate data available for systems with incomplete SAR or FYDP data. Both the Crusader and AAAV data was extracted from this source. For each system, the unit cost was recalculated in terms of year 2000 dollars. Appendix A shows the data used for each system.

B. METHODOLOGY

1. Cost Function Development

The principal characteristic of a cost function is that as more units of an item are produced, the average unit cost of that item decreases. In his book The Cost Analyst's Companion, David Lee relates that this phenomenon is called "cost progress" and that the cost-quantity relation for a given system in production is known as the "cost-progress curve," or "learning curve."
One of the most commonly used models of cost progress is the unit theory model. The form of this model is

\[ C(Q) = T_1 Q^\beta \]  

(1)

where \( C(Q) \) is the cost of the \( Q^{th} \) unit; \( T_1 \) and \( \beta \) are constants. The constant \( T_1 \) is the theoretical cost of the first unit produced. The constant \( \beta \) represents the “cost analyst’s slope” of the curve, or “slope of the learning curve,” defined by

\[ S \equiv \frac{\text{Cost at quantity } 2Q}{\text{Cost at quantity } Q} \]  

(2)

or

\[ S = 2^\beta \]  

(3)

The constant \( \beta \) is assumed to be negative, or the cost or each successive unit would increase, rather that decrease. All of the models developed for use in the scheduling tools are derived from the Unit Theory Model. Figure (3) shows a typical learning curve for a system with theoretical first unit cost \( T_1 \) of $10 and a learning curve slope of 80%.

![Learning Curve and Cumulative Cost Curve](image)

Figure 3. Typical Learning Curve and Cumulative Cost Curve for a Hypothetical System. First unit cost, \( T_1 \), is $10 and learning curve slope of 80% (\( \beta = -0.322 \)). The unit cost of the \( Q^{th} \) unit is measured on the left axis, the cumulative cost to the \( Q^{th} \) unit on the right axis.
2. Fitting the Data

The parameters of each cost function are estimated by fitting them to the data using least-squares techniques. When possible, data are transformed to log-space, and parameters are determined by linear regression. Otherwise, the nonlinear solver resident in Excel is used to find the best fit.

For five of the six cost functions developed herein, the dependent variable of the function is the cost of the $Q$th unit, $C(Q)$. Since the data is given for annual lots, however, the dependent variable to be used in place of $C(Q)$ is the average unit cost ($AUC$), which is simply defined as

$$AUC = \frac{\text{TOTAL LOT COST}}{\text{LOT SIZE}}$$

(4)

The independent variable to be used in place of $Q$ is the algebraic lot midpoint ($LMP$), the theoretical unit whose cost is equal to the $AUC$ for that lot on the learning curve. The $LMP$ can be approximated using the following rules (AFIT, 1997):

For the first lot (the lot starting at unit 1):

If Lot Size < 10, then $LMP = \text{Lot Size}/2$

If Lot Size $\geq$ 10, then $LMP = \text{Lot Size}/3$

For all other lots:

$$LMP = \frac{F + L + 2\sqrt{F \cdot L}}{4}$$

(5)

where $F$ is the first unit number in a lot, and $L$ is the last unit number in a lot.

The sixth cost function is fitted directly to the lot cost and lot quantity, so that expressing the data in terms of unit cost is unnecessary.

3. Filtering the Data

An important issue that must be addressed when considering cost functions that include rate of production is the treatment of the first and last production periods. Data typically shows a relatively smaller quantity of a system that is produced in the first and last periods. One of two interpretations must be assumed: either 1) the rate of production is lower in these years; or 2) the first and last lots are not produced over a full year. It is more realistic to expect that the second of these assumptions is correct. Therefore, since the fractional year is not given in the database, including the first and last production
periods in the data used for analysis is not usually appropriate for rate-based cost functions. For models of this type, we drop the last production period from our data. Unless the first production period is obviously a full lot, we drop it from the data also and model it as LRIP.

4. **Measures of Effectiveness**

A traditional measure of effectiveness for a linear or nonlinear model is the Coefficient of Determination, $R^2$, defined as

$$R^2 = 1 - \frac{(RESIDUAL SS)}{(CORRECTED TOTAL SS)}$$

where

$$RESIDUAL SS = \sum (Y_i - \hat{Y}_i)^2$$

$$CORRECTED TOTAL SS = \sum (Y - \bar{Y})^2$$

In principle, the $R^2$ could be negative if the model fits worse than the mean does. We calculate all $R^2$ in unit-space, and adjust them to take into account the complexity of the model relative to the complexity of the data as follows

$$R^2_{adj} = R^2 - \frac{K - 1}{n - K} (1 - R^2)$$

where $n$ is sample size and $K$ is the number of parameters in the model (Hamilton, 1992).

Since our interest is in how well the cost functions estimate the systems’ lot costs, it is the $R^2_{adj}$ of the fit of the lot costs that is our principal measure of effectiveness for a function.

C. **BASE MODEL**

1. **Characteristics and Assumptions**

The “Base Model” is simply the Unit Theory model presented in equation (1). As previously discussed, this model captures only the effect of learning on system cost. This model ignores the production rate. Therefore, whether all units are produced in the first period, or production is spread over several periods, the total system cost is the same.
2. Model Fit

The Base Model is the simplest function to fit. Since equation (1) is easily transformed to

\[ \ln C(Q) = \ln T_i + \beta \ln Q \] (10)

it is a simple matter to perform a linear regression to obtain the coefficients \( T_i \) and \( \beta \). With these coefficients, the annual costs of each system are readily determined. The median \( R^2_{\text{adj}} \) of the fit of the annual costs for this model over all systems is 0.75; the mean is 0.65. Parameters for each system and respective \( R^2_{\text{adj}} \) values are displayed in Appendix B. Figure (4) shows the data and fitted values for the BIFV unit and lot costs.

![Graph showing unit and lot costs over time]

Figure 4. Base Modeling of Bradley Infantry Fighting Vehicle.
The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a \( R^2_{\text{adj}} \) of 0.88.
D. MULTIPLICATIVE RATE MODEL

1. Characteristics and Assumptions

Since the Base Model doesn't reflect an increase in total cost as production is extended over time, we extend it to include the rate of production. Lee describes this as the production feature most commonly added, with this form

\[ C(Q) = T_i Q^\beta R^\chi \]  \hspace{1cm} (11)

where \( R \) denotes the number of units produced in a production period, and the constant \( \chi \) is the rate exponent of the curve. The quantity \( 2^\chi \) is the curve's "rate slope." As with \( \beta \), \( \chi \) is assumed to be negative. This implies that the unit cost of a system decreases as the rate at which it is produced increases. We refer to this model as the "Multiplicative Rate Model." Figure (5) shows this cost relationship for the Bradley Infantry Fighting Vehicle.

![Multiplicative Rate Cost Function](image)

Figure 5. Multiplicative Rate Cost Function

In the Multiplicative Rate Model, both the quantity produced and the rate of production affect the unit cost. The chart shows this relationship for the Bradley Infantry Fighting Vehicle, where \( T_i = 21.790; \beta = -0.136 \); and \( \chi = -0.250 \)
2. Model Fit

Fitting the Multiplicative Rate Model is done in a similar manner as with the Base Model, in that equation (11) is transformed to

$$\ln C(Q) = \ln T_1 + \beta \ln Q + \chi \ln R$$  \hspace{1cm} (12)

and a linear regression performed to obtain the coefficients $T_1$, $\beta$, and $\chi$. Results of the regressions for each of the systems are summarized in Appendix B. Predictably, the Multiplicative Rate Model more accurately fits the data, as compared to the Base Model. The median $R^2_{\text{adj}}$ for this model over all systems is 0.88; the mean is 0.82. Parameters for each system and respective $R^2_{\text{adj}}$ values are displayed in Appendix B. Figure (6) shows the data and fitted values for the BIFV unit and lot costs.

Figure 6. Multiplicative Rate Modeling of Bradley Infantry Fighting Vehicle
Bradley Infantry Fighting Vehicle plots of Data and Fitted Values. Estimated costs are determined by Multiplicative Rate Model. The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a $R^2_{\text{adj}}$ of 0.91.
E. IDA RATE-PENALTY MODEL

1. Characteristics and Assumptions

The Multiplicative Rate Model possesses the important characteristic that as the number of years spent in production is extended, the total cost of a system increases. An issue for consideration regarding the data, however, is that if the rate of production increases through the production periods, rate and quantity can be highly correlated. IDA chose not to use this extension to the Base Model due to these perceived statistical problems (Balut, 1988). Instead, IDA developed their own cost function, of the form

\[ C(Q) = T_i Q^\rho + \delta \frac{(R^* - R)^2}{R^*} \]

(13)

where \( \delta \) is the rate penalty parameter, \( R^* \) is the theoretically optimal rate of production for the system, and \( R \) is the actual rate of production for that period. This embellishment of the Base Model adds a penalty for production made at other than the optimal rate. This is referred to as the "IDA Model."

A useful feature of this relationship is that it does not assume that a higher production rate is necessarily better. This allows for the possibility of modeling the impact of paying overtime costs to increase production in a period, for example. Figure (7) shows the per unit and per lot penalties for the BIFV.

![Figure 7. IDA Rate-Penalty Model Unit and Annual Penalties for Different Rates of Production for the Bradley Infantry Fighting Vehicle](image)

The per unit penalty is \( \delta(R^* - R)^2/R^* \), where: \( R^* \), the theoretically optimal rate of production, is 190.55 per year; \( R \) is the actual rate of production; and \( \delta \), the penalty constant, is 0.021.
The per unit rate penalty in the IDA Model grows quadratically as the production rate departs from the optimal rate; that is, a deviation from the optimal production rate is penalized more if done in a single period than if divided over several periods. The predictable impact of this feature is that large deviations will be avoided. The optimal production schedule will tend to divide the total quantity of each system required evenly over the periods of procurement.

2. Model Fit

The IDA Model is more complicated to fit than either of the previous two developed thus far. The Excel solver to fit the function parameters in minimizing the sum of the squared differences between the data and the calculated average unit cost per period. The equation to calculate the average unit cost per period was derived as follows: the simple cost progress curve, equation (1), is integrated to yield the relationship between the total cost and cumulative quantity produced

$$T(Q) = \frac{T_i Q^{\beta+1}}{\beta + 1} \quad (14)$$

The SAR and DPP data provides the total cost for producing a given lot size. Therefore, the above equation is transformed to

$$TLC = \frac{T_i ((Q_U + 0.5)^{\beta+1} - (Q_L + 0.5)^{\beta+1})}{\beta + 1} \quad (15)$$

where $TLC$ is the total lot cost, $Q_U$ is the ending cumulative quantity once the lot is built, and $Q_L$ is the ending cumulative quantity of the previous lot. Adding 0.5 to the beginning and ending quantities provides a better approximation to the continuous cost function.

Dividing equation (14) by $R$, the size of the lot, yields the average unit cost. To this the rate penalty is added as follows

$$AUC = \frac{T_i ((Q_U + 0.5)^{\beta+1} - (Q_L + 0.5)^{\beta+1})}{(\beta + 1)R} + \delta \frac{(R^* - R)^2}{R^*} \quad (16)$$

where $R^*$ is the theoretically optimal rate of production for that system, recognizing that $R$ is equal to $(Q_U - Q_L)$. If $R^*$ is not explicitly available from the manufacturer, it is either determined by the solver or set at the maximum production rate, depending on what the data suggested was most appropriate. The median $R^2_{adj}$ for this model over all systems is 0.87; the mean is 0.77. Parameters for each system and respective $R^2_{adj}$ values
are displayed in Appendix B. Figure (8) shows the data and fitted values for the BIFV unit and lot costs.

![Graph showing unit and lot costs](image)

Figure 8. IDA Rate-Penalty Modeling of Bradley Infantry Fighting Vehicle
The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a $R_{adj}^2$ of 0.94.

F. LINEARIZED-UNIT-RATE-PENALTY (LURP) MODEL

1. Characteristics and Assumptions

An assumption that the per unit penalty should more correctly be modeled as linear leads to a variation of the IDA Model of the form

$$C(Q) = T_1Q^\theta + \delta|R^* - R|$$

(17)

Under this relationship, the same penalty is incurred for a deviation from the optimal production rate whether concentrated in one period or spread over several.

Figure (9) shows the per unit and per lot penalties for the BIFV.
Figure 9. LURP Model Unit and Annual Penalties for Different Rates of Production for the Bradley Infantry Fighting Vehicle
Bradley Infantry Fighting Vehicle penalties per unit and per lot for different rates of production. The per unit penalty is \( \delta|R^*-R| \), where: \( R^* \), the theoretically optimal rate of production, is 169.27; \( R \) is the actual rate of production; and \( \delta \), the penalty constant, is 0.008.

2. Model Fit

Fitting this model is accomplished in the same manner as the IDA Model, except that the penalty term in equation (16) is replaced with the penalty term in (17). The median \( R_{adj}^2 \) for this model over all systems is 0.84; the mean is 0.78. Parameters for each system and respective \( R_{adj}^2 \) values are displayed in Appendix B. Figures (10) and (11) show the data and fitted values for the BIFV unit and lot costs.

Figure 10. LURP Modeling of BIFV: Unit Costs
Bradley Infantry Fighting Vehicle plot of unit Data and Fitted Values. Estimated costs are determined by the LURP Model.
Figure 11. LURP Modeling of BIFV: Unit Costs
Bradley Infantry Fighting Vehicle plot of lot Data and Fitted Values. Estimated costs are determined by
the LURP Model. The fitted values of the plot correspond to a \( R^2_{adj} \) of 0.88.

G. RATE-CHANGE MODEL

1. Characteristics and Assumptions

One can argue that a plant will optimize its production at any rate, given time to adapt. Changes in the rate of production between periods, however, prevent this goal from being realized. Another Base Model extension that attempts to capture this behavior is

\[
C(Q) = T_i Q^\beta + \delta |\Delta R|
\]

where \( \Delta R \) is the change in rate from one period to the next. We refer to this model as the “Rate-Change Model.”

2. Model Fit

The parameters for the Rate-Change Model are determined in the same way as those of the rate-penalty models, except that the rate penalty term is replaced by the rate penalty term in equation (18). Values for \( \delta \) are constrained to be greater than or equal to 0.001 to ensure that a minimum penalty is assessed for changing production rate. For the first production period, \( \Delta R \) is defined to be zero. The median \( R^2_{adj} \) for this cost function over all systems is 0.73; the mean is 0.68. Parameters for each system and respective \( R^2_{adj} \) values are displayed in Appendix B. Figure (12) shows the data and fitted values for the BIFV unit and lot costs.
Figure 12. Rate Change Modeling of Bradley Infantry Fighting Vehicle
Bradley Infantry Fighting Vehicle plots of Data and Fitted Values. Estimated costs are determined by the Rate-Change Model. The plot on the top shows unit costs, the plot on the bottom shows lot costs. The fitted values of the bottom plot correspond to a $R^2_{adj}$ of 0.79.

H. BASE + OVERHEAD MODEL

1. Characteristics and Assumptions

All of the modifications to the Base Model presented thus far, in some form, attempt to capture the effect of production rate. One can argue, however, that the rate effect is being confounded with the effect of overhead costs. A low production rate extends procurement over more periods, causing overhead costs to be incurred for a longer time. The simplest extension of the Base Model uses the learning curve to determine the learning portion of each lot cost, and then adds an overhead term to each
lot cost proportional to the production period. This cost function estimates lot costs directly, then, rather than unit costs. The form of this "Base + Overhead Model" is

$$TLC = \frac{T_{l}(Q_{U})^{\beta+1} - (Q_{L})^{\beta+1}}{\beta + 1} + \Omega$$

(19)

where $TLC$ is the total lot cost, $Q_{U}$ is the ending cumulative quantity once the lot is built, $Q_{L}$ is the ending cumulative quantity of the previous lot, $\Omega$ is the overhead term, and $t$ is the fraction of the period in production. Since we cannot consider partial years of production, $t$ is always one in our application of the model. Multiple optimal schedules that vary the production rate may be determined with this model, but they will all ensure that the system is procured in the shortest time possible.

2. Model Fit

Since the lot costs are fitted directly for this model, continuity corrections are added to equation (19), and the function fitted to the data is simply

$$TLC = \frac{T_{l}(Q_{U} + .5)^{\beta+1} - (Q_{L} + .5)^{\beta+1}}{\beta + 1} + \Omega$$

(20)

The Excel Solver is used as with the rate penalty models to determine the parameters for each system. The overhead term, $\Omega$, is constrained to be greater than or equal to 1; this forces a minimum penalty for producing over an additional period. The median $R^{2}_{adj}$ for this cost function over all systems is 0.90; the mean is 0.86. Parameters for each system and respective $R^{2}_{adj}$ values are displayed in Appendix B. Figure (13) shows the data and fitted values for the BIFV unit and lot costs.

![Graph](image)

Figure 13. Base + Overhead Modeling of Bradley Infantry Fighting Vehicle BIFV plots of Data and Fitted Values. The fitted values of the plot correspond to a $R^{2}_{adj}$ of 0.93.
I. INITIAL COMPARISON OF MODELS

Using the median and mean $R_{\text{adj}}^2$ for each cost function, the models are compared. Figure (14), a series of box plots for each model, summarizes the models into a useful visual tool for determining which is the most suitable. The most recently presented model, Base + Overhead, exhibits the highest median $R_{\text{adj}}^2$ and also has the least variance. This can be considered the “best-fitting” model, followed, in order, by the Multiplicative Rate Model, the LURP Model, the IDA Rate-Penalty Model, the Base Model, and the Rate-Change Model. At this point, both the Rate-Change Model and the Base Model can be dropped from consideration for use as scheduling tools.

![Box Plots of the Systems’ $R_{\text{adj}}^2$ Values for Each of the Cost Models Explored](image)

Figure 14. Box Plots of the Systems’ $R_{\text{adj}}^2$ Values for Each of the Cost Models Explored Medians are labeled within the plots, means are labeled below the respective model name. The Base + Overhead model can be considered the “best-fitting” overall, as it yields the highest median $R_{\text{adj}}^2$, mean $R_{\text{adj}}^2$ and the lowest variance.
III. SPREADSHEET SCHEDULING TOOLS

A. INTRODUCTION

A worthwhile step before implementing a cost function in an optimizing program is to exercise it in a spreadsheet. In this manner we can confirm our beliefs as to how the function behaves, and perhaps gain further intuition as to how it will work in a math program. A strength of spreadsheets is that they can easily model complex relationships that are difficult, if not impossible, to implement in an optimization model. Elements of interest when employing our cost functions are: 1) how well the function estimates the current estimated total cost, given the current procurement schedule; 2) how the schedule “behaves” in moving towards optimality; and 3) to what degree the total cost is reduced when the schedule is “optimal.” As a baseline, refer to Figure (15), a depiction of current projected acquisition costs derived from the data. The plot shows the total yearly cost if the seventeen systems are produced on currently anticipated schedules with currently anticipated costs. These costs will be called “true” since none of the estimation models of Chapter II are involved.

![Procurement Costs Per Year](image)

**Figure 15. Cost of Current Procurement Schedule**
Cost of current procurement schedule, by system, over eighteen year planning horizon (taken from data). The total cost of all systems will be $158,483 (CY00$M). The plot provides a baseline from which the effect of using the cost functions of interest can be readily observed.
As the spreadsheet model does not automatically enforce budget and production constraints, the user must ensure that they are observed. Because future needs are unknown, imposing a yearly constraint may not necessarily be wise; nonetheless, budget limits are a key feature of our model. In the following experiments with each cost function, annual budget constraints are relaxed while observing the production constraints. Bounding annual production rate constraints requires careful judgment by the modeler. If these limits are not provided, as in this situation, they must be extrapolated from the data available (Appendix A). For each system, the highest observed annual quantity procured is used as the maximum production rate. The minimum production rate is chosen much more subjectively; this constraint is essentially a control over production stability. Table (1) lists the maximum and minimum annual production rates used in the modeling efforts for each system.

<table>
<thead>
<tr>
<th>System Rate/Year</th>
<th>AAAV</th>
<th>Abrams</th>
<th>Bradley</th>
<th>C-17</th>
<th>CH-47F</th>
<th>Crusader</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX</td>
<td>200</td>
<td>120</td>
<td>235</td>
<td>15</td>
<td>29</td>
<td>240</td>
</tr>
<tr>
<td>MIN</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>1</td>
<td>8</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Rate/Year</th>
<th>DDG-51</th>
<th>EELV</th>
<th>FA-18 E/F</th>
<th>JSOW</th>
<th>Minuteman III</th>
<th>NAVSTAR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX</td>
<td>3</td>
<td>14</td>
<td>48</td>
<td>900</td>
<td>80</td>
<td>3</td>
</tr>
<tr>
<td>MIN</td>
<td>1</td>
<td>4</td>
<td>12</td>
<td>100</td>
<td>10</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System Rate/Year</th>
<th>Osprey</th>
<th>SSN 774</th>
<th>Std Missile 2</th>
<th>T-45TS</th>
<th>Trident II</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAX</td>
<td>36</td>
<td>3</td>
<td>190</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>MIN</td>
<td>9</td>
<td>1</td>
<td>60</td>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Table 1. Estimated Maximum and Minimum Annual Production Rates

B. MULTIPLICATIVE RATE MODEL

Using the Multiplicative Rate Model to determine the costs of the current procurement schedule yields a total cost of $153,241 (CY00$M). This is an underestimate of the true cost by 3.31%. A plot of the procurement costs as determined by the model is virtually indistinguishable from the data plot in Figure (14).

Experimentation confirms the expectation that as the rate of production increases, unit costs decrease. The minimum total cost is achieved when each system is produced at
maximum rate, until the quantity demanded is satisfied. Furthermore, it is most beneficial to delay any reduction in production as long as possible. Disregarding annual budget limits, the optimal procurement schedule simply produces every system at maximum rate, except during the last period. Figure (16) shows the least costly procurement schedule as determined using the Multiplicative Rate Model. The total estimated cost of this schedule is $146,489 (CY00$M), a difference of $6,752 (CY00$M) from the estimated current cost. Since this schedule did not consider annual budget limits, this difference serves as an upper bound on the savings that could be expected.

![Procurement Costs Per Year](image)

**Figure 16. Cost of Optimal Procurement Schedule as Determined Using the Multiplicative Rate Model**

The model schedules production at the maximum rate, until demand has been satisfied. The total cost of all systems is estimated to be $146,489 (CY00$M). This represents a savings of $6,752 (CY00$M) compared with the current schedule.

### C. IDA RATE-PENALTY MODEL

The IDA Rate-Penalty Model yields a current-schedule total cost of $156,275 (CY00$M), an underestimate of the true cost by 1.39%. This is a slight improvement over the Multiplicative Rate Model. As mentioned earlier, when forced to produce at below optimum rate, this model seeks to divide the quantity required equally over the time allotted. Table (2) illustrates this property with a simple example.
Table 2. IDA Rate-Penalty Model Unit and Annual Rate Penalties

Unit and lot rate-penalties for a five period hypothetical procurement schedule. The total quantity procured is 400. In this situation, the system is limited to producing 90 per period vice its optimum of 100. Producing at this maximum for four periods exposes the schedule to a severe penalty in the fifth period. The least costly solution divides the penalty evenly over the five periods.

An adjustment that is used in the spreadsheet implementation of this model is to eliminate the rate-penalty term of the cost function for the last period of production. Otherwise, as just discussed, the system would never be produced at the optimal rate, even when unconstrained by budget or rate limits. By eliminating the penalty for the last period, it is tacitly assumed that the system is produced at the optimal rate during a fraction of this period.

The optimal schedule, then, produces each system at its optimal rate, thus incurring no rate-penalty. The total cost of such a schedule will be the same as if it had been modeled using the Base Model. The total cost of this schedule is $148,824 (CY00$M), allowing an upper bound of $7,451 (CY00$M) in savings from the estimated cost of the current schedule. The plot of the IDA Rate-Penalty optimal schedule is nearly identical to that of the Multiplicative Rate Model shown in Figure (16).

D. LURP MODEL

The LURP Model yields a current-schedule total cost of $155,674 (CY00$M). This is an underestimate of the true cost by 1.77%. As with the IDA Rate-Penalty Model, the penalty term is dropped from the last production period. Unlike the IDA Rate-Penalty Model, this model seeks to produce at as close to optimal as possible, for as many periods as possible; table (3) illustrates this property.
Table 3. LURP Model Unit and Annual Rate Penalties

<table>
<thead>
<tr>
<th>R* = 100</th>
<th>PERIODS</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>δ = 10</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Current Rate</td>
<td>90</td>
<td>90</td>
</tr>
<tr>
<td>Unit Penalty</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Lot Penalty</td>
<td>9000</td>
<td>9000</td>
</tr>
<tr>
<td>Current Rate</td>
<td>80</td>
<td>80</td>
</tr>
<tr>
<td>Unit Penalty</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>Lot Penalty</td>
<td>16000</td>
<td>16000</td>
</tr>
</tbody>
</table>

Unit and lot rate-penalties for a five period hypothetical procurement schedule, calculated with the LURP Model. The total quantity procured is 400. In this situation, the system is limited to producing 90 per period vice its optimum of 100. Unlike the IDA Rate-Penalty Model, this model seeks to produce at as close to optimal as possible, for as many periods as possible. Dividing the penalty evenly over the five periods, the preferred solution for the IDA Rate-Penalty Model, is more costly.

Producing at the optimal rate for each system and period, without regard to budget limits, provides the minimum achievable total cost. The plot of the LURP optimal schedule is also nearly identical to that of the Multiplicative Rate Model shown in Figure (16). The total estimated cost of this schedule is $143,965 (CY00$M), a difference of $11,709 (CY00$M) from the current cost estimated with the same function. Again, since this schedule did not consider annual budget limits, this difference serves as an upper bound on the savings that could be achieved.

E. BASE+OVERHEAD MODEL

The Base + Overhead Model yields a total cost of $159,683 (CY00$M). This is an overestimate of the true cost by only 0.76%, the closest of the four functions modeled with spreadsheets. This is also the simplest of the four functions. The same overhead cost is included in the lot cost if any units are procured, regardless of the quantity, including the last production period. A tacit assumption is made that the final production period incurs the same overhead as a full period. There is no unique optimal solution to this scheduling exercise, since any combination of quantities that completes production in the shortest amount of time will incur the same cost.

Figure (17) illustrates one optimal solution determined using this function. The total procurement cost over this schedule is $155,407 (CY00$M). This is only $4,276 (CY00$M) less than the estimated cost of the current schedule.
Figure 17. Cost of Optimal Procurement Schedule, Determined with the Base + Overhead Model

The same overhead cost is included in the lot cost if any units are procured, regardless of the quantity, including the last production period. There is no unique optimal solution to this scheduling exercise, since any combination of quantities that completes production in the shortest amount of time will incur the same cost. The total cost of all systems is estimated to be $155,407 (CY00$M). This represents a savings of $4,276 (CY00$M) compared with the current schedule.

F. FURTHER COMPARISON OF MODELS

Of the four functions implemented in spreadsheet applications, only the Base + Overhead Model overestimated the total cost of the current procurement schedule. The 0.76% error of this function’s estimate is the smallest of the group in absolute value. Additionally, the potential savings from optimal scheduling is the lowest. It is reasonable to argue that the Base + Overhead Model is the most conservative of the four. Table (3) summarizes the statistics of interest for each of these models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Multiplicative Rate</th>
<th>IDA Rate Penalty</th>
<th>LURP</th>
<th>Base + Overhead</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median $R^2_{adj}$</td>
<td>.88</td>
<td>.87</td>
<td>.84</td>
<td>.90</td>
</tr>
<tr>
<td>Mean $R^2_{adj}$</td>
<td>.82</td>
<td>.77</td>
<td>.78</td>
<td>.86</td>
</tr>
<tr>
<td>Cur. Sched Est Cost</td>
<td>153,241</td>
<td>156,275</td>
<td>155,674</td>
<td>159,683</td>
</tr>
<tr>
<td>$</td>
<td>Error</td>
<td>$</td>
<td>3.31%</td>
<td>1.39%</td>
</tr>
</tbody>
</table>

Table 3. Summary of Measures of Effectiveness of the Four Best Cost Models
G. "OPTIMAL" SPREADSHEET SOLUTION

The most accurate spreadsheet tool is selected based on the Base + Overhead cost function, for schedule planning. Using this tool, one can attempt to determine the optimal procurement schedule subject to any annual budget constraint. An estimated budget amount that will serve as a basis for later comparison, derived simply by estimating the largest annual cost from Figure (15), is $16,000 (CY00$M). Figure (18) shows the least costly schedule that the author was able to derive after several hours of experimentation and manipulation. The total cost of all systems on this schedule is estimated to be $159,159 (CY00$M), only $524 (CY00$M) less costly than the current schedule. Although the spreadsheet planner is useful for visualizing procurement costs, using it to find an optimal schedule is impractical and tedious.

![Chart showing procurement costs per year with best schedule at less than $16,000](chart.png)

Figure 18. Schedule "Optimized" using Spreadsheet Planning Tool.
The least costly schedule that the author was able to derive after several hours of experimentation and manipulation. The unconstrained optimal solution is used as a starting point, and lot quantities of various systems are changed until annual budget constraints are met. The total cost of all systems on this schedule is $159,159 (CY00$M), which is only $524 (CY00$M) less costly than the current schedule.
IV. OPTIMIZATION TOOL

The time required in manipulating the spreadsheet tool, coupled with the lack of improvement over the current schedule, reinforce the belief that a truly optimizing planning tool is required. All of the cost functions developed herein are inherently nonlinear, however, a hindrance to optimization modeling. Fortunately, in addition to being the best fitting of the cost functions explored, the Base + Overhead cost function is the simplest to approximate linearly. This cost function is implemented in an optimization model, henceforth called the Procurement Scheduling Optimization Model (PSOM).

A. LINEARIZING THE COST FUNCTION

The nonlinear portion of the Base + Overhead cost function, equation (19), is the learning effect term, which is linearized using a step function. This is accomplished for each system, using the following steps: 1) the cost of each unit is determined using the Base Model; 2) the mean cost of the units yet to be procured is determined; 3) this set of units is divided into two subsets: the first consisting of all units of above average cost; the second consisting of units of below average cost; 4) both of these subsets are further divided into two groups in the same manner, yielding a total of four groups; and, lastly, 5) the mean unit cost of each group is assigned as the price for every element in that group. This method of dividing the set of items to be acquired attempts to weigh the size of each group appropriately, contributing to a good deal less error than the standard technique of dividing them into groups of equal number. Although somewhat more of a demanding procedure, the task is easily accomplished in a spreadsheet. Figure (19) illustrates the process described above. Figure (20) shows the cumulative cost curve of a system priced using the Base Model versus the same system using a step function approximation. Implementing the step function approximation into a spreadsheet computes a cost of $160,546 for the current procurement schedule. This is a represents a slight increase in error, which is to be expected, an overestimate of the true cost by 1.30%.
Figure 19. Linearization of the Learning Curve Using a Step Function

The average unit cost of the systems yet to be procured is determined. The set of units to be procured is divided into two subsets: the first with units of above average cost, the second with units below average cost. These subsets are further divided in the same manner as the original set, yielding a total of four groups. The average unit cost of each group is assigned to every item in that group.

Figure 20. Cumulative Cost of a System: Base Model vs. Step Function

Comparison of cumulative system cost as determined by the Base Model and its step function approximation. The system shown has a T₁ of 20 and learning curve slope, exaggerated for clarity, of 71% (β = -0.5).
B. MODEL FORMULATION

1. Introduction

PSOM is an integer-linear program that recommends the quantity of each MDAP system to be procured in each year of the DPP. PSOM carries the same assumptions as the Base + Overhead cost which it employs; the most significant of these is that periods are measured in years, with partial periods of production not allowed. Thus, the overhead incurred when a system is in production is the same for each year.

The model uses both binary and continuous variables to achieve a balance of realism and solvability. Binary variables are used to indicate the start periods and periods of full rate production (FRP) for each system. Additional binary variables are used to control the step function approximation of the cost function. Although procurement quantities must be integers in reality, they are represented by continuous variables to expedite solution.

2. Formulation

**Indices**

\[ i \] system

\[ t, t' \] time period \( \{1 \ldots N\} \)

\[ p \] LRIP period \( \{1 \ldots M_i\} \)

\[ c \] cost level \( \{1 \ldots 4\} \)

**Data**

\[ N \] number of periods under consideration in this model

\[ M_i \] number of LRIP periods for system \( i \)

\[ QLRIP_{ip} \] number of units \( i \) in LRIP period \( p \)

\[ CLRIP_{ip} \] procurement cost of system \( i \) in LRIP period \( p \)

\[ PRICE_{ic} \] unit cost of system \( i \) at cost level \( c \)

\[ OVERHEAD_{Di} \] fixed cost incurred by system \( i \) if in FRP

\[ MINBUY_{ic} \] quantity of system \( i \) that must procured at cost level \( c \) before they may be procured at cost level \( c + 1 \)

\[ BUDGET_i \] annual budget constraint
\[ DEMAND_{it} \] required minimum quantities of system \( i \) by period \( t \)

\[ INV_i \] starting inventory of system \( i \) in period 1

\[ MINRATE_i \] minimum sustaining production rate for system \( i \) during FRP

\[ MAXRATE_i \] maximum production rate for system \( i \) during FRP

\[ EARLY_i \] earliest period in which FRP may begin for system \( i \) \((> M_i)\)

\[ LATE_i \] latest period in which FRP may begin for system \( i \) \((\leq N)\)

**Variables**

\[ q_{it} \] units of system \( i \) procured in period \( t \)

\[ start_{it} \] \{1 if system \( i \) begins FRP in period \( t \), else 0 \}

\[ frp_{it} \] \{1 if system \( i \) is in FRP in period \( t \), else 0 \}

\[ y_{itc} \] \{1 if system \( i \) is available at price level \( c \) in period \( t \), else 0 \}

\[ cumq_{it} \] cumulative units of system \( i \) procured by period \( t \)

\[ cost_{it} \] cost of all units of system \( i \) procured in period \( t \)

**Formulation**

MINIMIZE: \[ \sum_i \sum_t cost_{it} \]

SUBJECT TO:

**Budget**

\[ cost_{it} = \sum_c q_{itc} PRICE_{ic} + frp_{it} OVERHEAD_i \]

\[ + \sum_i \sum_{p=t}^{M_i} CLRIP_p \left( start_{it(1+r+p-M_i)} \right) \quad \forall i, t \] \hspace{1cm} (C1)

\[ BUDGET \geq \sum_i cost_{it} \quad \forall t \] \hspace{1cm} (C2)

**Demand**

\[ cumq_{it} = INV_i + \sum_{r=1}^t \sum_c q_{irtc} + \sum_{r=1}^t \sum_{p=t}^{M_i} QLRIP_p \left( start_{it(1+r+p-M_i)} \right) \quad \forall i, t \] \hspace{1cm} (C3)

\[ cumq_{it} \geq DEMAND_{it} \quad \forall i, t \] \hspace{1cm} (C4)
Production Rates
\[ \sum_{c} q_{n,c} \geq f_{n,n} \text{MINRATE}_i \quad \forall i, t \quad \text{(C5)} \]
\[ \sum_{c} q_{n,c} \leq f_{n,n} \text{MAXRATE}_i \quad \forall i, t \quad \text{(C6)} \]

FRP Start
\[ \sum_{i} t(\text{start}_n) \geq \text{EARLY}_i \quad \forall i \quad \text{(C7)} \]
\[ \sum_{i} t(\text{start}_n) \leq \text{LATE}_i \quad \forall i \quad \text{(C8)} \]

No Production Breaks
\[ \sum_{i} \text{start}_n = 1 \quad \forall i \quad \text{(C9)} \]

FRP Start/Production Status Integrity
\[ \text{start}_n \geq (f_{n,n} - f_{n,n-1}) \quad \forall it \quad \text{(C10)} \]

Step Function Approximation and Buy Limit per Cost Level
\[ y_{n,1} \text{MINBUY}_{i1} \leq \sum_{r=1}^{i} q_{n,r} \leq \text{MINBUY}_{i1} \quad \forall it \quad \text{(C11)} \]
\[ y_{n,2} \text{MINBUY}_{i2} \leq \sum_{r=1}^{i} q_{n,r} \leq \text{MINBUY}_{i2} y_{n,1} \quad \forall it \quad \text{(C12)} \]
\[ y_{n,3} \text{MINBUY}_{i3} \leq \sum_{r=1}^{i} q_{n,r} \leq \text{MINBUY}_{i3} y_{n,2} \quad \forall it \quad \text{(C13)} \]
\[ y_{n,4} \text{MINBUY}_{i4} \leq \sum_{r=1}^{i} q_{n,r} \leq \text{MINBUY}_{i4} y_{n,3} \quad \forall it \quad \text{(C14)} \]

Variables
\[ q_{n,c}, \text{cum}_n, \text{cost}_n \geq 0 \quad \forall i, t \quad \text{(C15)} \]
\[ f_{n,n}, \text{start}_n, y_{n,c} \in [0, 1] \quad \forall i, t \quad \text{(C16)} \]
3. **Explanation of Constraints**

Constraints (C1) define the cost of all units of system $i$ procured in period $t$, $cost_{it}$.

The most complicated term in these constraints is the "LRIP term"

$$
\sum_{i}^{M} \sum_{p=1}^{t} CLRIP_{ip} \left( start_{i+M,-p} \right) \quad \forall i, t
$$

which is the LRIP costs in time period $t$ due to LRIP for system $i$. A hypothetical system with three LRIP periods, a first LRIP period cost of $X$, and FRP beginning in period six provides an example (we omit the $i$ subscript for brevity):

$M = 3; \quad CLRIP_i = X; \quad start_6 = 1$

we see that in time period $t = 3$, the LRIP cost is

$$CLRIP_i \left( start_{i+3,-3} \right) = X (start_{i+3,2-t}) = X (start_6) = X \ast 1 = X$$

as is required. Similar results are obtained for LRIP periods 2 and 3 during time periods 4 and 5 respectively.

Constraints (C2) ensure that costs per year never exceed the annual budget.

Constraints (C3) define the cumulative quantity of each system procured by period $t$, and Constraints (C4) ensure that this quantity meets demand. Constraints (C5) and (C6) ensure that annual rates of production are maintained between their respective upper and lower limits. Constraints (C7) and (C8) imposed a window of allowable periods in which FRP may begin ($\sum t (start_{it})$ is simply the FRP start period for system $i$.)

Constraints (C9) make certain that there are no breaks in production. Constraints (C10) maintain consistency between the indicator variables for $start_{it}$ and $frp_{it}$. Constraints (C11) through (C14) enforce the step function approximation to the learning curve. The binary variables $y_{ic}$ ensure that all of the more expensive units are procured in the required quantity before allowing the next less expensive units to be procured. This is the same "toggling" technique used by Loerch (1997).

C. **IMPLEMENTATION AND ANALYSIS**

1. **Implementation**

PSOM is implemented in the General Algebraic Modeling System (GAMS) [Brooke et al. 1997] with the CPLEX solver, Version 6.5 [ILOG 1999]. Over an 18-year planning horizon seventeen MDAP systems are scheduled, at four cost levels. The model
has approximately 11,000 equations and 3,500 variables, of which 1,600 are binary. Appendix C is the GAMS implementation of PSOM.

PSOM is a mixed-integer linear program, and is solved by branch-and-bound enumeration. The time required to solve each run of the model is influenced by the relative integer termination tolerance; this is the difference between the best integer solution and the best known lower bound, divided by the absolute value of the best integer solution. With a relative tolerance of 0.001, PSOM generally runs in less than seven minutes on a personal computer equipped with a Pentium II 333MHZ processor and 296MB of RAM. With a relative tolerance of 0.0005, the mean time to solve is increased to 15 minutes. While a relative tolerance of one tenth of one percent certainly represents a higher fidelity and resolution than the underlying input data, we used a 0.0005 relative tolerance to more accurately compare model results at varying budget levels.

2. **Computational Results**

Since the seventeen systems modeled by PSOM are only a subset of the eighty-plus systems that comprise the MDAP population, there is no accurate budget amount to input to the model. We use the annual budget amount of $16,000 (CY00$M) derived earlier and used in our spreadsheet “optimization.” At this budget level, PSOM determines the optimal schedule depicted in Figure (21). PSOM minimizes cost by eliminating as much overhead as possible. Total cost of this schedule, as determined by GAMS, is $156,297 (CY00$M); this is 0.26% more than the spreadsheet-calculated cost of the same schedule, which we attribute to rounding and linearization error. Comparing the cost of the current to the optimal schedule, a potential savings of $3,792 (CY00$M) is revealed. Additionally, we notice that this is $3,268 (CY00$M) less than the least costly spreadsheet-determined schedule.
Figure 21. Optimal Schedule at Current Budget Level

The optimized procurement schedule at the current estimated annual budget limit. The optimized schedule represents a potential savings of $3,792 (CY00$M) over the current schedule.

PSOM can be used to construct a chart of the efficiency frontier, a plot of the minimum total cost of all systems at varying budget limits. This is built by repeatedly solving PSOM in a loop, with the budget decreasing after each iteration, from an amount in which the constraint is slack to the point at which the model becomes infeasible. A simple modification to PSOM, in which the budget is minimized vice the total cost of all systems, reveals the absolute minimum annual budget to be $12,714 (CY00$M). Figure (22) is the efficiency frontier for this procurement schedule. Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. The decision maker can use the information in many ways. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate
additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.

![Graph](image)

**Figure 22. Efficiency Frontier for MDAP Procurement Schedule**
Procurement schedules corresponding to all points above the line are sub-optimal. Schedules corresponding to points below the line are infeasible. Assuming that the current schedule is sub-optimal, the decision maker may choose to optimize the schedule for the current budget and thus reduce overall cost; or, given an allowable overall cost, the decision maker may choose to reduce the annual budget available. If the schedule is already optimal, the decision maker will readily appreciate the affect of changing the annual budget limit; the additional cost of a reduction in budget, or the potential long-term savings from an increased budget, are equally apparent.
V. CONCLUSION

Of the six cost functions derived from the Unit Theory Model (equation 1), the Base + Overhead Model is most suited to this purpose; it shows the best fit to the data, most accurate estimate of current total cost, and is the simplest to implement in an integer linear program. PSOM uses this relationship to determine the annual procurement costs of the MDAP systems that it schedules.

PSOM allows the analyst to specify: an annual budget limit; demand quantities for each system for all years in the planning horizon; minimum and maximum annual production rates; earliest and latest FRP start periods; and LRIP costs and quantities. PSOM determines the minimum cost procurement schedule given these constraints. Data input to the model requires a working knowledge of GAMS.

Perhaps the most illuminating product that PSOM can determine is the efficiency frontier for the schedule cost versus the annual budget limit. This allows a decision maker to visualize the tradeoff between total and annual costs.

PSOM can be easily expanded to include all 80+ MDAP systems, with some slowing of solution time. Solution times can be reduced by manipulating the model’s relative termination tolerance, while maintaining the required fidelity of the model solution. Thus, PSOM is a useful tool available to acquisition planners and decision makers. Expansion and use of PSOM or a similar optimization model is recommended for the upcoming and subsequent QDRs.
## APPENDIX A: DATA

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Table A1. Data for AAAV, Abrams, Bradley, C-17, CH-47F, and Crusader

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Table A3. Data for MV-22 Osprey, SSN 774, Std Missile 2, T45TS, and Trident II
All Cost CY00$M

45
### BASE MODEL

**MEASURE OF FIT AND PARAMETERS**

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<td>FA-18 E/F</td>
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<td>-0.12197</td>
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<td>2.550</td>
<td>-0.23668</td>
<td>-0.09960</td>
</tr>
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<td>Minuteman III</td>
<td>0.94</td>
<td>17.501</td>
<td>-0.01105</td>
<td>-0.44964</td>
</tr>
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<td>NAVSTAR</td>
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<td>-0.14171</td>
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<tr>
<td>MV-22 Osprey</td>
<td>0.95</td>
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<td>-0.15947</td>
<td>-0.16123</td>
</tr>
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<td>1967.715</td>
<td>-0.02312</td>
<td>-0.34216</td>
</tr>
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<td>Std Missile 2</td>
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<td>39.154</td>
<td>-0.47226</td>
<td>-0.13194</td>
</tr>
<tr>
<td>T-45TS</td>
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<td>60.082</td>
<td>-0.20338</td>
<td>0.01334</td>
</tr>
<tr>
<td>Trident II</td>
<td>0.75</td>
<td>151.321</td>
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<td>-0.31996</td>
</tr>
<tr>
<td><strong>MEDIAN</strong></td>
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Table B2. Multiplicative Rate Model $R^2_{adj}$ and Fitted Parameters
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<tr>
<th>System</th>
<th>Lot $R^2_{adj}$</th>
<th>$T_1$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$R^2$</th>
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<tbody>
<tr>
<td>AAAV</td>
<td>0.51</td>
<td>6.760</td>
<td>-0.04525</td>
<td>0.00553</td>
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</tr>
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<td>Abrams Upgrade</td>
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<td>17.338</td>
<td>-0.19414</td>
<td>0.04208</td>
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</tr>
<tr>
<td>Bradley Upgrade</td>
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<td>-0.11321</td>
<td>0.02098</td>
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</tr>
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<td>C-17</td>
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<td>322.087</td>
<td>-0.08388</td>
<td>19.90869</td>
<td>15.000</td>
</tr>
<tr>
<td>CH-47F</td>
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<tr>
<td>Crusader</td>
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</tr>
<tr>
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<td>1001.187</td>
<td>-0.06159</td>
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</tr>
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<td>EELV</td>
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</tr>
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<td>FA-18 E/F</td>
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<td>JSOW</td>
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<td>Minuteman III</td>
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</tr>
<tr>
<td>NAVSTAR</td>
<td>0.55</td>
<td>123.303</td>
<td>-0.16641</td>
<td>0.00000</td>
<td>3.000</td>
</tr>
<tr>
<td>MV-22 Osprey</td>
<td>0.95</td>
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<td>0.46394</td>
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</tr>
<tr>
<td>SSN 774</td>
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<td>1484.113</td>
<td>0.00000</td>
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<td>3.000</td>
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<tr>
<td>Std Missile 2</td>
<td>0.80</td>
<td>17.657</td>
<td>-0.46028</td>
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<tr>
<td>T-45TS</td>
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<td>-0.19889</td>
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<tr>
<td>Trident II</td>
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Table B3. IDA Rate-Penalty Model $R^2_{adj}$ and Fitted Parameters
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<th>System</th>
<th>Lot $R^2_{adj}$</th>
<th>$T_1$</th>
<th>$\beta$</th>
<th>$\delta$</th>
<th>$R^*$</th>
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<tbody>
<tr>
<td>AAAV</td>
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<td>6.753</td>
<td>-0.04671</td>
<td>0.00489</td>
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<tr>
<td>Abrams Upgrade</td>
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<td>16.985</td>
<td>-0.19390</td>
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<tr>
<td>Bradley Upgrade</td>
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<td>0.00765</td>
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<td>-0.12343</td>
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<tr>
<td>CH-47F</td>
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<td>Crusader</td>
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<td>0.01396</td>
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<tr>
<td>DDG 51</td>
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<td>990.690</td>
<td>-0.08644</td>
<td>130.04896</td>
<td>4.678</td>
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<tr>
<td>EELV</td>
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<td>-0.07373</td>
<td>0.00100</td>
<td>13.000</td>
</tr>
<tr>
<td>FA-18 E/F</td>
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<td>123.303</td>
<td>-0.16641</td>
<td>0.00000</td>
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</tr>
<tr>
<td>MV-22 Osprey</td>
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<tr>
<td>Std Missile 2</td>
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<td>-0.20159</td>
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Table B4. LURP Model $R^2_{adj}$ and Fitted Parameters
### RATE CHANGE MODEL

**MEASURE OF FIT AND PARAMETERS**

<table>
<thead>
<tr>
<th>System</th>
<th>$R^2_{adj}$</th>
<th>$T_1$</th>
<th>$\beta$</th>
<th>$\delta$</th>
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<td>Abrams Upgrade</td>
<td>-0.06</td>
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</tr>
<tr>
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<td>C-17</td>
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<td>0.00100</td>
</tr>
<tr>
<td>CH-47F</td>
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<td>-0.19727</td>
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</tr>
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</tr>
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<td>Trident II</td>
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Table B5. Rate-Change Model $R^2_{adj}$ and Fitted Parameters
## BASE + OVERHEAD MODEL
### MEASURE OF FIT AND PARAMETERS

<table>
<thead>
<tr>
<th>System</th>
<th>Lot $R^2_{adj}$</th>
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<th>$\beta$</th>
<th>$\Omega$</th>
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<tbody>
<tr>
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<td>7.003</td>
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<tr>
<td>C-17</td>
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<td>1.000</td>
</tr>
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<td>Std Missile 2</td>
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</tbody>
</table>

Table B6. Base + Overhead Model $R^2_{adj}$ and Fitted Parameters
APPENDIX C: GAMS IMPLEMENTATION

**PROCUREMENT SCHEDULE OPTIMIZATION MODEL (PSOM)**

*-----DEFAULTS-----------------------------------------------
$OFFUPPER OFFSYMREF INLINECOM{ }

OPTIONS RESLIM = 100000
     ITERLIM = 100000
     LIMCOL = 0
     LIMROW = 0
     DECIMALS = 2
     SOLPRINT = OFF
     MIP = CPLEX
     SYSOUT = OFF
     OPTCR = .0005
;
*-------------------------------------------------------------
SOMETEXT
Original:  9/28/00
Author:  Donald E. Humpert

Description:  Base + Overhead Model
              Time period one = current year = 2000;
              Time period 19 = 2018;

SOMETEXT
*---INDICES---------------------------------------------------

SET
 i      system
     /AAAV
     ABRAMS
     BRADLEY
     CI7
     CH47F
     CRUSADER
     DDG51
     EELV
     FA18EF
     JSOW
     MMIII
     NAVSTAR
     OSPREY
     SSN774
     STDM5L2
     T45TS
     TRIDENT2/

 t     time periods / 1*19 /

 p     LRIP time periods / 1*3 /

 c     cost level /1*4/
 ;

ALIAS (t, tp);

*---DATA------------------------------------------------------

PARAMETERS
 M(i)   LRIP time periods for system i
       /AAAV 1
       CRUSADER 1/

 QLRIP(i,p) number of units i in LRIP period p
           /AAAV .1 38
           CRUSADER .1 62/
CLRIP(i,p) procurement cost of system i in LRIP period p
/AAAV .1 2.494
CRUSADER .1 5.830/

INV(i) starting inventory of system i in period 1
/AAAV 0
ABRAMS 848
BRADLEY 229
C17 85
CH47F 0
CRUSADER 0
DDG51 48
EELV 1
FA18EF 98
JSOW 1017
MMII 148
NAVSTAR 0
OSPREY 30
SSN774 3
STDMSL2 294
T45TS 153
TRIDENT2 35/

MINR(i) min sustaining production rate for system i during FRP
/AAAV 20
ABRAMS 20
BRADLEY 20
C17 1
CH47F 8
CRUSADER 20
DDG51 1
EELV 4
FA18EF 12
JSOW 100
MMII 10
NAVSTAR 3
OSPREY 9
SSN774 1
STDMSL2 60
T45TS 1
TRIDENT2 5/

MAXR(i) maximum production rate for system i during FRP
/AAAV 200
ABRAMS 120
BRADLEY 235
C17 15
CH47F 29
CRUSADER 240
DDG51 3
EELV 14
FA18EF 48
JSOW 900
MMII 80
NAVSTAR 3
OSPREY 36
SSN774 3
STDMSL2 190
T45TS 15
TRIDENT2 12/
EARLY(i) earliest period in which FRP may begin for system i
/AAAV  3
ABRAMS  2
BRADLEY  2
C17  2
CH47F  2
CRUSADER  3
DDG51  2
EELV  2
FA18EF  2
JSOW  2
MMI1  2
NAVSTAR  2
OSPREY  2
SSN774  2
STDMSL2  2
T45TS  2
TRIDENT2  2/

LATE(i) latest period in which FRP may begin for system i
/AAAV  16
ABRAMS  2
BRADLEY  2
C17  2
CH47F  14
CRUSADER  18
DDG51  2
EELV  2
FA18EF  2
JSOW  2
MMI1  2
NAVSTAR  17
OSPREY  2
SSN774  2
STDMSL2  2
T45TS  2
TRIDENT2  2/

D(i,t) required quantities of system i by period t
/AAAV .10  950
ABRAMS .7  1131
BRADLEY .13  1602
C17 .5  134
CH47F .14  300
CRUSADER .18  1630
DDG51 .6  58
EELV .19  181
FA18EF .11  548
JSOW .14  10000
MMI1 .8  652
NAVSTAR .17  42
OSPREY .14  408
SSN774 .15  30
STDMSL2 .11  1500
T45TS .5  169
TRIDENT2 .6  88/
FIXED(i) fixed costs for system i incurred if in fsp
/AAA V 37.372
ABRAMS 219.906
BRADLEY 53.401
C17 831.878
CH47F 1.000
CRUSADER 90.972
DDG51 980.791
EELV 1.000
FA18EF 1106.798
JSOW 4.665
MMIII 34.847
NAVSTAR 1.000
OSPREY 197.694
SSN774 414.191
STDMSL2 16.248
T45TS 1.000
TRIDENT2 92.509;

TABLE PRICEL(i,c) price of system i at cost level c

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| TRIDENT2| 27.7030| 25.6831| 24.0186| 22.6609;

TABLE LIMITL(i,c) min number of system i that must be procured at
price c before any can be procured at price next c

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</tbody>
</table>
| TRIDENT2| 11    | 13    | 14    | 15;

SCALAR BGT annual budget /25000/;
*---VARIABLES---------------------------------------------------------------

VARIABLE
  TOTAL   total procurement costs for all systems over all periods

POSITIVE VARIABLES
  q(i,t,c)  units of system i procured in period t at cost level c
  cumq(i,t) cumulative units of system i procured by period t
  cost(i,t) cost of units of system i procured in period t

BINARY VARIABLES
  start(i,t)  1 if system i begins FRP in period t else 0
  frp(i,t)    1 if system i is in FRP in period t else 0
  y(i,t,c)    1 if system i available at price level c in period t else 0

*---EQUATIONS---------------------------------------------------------------

EQUATIONS
  OBJ       Objective Function
  PAYOUT(i,t)       cost(i,t) = sum(c, q(i,t,c)*PRICE(i,c)) + frp(i,t)*FIXED(i)  
                 + sum(p$ (ord(p)<=M(i)), CLRIP(i,p)*start(i,t+(1+M(i)-ord(p))))
  BUDGET(t)       BGT = sum(i, cost(i,t))
  CUMQUANT(i,t)   cumq(i,t) = INV(i)  
                 + sum(tp$ (ord(tp)<=ord(t)), sum(c, q(i,tp,c)))  
                 + sum(tp$ (ord(tp)<=ord(t)), sum(p$ (ord(p)<=M(i)),  
                 QLRIP(i,p)*start(i,tp+(1+M(i)-ord(p)))))
  DEMAND(i,t)     cumq(i,t) = D(i,t)
  MINPROD(i,t)    sum(c, q(i,t,c)) = frp(i,t)*MINR(i)
  MAXPROD(i,t)    sum(c, q(i,t,c)) = frp(i,t)*MAXR(i)
  EARLIEST(i)     sum(t, ord(t)*start(i,t)) = EARLY(i)

*---OBJECTIVE FUNCTION------------------------------------------------------

OBJ..       TOTAL =e= sum(i, sum(t, cost(i,t)))

*---CONSTRAINTS------------------------------------------------------------

PAYOUT(i,t)..  cost(i,t) =e= sum(c, q(i,t,c)*PRICE(i,c)) + frp(i,t)*FIXED(i)  
                + sum(p$ (ord(p)<=M(i)), CLRIP(i,p)*start(i,t+(1+M(i)-ord(p))))
BUDGET(t)..    BGT =g= sum(i, cost(i,t))
CUMQUANT(i,t)..  cumq(i,t) =e= INV(i)  
                + sum(tp$ (ord(tp)<=ord(t)), sum(c, q(i,tp,c)))  
                + sum(tp$ (ord(tp)<=ord(t)), sum(p$ (ord(p)<=M(i)),  
                QLRIP(i,p)*start(i,tp+(1+M(i)-ord(p)))))
DEMAND(i,t)..  cumq(i,t) =g= D(i,t)
MINPROD(i,t)..  sum(c, q(i,t,c)) =g= frp(i,t)*MINR(i)
MAXPROD(i,t)..  sum(c, q(i,t,c)) =l= frp(i,t)*MAXR(i)
EARLIEST(i)..  sum(t, ord(t)*start(i,t)) =g= EARLY(i)
LATEST(i) . . \[ \text{sum}(t, \text{ord}(t) \times \text{start}(i,t)) = \text{LATE}(i); \]

NOBREAKS(i) . . \[ \text{sum}(t, \text{start}(i,t)) = e = 1; \]

FRPSTART(i,t) . . \[ \text{start}(i,t) = g = (\text{frp}(i,t) - \text{frp}(i,t-1)); \]

STEP1LHS(i,t,c) . . \[ \text{LIMIT}(i, "1") \times y(i,t,"1") = l = \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"1")); \]

STEP2LHS(i,t,c) . . \[ \text{LIMIT}(i, "2") \times y(i,t,"2") = l = \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"2")); \]

STEP3LHS(i,t,c) . . \[ \text{LIMIT}(i, "3") \times y(i,t,"3") = l = \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"3")); \]

STEP1RHS(i,t,c) . . \[ \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"1")) = L = \text{LIMIT}(i, "1"); \]

STEP2RHS(i,t,c) . . \[ \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"2")) = L = \text{LIMIT}(i, "2") \times y(i,t,"1"); \]

STEP3RHS(i,t,c) . . \[ \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"3")) = L = \text{LIMIT}(i, "3") \times y(i,t,"2"); \]

STEP4RHS(i,t,c) . . \[ \text{sum}(\text{tp}(\text{ord}(\text{tp}) \leq \text{ord}(t)), q(i,\text{tp},"4")) = L = \text{LIMIT}(i, "4") \times y(i,t,"3"); \]

***************************************************************************

MODEL PSOM/ ALL /;
file report1:put report1:report1:pc=5;
put report1;
put 'ANNUAL BUDGET', 'TOTAL COST'/;
put 'SYSTEM QUANTITIES PER PERIOD, PER BUDGET'/;
for(BIT = 22000 downto 12750 by 250,
    Solve PSOM using MIP MINIMIZING TOTAL;
    DISPLAY BUDGET.l;
    DISPLAY q.l;
    put report1;
    put BGT;
    put TOTAL.l;
    put/;
);
LIST OF REFERENCES

Air Force Institute of Technology, 1997, Course Reference for Intermediate Cost Analysis Course (BCE-204), School of Systems and Logistics, Dept of Quantitative Management, Wright-Patterson AFB, Ohio, Chapter 17.


Rogers, P., 1999, Conversation between Phil Rogers, OUSD(AT&L), and the author, November 1999.

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