NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE 2643

SPAN LOAD DISTRIBUTIONS RESULTING FROM ANGLE OF ATTACK,
ROLLING, AND PITCHING FOR TAPERED SWEPTBACK WINGS
WITH STREAMWISE TIPS

SUPersonic LEADING AND TRAILING EDGES

By John C. Martin and Isabella Jeffreys

Langley Aeronautical Laboratory
Langley Field, Va.

DISTRIBUTION STATEMENT A
Approved for Public Release
Distribution Unlimited

NACA
Washington
July 1952

Reproduced From
Best Available Copy
SPAN LOAD DISTRIBUTIONS RESULTING FROM ANGLE OF ATTACK,
ROLLING, AND PITCHING FOR TAPERED SWEEPBACK WINGS
WITH STREAMWISE TIPS

SUPersonic Leading AND TraLLIng EdGES

By John C. Martin and Isabella Jeffreys

SUMMARY

On the basis of the linearized supersonic-flow theory the span load
distributions resulting from constant angle of attack, from steady rolling,
and from steady pitching were calculated for a series of thin sweepback
tapered wings with streamwise tips and with supersonic leading and trailing
edges. The results are valid for the Mach number range for which the
Mach line from either wing tip does not intersect the remote half-wing.

The results of the analysis are presented as a series of design
charts. Some illustrative variations of the spanwise distribution of
circulation with the various design parameters are also presented.

INTRODUCTION

A knowledge of aerodynamic spanwise loading is of great value in
performing aerodynamic calculations. In references 1 to 4 the linearized
upwash behind a lifting wing is shown to be largely determined by the
spanwise loading except for the region close to the trailing edge. It
may also be demonstrated that, except in the vicinity of the trailing
edge, the sidewash velocity component is also largely determined by the
spanwise loading. The aim of the present paper is to determine spanwise
loadings for a series of thin sweepback tapered wings with streamwise
tips and with supersonic leading and trailing edges. These spanwise
loadings can be utilized in connection with the estimation of flow fields
although the results of the analysis may also be applied to problems in
aerodynamic loads and aerelasticity.

The spanwise distribution of circulation resulting from a constant
angle of attack was evaluated chiefly because of the significance of the
downwash induced by the wing on the horizontal tail surfaces. Similarly, the spanwise distribution of circulation resulting from a constant rate of roll was evaluated principally because of the significance of the velocities induced by the wing on the tail-surface contribution to stability and damping. The spanwise distribution of circulation resulting from a constant rate of pitch was evaluated because of the possible importance of the downwash induced by the pitching wing on the horizontal tail surfaces and because the downwash resulting from a pitching wing is one component of the downwash induced by a wing with a constant vertical acceleration. (See reference 5.)

This paper presents calculated curves for the spanwise distribution of circulation (the spanwise distribution of circulation is proportional to the spanwise loading) resulting from a constant angle of attack, a constant rate of roll, and a constant rate of pitch. The wings considered have an arbitrary taper ratio, leading and trailing edges that are straight across the semispan and swept at a constant angle, and tips that are parallel to the free-stream direction. The results are valid for the range of Mach number for which the leading and trailing edges are supersonic.

The results of the analysis are given in the form of generalized equations for the spanwise distribution of circulation resulting from a constant angle of attack, a constant rate of roll, and a constant rate of pitch. A series of design curves is presented from which rapid estimation of the spanwise distributions of circulation can be made for given values of aspect ratio, taper ratio, Mach number, and leading-edge sweep. Some illustrative variations of the spanwise distributions of circulation are also presented.

SYMBOLS

A aspect ratio

\[ B = \sqrt{M^2 - 1} \]

h spanwise coordinate of intersection of trailing edge of wing and Mach line from wing tip

c section lift coefficient

\[ \Delta C_p \] pressure-difference coefficient

c chord (subscript r refers to root chord)

\bar{c} mean aerodynamic chord
\( d \)  \( \text{spanwise coordinate of intersection of trailing edge of wing and Mach line reflected from wing tip} \)

\( e \)  \( \text{spanwise coordinate of intersection of trailing edge of wing and Mach line from leading edge of wing} \)

\( g(x_1) \)  \( \text{expression for that part of boundary of area } S_1 \text{ not made up of Mach lines from point } (x,y) \text{(see fig. 3)} \)

\( g_1(x,y), g_2(x,y) \)  \( \text{limits of integration (see fig. 3)} \)

\( H \)  \( \text{distance in chord lengths from wing apex to center-of-gravity location for wing with a static margin of 0.058} \)

\( b \)  \( \text{wing span} \)

\( k = \frac{\cot \Lambda_{TE}}{\cot \Lambda} = \frac{AB(1 + \lambda)}{AB(1 + \lambda) - \frac{1}{2}MB(1 - \lambda)} \)

\( \dot{c} \)  \( \text{spanwise loading} \)

\( M \)  \( \text{Mach number} \)

\( m \)  \( \cot \Lambda \)

\( P \)  \( \text{defined by equation (6)} \)

\( p \)  \( \text{rate of roll} \)

\( q \)  \( \text{rate of pitch} \)

\( S_1 \)  \( \text{area of integration} \)

\( V \)  \( \text{free-stream velocity} \)

\( x, y, z \)  \( \text{rectangular coordinates (x-axis parallel to free-stream direction)} \)

\( x_1, y_1 \)  \( \text{auxiliary rectangular coordinates} \)

\( \alpha \)  \( \text{angle of attack} \)

\( \Gamma \)  \( \text{spanwise distribution of circulation (defined by equation (2))} \)
\( \phi \) velocity potential on wing upper surface

\( \Lambda \) sweep of wing leading edge (see fig. 1)

\( \Lambda_{TE} \) sweep of wing trailing edge (see fig. 1)

\( \lambda \) taper ratio

\( \oint \) indicates a closed line integral

Subscript:

TE refers to wing trailing edge

ANALYSIS

Scope

The analysis is limited to calculations of the spanwise distributions of circulation for wings of vanishingly small thickness that have zero camber. The results are valid for a range of supersonic speeds for which the leading and trailing edges are supersonic (the components of free-stream velocity normal to the edges are supersonic). The wing configurations considered are defined by the information and sketches given in figure 1. These wings have an arbitrary taper ratio, streamwise tips, and sweptback leading edges, although the trailing edges may be either sweptback or sweptforward. A further restriction is that the Mach line from either tip may not intersect the remote half-wing.

Method

Basic considerations. - The evaluation of the spanwise loadings generally requires the knowledge of the pressure distribution on the wing surface or the knowledge of the perturbation velocity potential along the wing trailing edge. These two quantities are related by the following expression:

\[ c_{ci} = \int_{LE}^{TE} \Delta C_p \, dx = \frac{h}{V} \phi_{TE} \]  \tag{1}
The spanwise distribution of circulation is related to the spanwise loading and the trailing-edge potential by the following equation:

\[ \Gamma = 2 \int_{LE}^{TE} \phi_x \, dx = 2 \phi_{TE} = \frac{V}{2} \alpha c \gamma \]  \hspace{1cm} (2)

In the remaining sections the spanwise distribution of circulation will be used in preference to the spanwise loading since flow-field calculations are generally set up in terms of the spanwise distribution of circulation.

**Determination of the trailing-edge potential.** - The potential function \( \phi \) must satisfy the linearized partial-differential equation of steady flow and the boundary conditions that are associated with the wing in its prescribed motion. The boundary conditions on a wing performing the motions considered here are:

For a constant angle of attack,

\[ \phi_z = -\alpha V \quad (z = 0) \]  \hspace{1cm} (3a)

For a constant rate of roll,

\[ \phi_z = -\rho y \quad (z = 0) \]  \hspace{1cm} (3b)

For a constant rate of pitch,

\[ \phi_z = -\rho x \quad (z = 0) \]  \hspace{1cm} (3c)

Note that, within the framework of the linearized theory, the boundary condition for a wing with a constant rate of roll is also the boundary condition for a wing which has a linear lateral twist and that the boundary condition on a wing with a constant rate of pitch is also the boundary condition on a wing which has linear camber.

The potential along the wing trailing edge can be determined by Eevard's method (reference 6). From this reference the potential at any point on the upper surface of the wing may be expressed as

\[ \phi(x,y) = -\frac{1}{\pi} \iint_{S_1} \frac{\phi_z}{\sqrt{(x - x_1)^2 + B^2(y - y_1)^2}} \, dx_1 \, dy_1 \]  \hspace{1cm} (4)
The area of integration $S_1$ is the area of the wing plan form within the "effective" forward Mach cone from the point $(x,y)$. Figure 2 shows such a region of integration. For the motions of the wing considered herein, the potential on the upper surface of the wing may be obtained by substituting equations (3) into equation (4) and performing the indicated integrations.

The evaluation of the integrals involved in finding the potential can, however, be simplified by making use of the well-known relation (reference 7, p. 181)

$$
\int \int \frac{\partial P(x_1, y_1)}{\partial y_1} \, dx_1 \, dy_1 = - \oint P(x_1, y_1) \, dx_1
$$

(5)

From a comparison of equations (4) and (5) the function $P(x_1, y_1)$ is seen to be given by

$$
P(x_1, y_1) = - \frac{1}{\pi} \int \frac{\phi_z}{\sqrt{(x - x_1)^2 - B^2(y - y_1)^2}} \, dy_1
$$

(6)

Hence, from equations (3), (4), (5), and (6) the potential on the upper surface of a wing is as follows:

For a constant angle of attack,

$$
\phi(x,y) = \frac{\pi V}{\pi B} \oint_{S_1} \sin^{-1} \frac{B(y - y_1)}{x - x_1} \, dx_1
$$

(7)

For a constant rate of roll,

$$
\phi(x,y) = \frac{p}{\pi B} \oint_{S_1} \left[ \frac{1}{B} \sqrt{(x - x_1)^2 - B^2(y - y_1)^2} + y \sin^{-1} \frac{B(y - y_1)}{x - x_1} \right] \, dx_1
$$

(8)
For a constant rate of pitch,

$$\varphi(x,y) = \frac{q}{\pi B} \oint_{S_1} x_1 \sin^{-1} \frac{B(y - y_1)}{x - x_1} \, dx_1$$ \hspace{1cm} (9)

The line integrals in equations (7) to (9) along the Mach lines from the point \((x,y)\) can be easily evaluated. The following expressions for the potentials are obtained:

For a constant angle of attack,

$$\varphi(x,y) = \frac{\alpha y}{2B} \left[ 2x - \varepsilon_1(x,y) - \varepsilon_2(x,y) \right] + \frac{\alpha y}{\pi B} \oint_{g_1(x)} \sin^{-1} \frac{B(y - g(x_1))}{x - x_1} \, dx_1$$ \hspace{1cm} (10)

For a constant rate of roll,

$$\varphi(x,y) = \frac{p y}{2B} \left[ 2x - \varepsilon_1(x,y) - \varepsilon_2(x,y) \right] + \frac{p}{\pi B} \oint_{g_1(x)} \left\{ \frac{\sqrt{(x - x_1)^2 - B^2 \left[ y - g(x_1) \right]^2}}{y \sin^{-1} \frac{B[y - g(x_1)]}{x - x_1}} \right\} \, dx_1$$ \hspace{1cm} (11)

For a constant rate of pitch,

$$\varphi(x,y) = \frac{q}{2B} \left\{ x^2 - \frac{\varepsilon_1(x,y)^2}{2} - \frac{\varepsilon_2(x,y)^2}{2} \right\} + \frac{q}{\pi B} \oint_{g_1(x)} x_1 \sin^{-1} \frac{B[y - g(x_1)]}{x - x_1} \, dx_1$$ \hspace{1cm} (12)
where $g(x_1)$ is the expression for that part of the boundary of the area $S_1$ that does not contain the Mach lines from the point $(x, y)$, and where $g_1(x)$ and $g_2(x)$ are the limits of integration which are actually the end points of the $g(x_1)$ boundary. (See fig. 3.) Note that equations (10), (11), and (12) are applicable to any plan form to which Evard's method can be applied. Since there are no singularities in the integrands of these equations they can be evaluated numerically without difficulty.

RESULTS

Expressions for the trailing-edge potentials were either taken from table I of reference 8 or found by the use of equations (10), (11), and (12). The spanwise distribution of circulation was expressed as a function of $\gamma$ by substituting the equation for the trailing edge into the expressions for the potential difference between the upper and lower surfaces of the wing. These expressions for the spanwise distributions of circulation are presented for constant angle of attack in table I, for constant rate of roll in table II, and for constant rate of pitch in table III. The formulas are valid for either sweptforward or sweptback trailing edges, the proper applications depending on the sign of $k$.

The results of the calculations for the spanwise distribution of circulation for wings with a constant angle of attack are presented in figures 4 to 9. An index of these figures is given in table IV. Similar results for constant rate of roll are plotted in figures 10 to 15 and for constant rate of pitching about the wing apex in figures 16 to 21, with indexes of figures for the two types of motions listed in tables V and VI. These figures are equally applicable for sweptback or sweptforward trailing edges.

The results of the calculations presented in figures 16 to 21 are for wings pitching about their apex. The spanwise distribution of circulation for a wing pitching about an arbitrary point located a distance $x_d$ downstream of the wing apex is given by

$$\Gamma'_d = \Gamma_d - \frac{q x_d}{\alpha V} \Gamma_\alpha$$

(13)

where the subscript $d$ indicates the spanwise distribution of circulation associated with a pitching wing and the subscript $\alpha$ indicates the spanwise distribution of circulation associated with a wing at a constant angle of attack.
Note that, if the total circulation of a wing is divided among several lifting lines, the distribution of circulation associated with each lifting line can be determined by the superposition of the distributions of circulation associated with a number of wings. For this reason the calculations were extended to rather large values of the parameter AB.

Figures 4 to 21 indicate that in many cases the spanwise distribution of circulation can be approximated very closely by simple curves. Thus it is to be expected that for these cases the flow field behind the wings could be calculated approximately by making use of these simple curves that approximate the actual spanwise distribution of circulation.

Illustrative curves of the spanwise distribution of circulation for wings with a constant angle of attack, a constant rate of roll, and a constant rate of pitch are presented in figures 22, 23, and 24, respectively. In figure 24 the values presented were calculated by equation (13) for center of gravity located to provide a static margin of 0.05%. These figures show the effect on the spanwise distribution of circulation of varying each of the parameters - aspect ratio, taper ratio, Mach number, and leading-edge sweep - separately. Some specific variations of the spanwise distribution of circulation with the position of the axis of pitch are presented in figure 25.

CONCLUDING REMARK

On the basis of the steady linearized supersonic-flow theory the spanwise distribution of circulation resulting from constant angle of attack, from steady rolling, and from steady pitching was determined for a series of thin sweptback tapered wings with streamwise tips and with supersonic leading and trailing edges.

Langley Aeronautical Laboratory
National Advisory Committee for Aeronautics
Langley Field, Va., October 24, 1951
REFERENCES


TABLE I.- CIRCULATION EXPRESSIONS FOR CONSTANT ANGLE OF ATTACK

<table>
<thead>
<tr>
<th>Range of $\frac{x}{b/2}$</th>
<th>Expression for circulation along the span; Mach line coincident with leading edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{k \rho b \nu}{\rho b (1 + \lambda)} \left( 1 - \frac{x}{b/2} \right) \sqrt{\left( 1 + \frac{\lambda}{b/2} \right) \left( \frac{AB(1 + \lambda)(1 + k - \frac{x}{b/2})}{b/2} + \text{Mach} \right)}$</td>
</tr>
<tr>
<td>$\frac{b}{b/2} \leq \frac{x}{b/2} \leq 1$</td>
<td>$\frac{k \rho b \nu}{\rho b (1 + \lambda)} \left( \frac{b}{b/2} - \frac{x}{b/2} \right) \sqrt{\left( 1 + \frac{\lambda}{b/2} \right) \left( \frac{AB(1 + \lambda)(1 + k - \frac{x}{b/2})}{b/2} + \text{Mach} \right)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of $\frac{x}{b/2}$</th>
<th>Expression for circulation along the span; Mach line intersecting tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{k \rho b \nu}{\rho b (1 + \lambda)} \left( \frac{b}{b/2} - \frac{x}{b/2} \right) \sqrt{\left( 1 + \frac{\lambda}{b/2} \right) \left( \frac{AB(1 + \lambda)(1 + k - \frac{x}{b/2})}{b/2} + \text{Mach} \right)}$</td>
</tr>
<tr>
<td>$\frac{b}{b/2} \leq \frac{x}{b/2} \leq 1$</td>
<td>$\frac{k \rho b \nu}{\rho b (1 + \lambda)} \left( 1 - \frac{x}{b/2} \right) \sqrt{\left( 1 + \frac{\lambda}{b/2} \right) \left( \frac{AB(1 + \lambda)(1 + k - \frac{x}{b/2})}{b/2} + \text{Mach} \right)}$</td>
</tr>
</tbody>
</table>
TABLE I.- CIRCULATION EXPRESSIONS FOR CONSTANT ANGLE OF ATTACK - Continued

<table>
<thead>
<tr>
<th>Range of ( \dfrac{x}{b/2} )</th>
<th>Expression for circulation along the span; Mach line intersecting tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \dfrac{2b/2}{b/2} \leq \dfrac{x}{b/2} \leq 1 )</td>
<td></td>
</tr>
<tr>
<td>( \dfrac{2b/2}{b/2} \leq \dfrac{x}{b/2} \leq 1 )</td>
<td></td>
</tr>
<tr>
<td>( \dfrac{b}{b/2} \leq \dfrac{x}{b/2} \leq \dfrac{h}{b/2} )</td>
<td></td>
</tr>
<tr>
<td>( \dfrac{b}{b/2} \leq \dfrac{x}{b/2} \leq \dfrac{h}{b/2} )</td>
<td></td>
</tr>
</tbody>
</table>

**Figure:** Diagram illustrating the geometric relationships for the expression of circulation along the span with the Mach line intersecting the tip. The diagram shows the geometry of the airfoil and the mach line, with some key points labeled for clarity.

**Legend:**
- **AB:** Distance between two points on the airfoil
- **\( h \):** Distance from the airfoil to the Mach line
- **\( k \):** Mach number
- **\( \alpha \):** Angle of attack

**Equations:**

- For \( \dfrac{x}{b/2} \leq 1 \):
  \[ \text{Expression} = \frac{\text{Function}}{\text{Determined Function}} \]

- For \( \dfrac{b}{b/2} \leq \dfrac{x}{b/2} \leq \dfrac{h}{b/2} \):
  \[ \text{Expression} = \frac{\text{Function}}{\text{Determined Function}} \]

- For \( \dfrac{b}{b/2} \leq \dfrac{x}{b/2} \leq \dfrac{h}{b/2} \):
  \[ \text{Expression} = \frac{\text{Function}}{\text{Determined Function}} \]

- For \( \dfrac{2b/2}{b/2} \leq \dfrac{x}{b/2} \leq 1 \):
  \[ \text{Expression} = \frac{\text{Function}}{\text{Determined Function}} \]

**Notes:**
- The expressions are derived based on the geometric and aerodynamic principles applicable to the given range of the variable \( \dfrac{x}{b/2} \).
- Each range corresponds to a specific geometric configuration affecting the circulation along the span.

**Further Reading:**
- Detailed aerodynamic analysis of airfoils and their performance under varying conditions.
- Additional tables and figures may be found in the comprehensive report on airfoil design and performance.
TABLE I. - CIRCULATION EXPRESSIONS FOR CONSTANT ANGLE OF ATTACK - Continued

### Expression for circulation along the span; Mach line from center intersecting trailing edge

<table>
<thead>
<tr>
<th>Range of $\frac{Y}{b/2}$</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{Y}{b/2} \leq \frac{a}{b/2}$</td>
<td>$\frac{2\pi ab/2}{nAB(1 + \lambda)(\beta^2 - 1)} \left( \frac{4\pi bK + AB(1 + \lambda)(1 - k)Y}{b/2} \right) \cos^{-1} \left( \frac{AB(1 + \lambda)(1 - B^2\pi^2)Y}{b/2} + 4\pi bK \right)$ + $\frac{4\pi bK + AB(1 + \lambda)(1 + k)Y}{b/2} \cos^{-1} \left( \frac{AB(1 + \lambda)(1 + B^2\pi^2)Y}{b/2} + 4\pi bK \right)$</td>
</tr>
<tr>
<td>$\frac{a}{b/2} \leq \frac{Y}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{2\pi ab/2}{nAB(1 + \lambda)(\beta^2 - 1)} \left( \frac{4\pi bK + AB(1 + \lambda)(1 - k)Y}{b/2} \right) \cos^{-1} \left( \frac{AB(1 + \lambda)(1 - B^2\pi^2)Y}{b/2} + 4\pi bK \right)$ + $\frac{4\pi bK + AB(1 + \lambda)(1 + k)Y}{b/2} \cos^{-1} \left( \frac{AB(1 + \lambda)(1 + B^2\pi^2)Y}{b/2} + 4\pi bK \right)$</td>
</tr>
<tr>
<td>$\frac{b}{b/2} \leq \frac{Y}{b/2} \leq 1$</td>
<td>$\frac{2\pi bK - \frac{1}{\sqrt{b}}} {nAB(1 + \lambda)(\beta^2 - 1)} \left( \frac{4\pi bK + AB(1 + \lambda)(1 - k)Y}{b/2} \right) \cos^{-1} \left( \frac{AB(1 + \lambda)(1 - B^2\pi^2)Y}{b/2} + 4\pi bK \right)$ + $\frac{4\pi bK + AB(1 + \lambda)(1 + k)Y}{b/2} \cos^{-1} \left( \frac{AB(1 + \lambda)(1 + B^2\pi^2)Y}{b/2} + 4\pi bK \right)$</td>
</tr>
</tbody>
</table>

### Expression for circulation along the span; Mach line coincident with leading edge; unswept trailing edge

<table>
<thead>
<tr>
<th>Range of $\frac{Y}{b/2}$</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{Y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$\frac{4\pi ab/2}{n} \sqrt{\frac{\pi^2}{(b/2)^2} + \frac{16}{A^2b^2(1 + \lambda)^2}}$</td>
</tr>
<tr>
<td>$\frac{h}{b/2} \leq \frac{Y}{b/2} \leq 1$</td>
<td>$\frac{4\pi ab/2}{n} \sqrt{\frac{(1 - \frac{Y}{b/2})^2}{b/2} + \frac{h}{b/2} \frac{1}{AB(1 + \lambda)}}$</td>
</tr>
</tbody>
</table>
### Table I - Circulation Expressions for Constant Angle of Attack - Continued

![Graphical representation of the angle of attack and circulation expressions]

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line intersecting tip; unswept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$- \frac{2V_{ab}b/2}{\sqrt{b^2 - y^2}} \left( \frac{y}{b/2} - \frac{4mb}{AB(1 + \lambda)} \cos^{-1} \left[ \frac{h - BnAB(1 + \lambda)\frac{y}{b/2}}{4mb - AB(1 + \lambda)\frac{y}{b/2}} \right] \right)$</td>
</tr>
<tr>
<td>$\frac{h}{b/2} \leq \frac{y}{b/2} \leq \frac{d}{b/2}$</td>
<td>[ \frac{2V_{ab}b/2}{\pi} \left[ \frac{1}{Bn - \frac{1}{2}} - \frac{\frac{y^2}{b^2}}{(b/2)^2} \right] \frac{Bn + \frac{y}{b/2} \left( \frac{4mb}{AB(1 + \lambda)} + 1 + 2Bn \right) + \frac{4mb}{AB(1 + \lambda)} - Bn - 1 + \frac{1}{\sqrt{b^2 - y^2}} \left( \frac{4mb}{AB(1 + \lambda)} - \frac{y}{b/2} \right) \cos^{-1} \left[ \frac{BnAB(1 + \lambda)\frac{y}{b/2} - 1}{4mb - AB(1 + \lambda)\frac{y}{b/2}} \right] + \cos^{-1} \left[ \frac{AB(1 + \lambda)(1 + 2Bn)\frac{y}{b/2} + 4mb - CAB(1 + \lambda)(2Bn + 1)}{4mb - AB(1 + \lambda)\frac{y}{b/2} + 4mb} \right] \right] + \left[ \frac{y}{b/2} + \frac{4mb}{AB(1 + \lambda)} \cos^{-1} \left[ \frac{BnAB(1 + \lambda)\frac{y}{b/2} + 1}{AB(1 + \lambda)\frac{y}{b/2} + 4mb} \right] \right] ]</td>
</tr>
<tr>
<td>$\frac{d}{b/2} \leq \frac{y}{b/2} \leq 1$</td>
<td>[ \frac{2V_{ab}b/2}{\pi} \left[ \frac{1}{Bn + 1} \left( \frac{y}{b/2} + \frac{4mb}{AB(1 + \lambda)} \right) \cos^{-1} \left[ \frac{AB(1 + \lambda)(2Bn - 1)\frac{y}{b/2} + 4mb - CAB(1 + \lambda)(2Bn - 1)}{AB(1 + \lambda)\frac{y}{b/2} + 4mb} \right] + \frac{2}{(Bn + 1)^2} \left( Bn + \frac{y}{b/2} \left( \frac{4mb}{AB(1 + \lambda)} + 1 + 2Bn \right) + \frac{4mb}{AB(1 + \lambda)} + 1 - Bn \right) \right] ]</td>
</tr>
</tbody>
</table>
Table I: Circulation Expressions for Constant Angle of Attack - Continued

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge and intersecting Mach line from tip; unswept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$-\frac{2Vab/2}{y(b^2/2 - 1)} \left( \frac{-y}{b/2} - \frac{4nB}{AB(1 + \lambda)} \right)^{\cos^{-1} \left( \frac{\frac{4nB}{AB(1 + \lambda)} - \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}}{\frac{4nB}{AB(1 + \lambda)\frac{y}{b/2}} + \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}} \right)}$</td>
</tr>
<tr>
<td>$\frac{h}{b/2} \leq \frac{y}{b/2} \leq \frac{e}{b/2}$</td>
<td>$\frac{2Vab/2}{x} \left[ \frac{2}{(b/2)^2 - 1} \left( \frac{y^2}{b/2} - \frac{4nB}{AB(1 + \lambda)} \right)^{\cos^{-1} \left( \frac{\frac{4nB}{AB(1 + \lambda)} - \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}}{\frac{4nB}{AB(1 + \lambda)\frac{y}{b/2}} + \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}} \right)}$</td>
</tr>
<tr>
<td>$\frac{e}{b/2} \leq \frac{y}{b/2} \leq 1$</td>
<td>$-\frac{2Vab/2}{y(b^2/2 - 1)} \left( \frac{-y}{b/2} - \frac{4nB}{AB(1 + \lambda)} \right)^{\cos^{-1} \left( \frac{\frac{4nB}{AB(1 + \lambda)} - \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}}{\frac{4nB}{AB(1 + \lambda)\frac{y}{b/2}} + \frac{4nB}{4nB - AB(1 + \lambda)\frac{y}{b/2}}} \right)}$</td>
</tr>
</tbody>
</table>

\[ \frac{h}{b/2} = \frac{AB(1 + \lambda)(8m + 1) - 4nB}{8mAB(1 + \lambda)} \]

\[ \frac{e}{b/2} = \frac{4nB}{AB(1 + \lambda)} \]
TABLE 1. CIRCULATION EXPRESSIONS FOR CONSTANT ANGLE OF ATTACK - Concluded

\[ \frac{\alpha}{b/2} = \frac{h}{AB(1 + \lambda)} \]

\[ \frac{h}{b/2} = \frac{AB(1 + \lambda)(3m + 1) - 4mb}{4mAB(1 + \lambda)} \]

<table>
<thead>
<tr>
<th>Range of ( \frac{y}{b/2} )</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge; unswept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{y}{b/2} \leq \frac{c}{b/2} )</td>
<td>[ \frac{-2mb/2}{\sqrt{b^2 - 1}} \left( \frac{y}{b/2} - \frac{4mb}{AB(1 + \lambda)} \right) \cos^{-1} \left( \frac{4mb - b^2AB(1 + \lambda) + \frac{Y_{b/2}}{2}}{4mb - b^2AB(1 + \lambda) - \frac{Y_{b/2}}{2}} \right) ]</td>
</tr>
<tr>
<td>( \frac{c}{b/2} \leq \frac{y}{b/2} \leq \frac{h}{b/2} )</td>
<td>[ \frac{-2mb/2}{\sqrt{b^2 - 1}} \left( \frac{y}{b/2} - \frac{4mb}{AB(1 + \lambda)} \right) \cos^{-1} \left( \frac{AB(1 + \lambda)(1 + 2mb) + 4mb - 2AB(1 + \lambda)(mb - 1)}{4mb - AB(1 + \lambda) - \frac{Y_{b/2}}{2}} \right) ]</td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{y}{b/2} \leq \frac{1}{2} )</td>
<td>[ \frac{2mb}{(b/2)^2} \left( \frac{h}{b/2} - \frac{Y_{b/2}}{b/2} \right) \frac{4mb}{AB(1 + \lambda)} + 1 + 3mb \left( \frac{4mb}{AB(1 + \lambda)} - \frac{h}{b/2} \right) ]</td>
</tr>
</tbody>
</table>

Range of \( \frac{x}{b/2} \) | Expression for circulation along the span; unswept leading edge; Mach line from tip does not intersect remote half-wing |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{h}{b/2} )</td>
<td>[ \frac{2mb/2}{b/2} \left( \frac{1 - (1 - \lambda)x}{b/2} \right) \cos^{-1} \left( \frac{AB(1 + \lambda)(1 - \lambda) - \left( \frac{x}{b/2} \right)^2}{AB(1 + \lambda)(1 - \lambda) - \left( \frac{x}{b/2} \right)^2} \right) ]</td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{x}{b/2} \leq \frac{1}{2} )</td>
<td>[ \frac{2mb}{AB(1 + \lambda)} \left( 1 - \frac{x}{b/2} \right) \left( \frac{x}{b/2} - \frac{y}{b/2} \right) + \frac{\left( 1 - (1 - \lambda)x_{b/2} \right)}{b/2} ]</td>
</tr>
</tbody>
</table>
TABLE II.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF ROLL

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line coincident with leading edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$\frac{\partial p(b/2)^2}{3nAB(1 + \lambda)} \sqrt{\frac{x}{(b/2)^2} \left[ \frac{\Delta p^2(1 + \lambda)^2(1 - k^2)}{(b/2)^2} + 8kAB(1 + \lambda) y \frac{b/2}{b/2} + 16k^2 \right] \frac{1}{x^2}}$</td>
</tr>
<tr>
<td>$\frac{h}{b/2} \leq \frac{y}{b/2} \leq 1$</td>
<td>$\frac{\partial p(b/2)^2}{3nAB(1 + \lambda)} \sqrt{\frac{x}{(b/2)^2} \left[ \frac{(1 - 3k) - 2x + 4k}{AB(1 + \lambda)} \right] \left[ \frac{x}{(b/2)^2} \left( 1 + k \right) + \frac{4k}{AB(1 + \lambda)} \right]}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line intersecting tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$\frac{\partial p(b/2)^2}{km(B_m^2 - 1)} \left[ \frac{y}{(b/2)^2} \left( 1 - B_m^2 \frac{y}{b/2} \right) + \frac{s_n sk(k - k)}{AB(1 + \lambda)} \frac{y}{b/2} + \frac{16s_n sk(k - k)}{AB(1 + \lambda)} \right]$</td>
</tr>
<tr>
<td>$\frac{1}{2k(B_m^2 - 1)} \left( \frac{y}{(b/2)^2} \left( 1 - k \right) \left( 2B_m^2 \frac{y}{b/2} - 1 - k \right) + \frac{s_n sk(k - k)}{AB(1 + \lambda)} \frac{y}{b/2} \right)$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$\frac{16s_n sk(k - k)}{A^2 B_m^2 (1 + \lambda)^2} \cos^{-1} \left[ \frac{\left( AB(1 + \lambda) (1 - B_m^2 \frac{y}{b/2}) \right)}{\frac{B_m}{b/2} + 4s_n sk} \right]$</td>
</tr>
<tr>
<td>$\frac{1}{2k(B_m^2 - 1)} \left( \frac{y}{(b/2)^2} \left( 1 - k \right) \left( 2B_m^2 \frac{y}{b/2} - 1 - k \right) + \frac{s_n sk(k - k)}{AB(1 + \lambda)} \frac{y}{b/2} \right)$</td>
<td></td>
</tr>
<tr>
<td>$0 \leq \frac{y}{b/2} \leq \frac{h}{b/2}$</td>
<td>$\frac{16s_n sk(k - k)}{A^2 B_m^2 (1 + \lambda)^2} \cos^{-1} \left[ \frac{\left( AB(1 + \lambda) (1 + B_m^2 \frac{y}{b/2}) \right)}{\frac{B_m}{b/2} + 4s_n sk} \right]$</td>
</tr>
<tr>
<td>Range of $y/b/2$</td>
<td>Expression for circulation along the span; Each line intersecting tip</td>
</tr>
<tr>
<td>------------------</td>
<td>---------------------------------------------------------------------</td>
</tr>
<tr>
<td>$y/b/2 &lt; 1$</td>
<td>$\frac{Sp(b/2)^2}{\kappa x (\rho^2 - 1)^{3/2}} \left[ \frac{y^2}{(b/2)^2} (1 + k - \frac{2\rho^2 x y}{b/2}) \right. + \frac{3\rho x}{b/2} + \frac{(\rho^2 x y)^2}{A^2 (1 + \lambda)^2} \right]$</td>
</tr>
<tr>
<td>$1 &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$y/b/2 &lt; \frac{4}{b/2}$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} \leq y/b/2 \leq 1$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$y/b/2 \leq 1$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$1 &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$y/b/2 &lt; \frac{4}{b/2}$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} \leq y/b/2 \leq 1$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$y/b/2 \leq 1$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$1 &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$y/b/2 &lt; \frac{4}{b/2}$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} \leq y/b/2 \leq 1$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
<tr>
<td>$\frac{4}{b/2} &lt; y/b/2 &lt; 4$</td>
<td>$\frac{1}{2k} \left[ \frac{y^2}{(b/2)^2} (1 + k - \rho^2 x y + k - 1) - \frac{3\rho x (\rho^2 x y + k - 1)}{b/2} \right]$</td>
</tr>
</tbody>
</table>
TABLE II.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF ROLL - Continued

<table>
<thead>
<tr>
<th>Range of ( \frac{y}{b/2} )</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge and intersecting Mach line from tip</th>
</tr>
</thead>
</table>
| \( 0 \leq \frac{y}{b/2} \leq \frac{h}{b/2} \) | \[
\frac{2h}{\kappa_1 h (b/2)^2 + 1} \frac{y}{b/2} \left[ \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \right] \frac{Y}{b/2} \left[ \frac{1}{(b/2)^2} \right] \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} + \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} \]
| \( \frac{h}{b/2} \leq \frac{y}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{2h}{\kappa_1 h (b/2)^2 + 1} \frac{y}{b/2} \left[ \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \right] \frac{Y}{b/2} \left[ \frac{1}{(b/2)^2} \right] \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} + \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} \]
| \( \frac{b}{b/2} \leq \frac{y}{b/2} \leq \frac{c}{b/2} \) | \[
\frac{2h}{\kappa_1 h (b/2)^2 + 1} \frac{y}{b/2} \left[ \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \right] \frac{Y}{b/2} \left[ \frac{1}{(b/2)^2} \right] \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} + \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} \]
| \( \frac{c}{b/2} \leq \frac{y}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{2h}{\kappa_1 h (b/2)^2 + 1} \frac{y}{b/2} \left[ \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \right] \frac{Y}{b/2} \left[ \frac{1}{(b/2)^2} \right] \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} + \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \frac{Y}{b/2} \]

\( h \) = \( \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \) \( \frac{Y}{b/2} \) \( \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \)

\( c \) = \( \frac{\kappa_1 h (b/2)^2 + 1 - h}{\kappa_1 h (b/2)^2 + 1 + k} \) \( \frac{Y}{b/2} \) \( \frac{16\kappa_1 h (b/2)^2}{\kappa_1 h (b/2)^2 + 1 + k} \)
TABLE II.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF ROLL - Continued

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge and intersecting Mach line from tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{b/2}{b/2} \leq 1$</td>
<td>$-\frac{p(b/2)^2}{2x^2(1+b^2)-1} \left( \frac{y^2}{(b/2)^2} - \frac{k(b/2)^2}{2(1-k)(1+k)} + \frac{8\text{MK}(b^2-b^2)(1-k)}{\text{AB}(1+k)} \frac{y}{b/2} \right) - \frac{16\text{MK}(b^2-b^2)^2}{\text{A}^2\text{B}^2(1+k)} \left( \text{AB}(1+k)(1+k) \frac{y}{b/2} + 4\text{MK}(1+k) \text{BnK} \right) - 2\text{MK}(1+k)(1+k) \frac{y}{b/2} + 4\text{MK}(1+k) \text{BnK} $</td>
</tr>
</tbody>
</table>

\[ \frac{y}{b/2} = \frac{\text{BnK}}{\text{AB}(1+k)(\text{BnK} - 1)} \]

\[ \frac{y}{b/2} = \frac{k(\text{AB}(1+k)(\text{BnK} - 1) - 4\text{MK})}{\text{AB}(1+k)(\text{BnK} + 1)} \]

<table>
<thead>
<tr>
<th>Range of $\frac{y}{b/2}$</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{b/2}{b/2} \leq 1$</td>
<td>$\frac{2\text{MK}(b/2)^2}{3\text{MKnK}} \left( \frac{y^2}{(b/2)^2} - \frac{\text{MK}}{\text{AB}(1+k)} \frac{y}{b/2} + 2\text{MK} + \frac{16\text{MK}(b^2-b^2)^2}{\text{A}^2\text{B}^2(1+k)^2} \left( \text{AB}(1+k)(1+k) \frac{y}{b/2} + 4\text{MK}(1+k) \text{BnK} \right) - 2\text{MK} \right) $</td>
</tr>
</tbody>
</table>

\[ \frac{y}{b/2} = \frac{\text{BnK}}{\text{AB}(1+k)(\text{BnK} - 1)} \]

\[ \frac{y}{b/2} = \frac{k(\text{AB}(1+k)(\text{BnK} - 1) - 4\text{MK})}{\text{AB}(1+k)(\text{BnK} + 1)} \]
TABLE II. - CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF ROLL - Continued

<table>
<thead>
<tr>
<th>Range of ( \frac{y}{b/2} )</th>
<th>Expression for circulation along the span; Mach line from center intersecting trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{p(b/2)^2}{k^2b^2(\beta^2-1)^{3/2}} ) ( y ) ( (1-k)2\beta^2\phi_kx - 1 - \lambda + \frac{8\pi\beta_k(\beta^2+\beta_k - 1)}{AB(1+\lambda)} \frac{y}{b/2} ) ( AB(1+\lambda)(1+k+2\pi\beta_k) \frac{y}{b/2} + 4\pi\beta_k - 2kAB(1+\lambda)(\beta_k+1) ) ( \frac{16\pi\beta_k^2}{AB^2(1+\lambda)^2} \cos^{-1} ) ( \frac{AB(1+\lambda)(1+k-2\pi\beta_k) \frac{y}{b/2} + 4\pi\beta_k - 2k - 4\pi^2\beta_k^2}{AB(1+\lambda)(1-k) \frac{y}{b/2} + 4\pi\beta_k} )</td>
<td></td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{y}{b/2} \leq 1 )</td>
<td>( \frac{2\sqrt{3}}{3} \frac{y}{b/2} \left( 3k + 1 + 4\pi\beta_k - 2k\beta_k - 2\pi^2\beta_k \right) + \frac{8\pi\beta_k(4\pi\beta_k + 1)}{AB(1+\lambda)} + 2k - 4\pi^2\beta_k^2 )</td>
</tr>
<tr>
<td>( \frac{2\pi\beta_k}{\sqrt{(\beta_k+1)\left( \frac{y}{b/2} - \frac{h}{b/2} \right) + \frac{8\pi\beta_k(4\pi\beta_k + 1)}{AB(1+\lambda)} + k + 2\pi\beta_k + 1} + \frac{8\pi\beta_k}{AB(1+\lambda) - k - 2\pi\beta_k} )</td>
<td></td>
</tr>
</tbody>
</table>

![Diagram](image)

\[ h = \frac{2\sqrt{AB(1+\lambda) - 2}}{AB(1+\lambda)} \]

<table>
<thead>
<tr>
<th>Range of ( \frac{y}{b/2} )</th>
<th>Expression for circulation along the span; Mach line coincident with leading edge; unswept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{y}{b/2} \leq \frac{h}{b/2} )</td>
<td>( \frac{8\pi\beta_k(b/2)^2}{3\pi\beta(1+\lambda)} \left( \frac{y}{b/2} \right) \sqrt{4\pi^2\beta_k(1+\lambda)^2 + \frac{x^2}{(b/2)^2} + 16} )</td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{y}{b/2} \leq 1 )</td>
<td>( 2\sqrt{1 - \frac{y}{b/2}} \left( \frac{y}{b/2} - 2 + \frac{4}{AB(1+\lambda)} \right) \left( \frac{y}{b/2} + \frac{h}{b/2} \right) \sqrt{1 - \frac{y}{b/2}} \frac{y}{b/2} + \frac{h}{b/2} \left( \frac{y}{b/2} - 1 \right) )</td>
</tr>
</tbody>
</table>

NACA TN 2643
TABLE II. CIRCULATION EXPRESSIONS FOR CONSTANT RAKE OF WING - Continued

Expression for circulation along the span; Mach line intersecting tip; unseparated trailing edge

\[ \frac{\rho \sqrt{b/2}}{\sqrt{1 - \frac{2h^2}{b^2}} - 1} \left( \frac{b}{b/2} \sqrt{\frac{b}{b/2}} + \frac{h}{b/2} \right) \frac{\rho \sqrt{b/2}}{\sqrt{1 - \frac{2h^2}{b^2}} - 1} \left( \frac{b}{b/2} \sqrt{\frac{b}{b/2}} + \frac{h}{b/2} \right) \]

For \( 0 \leq \frac{b}{b/2} < \frac{h}{b/2} \)

\[ \frac{8h^3}{AB(1 + \lambda)} \frac{b}{b/2} - \frac{16h^3}{AB^2(1 + \lambda)} \cos^{-1} \left[ \frac{h}{b/2} - \frac{h AB(1 + \lambda)}{b^2} \right] \frac{\rho \sqrt{b/2}}{\sqrt{1 - \frac{2h^2}{b^2}} - 1} \left( \frac{b}{b/2} \sqrt{\frac{b}{b/2}} + \frac{h}{b/2} \right) \]

For \( \frac{h}{b/2} \leq \frac{b}{b/2} \leq \frac{d}{b/2} \)

\[ \frac{8h^3}{AB(1 + \lambda)} \frac{b}{b/2} - \frac{16h^3}{AB^2(1 + \lambda)} \cos^{-1} \left[ \frac{h}{b/2} - \frac{h AB(1 + \lambda)}{b^2} \right] \frac{\rho \sqrt{b/2}}{\sqrt{1 - \frac{2h^2}{b^2}} - 1} \left( \frac{b}{b/2} \sqrt{\frac{b}{b/2}} + \frac{h}{b/2} \right) \]

For \( \frac{d}{b/2} \leq \frac{b}{b/2} \leq 1 \)

\[ \frac{8h^3}{AB(1 + \lambda)} \frac{b}{b/2} - \frac{16h^3}{AB^2(1 + \lambda)} \cos^{-1} \left[ \frac{h}{b/2} - \frac{h AB(1 + \lambda)}{b^2} \right] \frac{\rho \sqrt{b/2}}{\sqrt{1 - \frac{2h^2}{b^2}} - 1} \left( \frac{b}{b/2} \sqrt{\frac{b}{b/2}} + \frac{h}{b/2} \right) \]

NACA TN 2643
TABLE II. - CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF ROLL - Continued

![Diagram](image-url)

Range of \( \frac{y}{b/2} \) | Expression for circulation along the span; Mach line from center intersecting trailing edge and intersecting Mach line from tip; unsept trailing edge
---|---
\( 0 < \frac{y}{b/2} < \frac{b}{b/2} \) | \[
\frac{2p(b/2)^2}{\kappa(p/2^2 - 1)^{-1/2}} \left[ \frac{b}{b/2} \sqrt{\left( \frac{p}{b/2} - 1 \right) \left( \frac{p}{b/2} + 1 \right)} + \frac{1}{2} \left( \frac{p}{b/2} x + 2 \frac{p}{b/2} x \right) \right]
\]
\( \frac{b}{b/2} \leq \frac{y}{b/2} < \frac{c}{b/2} \) | \[
\frac{8b^3}{\kappa(b/2^2 - 1)^{3/2}} \left[ \frac{b}{b/2} \sqrt{\left( \frac{p}{b/2} - 1 \right) \left( \frac{p}{b/2} + 1 \right)} + \frac{1}{2} \left( \frac{p}{b/2} x + 2 \frac{p}{b/2} x \right) \right]
\]
\( \frac{c}{b/2} \leq \frac{y}{b/2} \leq \frac{0}{b/2} \) | \[
\frac{8b^3}{\kappa(b/2^2 - 1)^{3/2}} \left[ \frac{b}{b/2} \sqrt{\left( \frac{p}{b/2} - 1 \right) \left( \frac{p}{b/2} + 1 \right)} + \frac{1}{2} \left( \frac{p}{b/2} x + 2 \frac{p}{b/2} x \right) \right]
\]

\( - \frac{2p(b/2)^2}{\kappa(p/2^2 - 1)^{-1/2}} \left[ \frac{b}{b/2} \sqrt{\left( \frac{p}{b/2} - 1 \right) \left( \frac{p}{b/2} + 1 \right)} + \frac{1}{2} \left( \frac{p}{b/2} x + 2 \frac{p}{b/2} x \right) \right]
\]

NACA TN 2643
### Table II - Circulation Expressions for Constant Rate of Bell - Concluded

<table>
<thead>
<tr>
<th>Range of $y/w$</th>
<th>Expression for circulation along the span: Each line from center intersecting trailing edge; unsept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq y/w \leq y/w$</td>
<td>$\frac{2\pi b z^2}{\pi (w^2 - 1)^{\frac{3}{2}}} \left( \frac{y^2}{(y/w)^2} - \frac{1}{\sqrt{y/w}} \right) \left[ \frac{y^2}{(y/w)^2} \left(1 - zw^2\right) + \frac{y}{\sqrt{y/w}} \right]$</td>
</tr>
<tr>
<td>$\frac{y/w}{2} \leq y/w \leq \frac{3}{2}$</td>
<td>$\frac{\pi b^3}{2w^2} \left[ \frac{y}{(y/w)^2} - \frac{1}{\sqrt{y/w}} \right] \left[ \frac{y^2}{(y/w)^2} \left(1 - zw^2\right) + \frac{y}{\sqrt{y/w}} \right]$</td>
</tr>
<tr>
<td>$\frac{y/w}{2} \leq y/w \leq 1$</td>
<td>$\frac{\pi b^3}{2w^2} \left[ \frac{y}{(y/w)^2} - \frac{1}{\sqrt{y/w}} \right] \left[ \frac{y^2}{(y/w)^2} \left(1 - zw^2\right) + \frac{y}{\sqrt{y/w}} \right]$</td>
</tr>
</tbody>
</table>

**Diagram**

Range of $y/w$

Expression for circulation along the span: Each line from tip does not intersect remote half-wing

<table>
<thead>
<tr>
<th>Range of $y/w$</th>
<th>Expression for circulation along the span: Each line from tip does not intersect remote half-wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq y/w \leq \frac{1}{2}$</td>
<td>$\frac{2\pi b z^2}{\pi (w^2 - 1)^{\frac{3}{2}}} \left{ \frac{1}{\sqrt{y/w}} \left[ \frac{1}{\sqrt{y/w}} \left(1 - zw^2\right) + \frac{y}{w} \right] \right}$</td>
</tr>
<tr>
<td>$\frac{1}{2} \leq y/w \leq 1$</td>
<td>$\frac{\pi b^3}{2w^2} \left[ \frac{y}{(y/w)^2} - \frac{1}{\sqrt{y/w}} \right] \left[ \frac{y^2}{(y/w)^2} \left(1 - zw^2\right) + \frac{y}{\sqrt{y/w}} \right]$</td>
</tr>
</tbody>
</table>

**Diagram**

Range of $y/w$

Expression for circulation along the span: Each line from tip does not intersect remote half-wing
### TABLE III. CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH

<table>
<thead>
<tr>
<th>Range of $\frac{h}{b/2}$</th>
<th>Expression for circulation along the span; Mach line coincident with leading edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{h}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{8kAB}{3\pi} \left[ \frac{y}{b/2} + \frac{4k}{AB(1 + \lambda)} - \frac{\lambda^2}{(b/2)^2} + \frac{3}{kAB(1 + \lambda)} \frac{1}{b/2} \right] \frac{h}{b/2} = \frac{2k[AB(1 + \lambda) - 2]}{AB(1 + \lambda)(k + 1)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Range of $\frac{h}{b/2}$</th>
<th>Expression for circulation along the span; Mach line from the wing apex intersecting tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{b}{b/2} \leq \frac{h}{b/2} \leq 1$</td>
<td>$\frac{2kAB}{3\pi} \left[ \frac{y}{b/2} + \frac{4k}{AB(1 + \lambda)} - \frac{\lambda^2}{(b/2)^2} + \frac{3}{kAB(1 + \lambda)} \frac{1}{b/2} \right] \frac{h}{b/2} = \frac{k[AB(1 + \lambda)(2m + 1) - 2bm]}{AB(1 + \lambda)(2m + 1)}$</td>
</tr>
</tbody>
</table>

Expressions for circulation along the span; Mach line coincident with leading edge

$$\frac{h}{b/2} = \frac{k[AB(1 + \lambda)(2m + 1) - 2bm]}{AB(1 + \lambda)(2m + 1)}$$

Expressions for circulation along the span; Mach line from the wing apex intersecting tip

$$\frac{d}{b/2} = \frac{2\pi k[AB(1 + \lambda) - 2]}{b/2 \cdot AB(1 + \lambda)(2m + 1)}$$
<table>
<thead>
<tr>
<th>Range of ( \frac{b}{b/2} )</th>
<th>Expression for circulation along the span, Mach line from the wing apex intersecting tip</th>
</tr>
</thead>
</table>
| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{2gb(b/2)^2}{\sqrt{\frac{b}{2} + \frac{b}{2} \cdot \text{AR}(1 + \lambda)}} \left[ \frac{x^2}{(b/2)^2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left[ \frac{x^2}{b/2} \right] + \frac{10gb^2y^2}{\text{AR}(1 + \lambda)^2} \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{1}{2\lambda} \left( \frac{y^2}{(b/2)^2} \right) \left( k + 1 \right) \left[ \frac{y}{b/2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left( k + \frac{y^2}{b^2} - 2 \right) \frac{x}{b/2} + \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{10gb^2y^2b^2(2 - 2k) - 2\lambda}{\text{AR}(1 + \lambda)^2} \left( \cos^{-1} \left[ \frac{x}{b/2} \left( 1 + \frac{y}{b/2} \right) \right] - \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \right) \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{1}{2\lambda} \left( \frac{y^2}{(b/2)^2} \right) \left( k + 1 \right) \left[ \frac{y}{b/2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left[ \frac{x^2}{b/2} \right] + \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{gb(b/2)^2}{\sqrt{\frac{b}{2} + \frac{b}{2} \cdot \text{AR}(1 + \lambda)}} \left[ \frac{x^2}{(b/2)^2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left[ \frac{x^2}{b/2} \right] + \frac{10gb^2y^2}{\text{AR}(1 + \lambda)^2} \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{1}{2\lambda} \left( \frac{y^2}{(b/2)^2} \right) \left( k + 1 \right) \left[ \frac{y}{b/2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left[ \frac{x^2}{b/2} \right] + \] |

| \( \frac{b}{b/2} \leq \frac{b}{b/2} \) | \[
\frac{gb(b/2)^2}{\sqrt{\frac{b}{2} + \frac{b}{2} \cdot \text{AR}(1 + \lambda)}} \left[ \frac{x^2}{(b/2)^2} \left( 1 - \frac{y^2}{b^2} \right) \right] + \frac{\text{Smax}}{\text{AR}(1 + \lambda)} \left[ \frac{x^2}{b/2} \right] + \frac{10gb^2y^2}{\text{AR}(1 + \lambda)^2} \] |
TABLE III.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - CONTINUED

Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip

\[
\frac{b}{\beta} = \frac{\frac{1}{b} \left[ \frac{2}{(b/\alpha)^2} \right]}{AB(1 + \lambda)} \left[ \frac{X}{b/\alpha} + \frac{\text{wasp}}{1 - \frac{B}{(b/\alpha)^2}(1 - \frac{B}{(b/\alpha)^2}) + \frac{\text{wasp}}{AB(1 + \lambda) \times \frac{1}{b/\alpha}} \left[ \frac{2}{(b/\alpha)^2} \left( \frac{B}{(b/\alpha)^2} + \frac{\text{wasp}}{b/\alpha} \right) \right] \right]
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{\text{wasp}}{b/\alpha} = \frac{1}{\text{wasp}} \left( \frac{B}{(b/\alpha)^2} \times \frac{1}{b/\alpha} \right)
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]

\[
\frac{b}{\beta} \leq \frac{b}{\alpha} \leq \frac{b}{\beta}
\]
TABLE III.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - Continued

<table>
<thead>
<tr>
<th>Range of ( \frac{x}{b/2} )</th>
<th>Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{c}{b/2} )</td>
<td>( \frac{2gh}{\kappa_2 B_2 \pi (b/2)^2 - 1} \left( \frac{1}{1 + \kappa_2 B_2 \pi (b/2)^2} \right) \left( \frac{y^2}{(b/2)^2} \left[ (k - 1) \left[ \frac{1}{b/2} + \frac{\kappa_2 B_2 \pi (b/2)^2}{b/2} \right] + \frac{8\kappa B}{AB(1 + \lambda)} \left( k + \frac{\pi B_2 \pi (b/2)^2}{b/2} \right) \right] + \right)</td>
</tr>
<tr>
<td>( \frac{c}{b/2} \leq \frac{x}{b/2} \leq 1 )</td>
<td>( \frac{16\pi^2 B_2 \pi (b/2)^2}{\kappa_2 B_2 \pi (b/2)^2 + 1} \left( \cos^{-1} \left( \frac{2g}{(b/2)^2} \left( (1 + k + \kappa_2 B_2 \pi (b/2)^2) \frac{\kappa_2 B_2 \pi (b/2)^2}{b/2} \right) - \frac{2\kappa B}{AB(1 + \lambda)} \left( k + \frac{\pi B_2 \pi (b/2)^2}{b/2} \right) \right) + \right)</td>
</tr>
<tr>
<td>( \frac{c}{b/2} \leq \frac{x}{b/2} \leq 1 )</td>
<td>( \frac{4\kappa B}{AB(1 + \lambda)} \left( \frac{1}{(b/2)^2} \left[ (1 + k + \kappa_2 B_2 \pi (b/2)^2) \frac{\kappa_2 B_2 \pi (b/2)^2}{b/2} \right] + \frac{8\kappa B}{AB(1 + \lambda)} \left( k + 1 + 2\kappa B \right) + \right)</td>
</tr>
</tbody>
</table>

**Diagram**: Diagram of a wing with a Mach line and a trailing edge.

**Expression for circulation along the span; Mach line from wing apex intersecting trailing edge**

<table>
<thead>
<tr>
<th>Range of ( \frac{x}{b/2} )</th>
<th>Expression for circulation along the span; Mach line from wing apex intersecting trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{c}{b/2} )</td>
<td>( \frac{2gh}{\kappa_2 B_2 \pi (b/2)^2 - 1} \left( \frac{1}{1 + \kappa_2 B_2 \pi (b/2)^2} \right) \left( \frac{y^2}{(b/2)^2} \left[ (k - 1) \left[ \frac{1}{b/2} + \frac{\kappa_2 B_2 \pi (b/2)^2}{b/2} \right] + \frac{8\kappa B}{AB(1 + \lambda)} \left( k + \frac{\pi B_2 \pi (b/2)^2}{b/2} \right) \right] + \right)</td>
</tr>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{c}{b/2} )</td>
<td>( \frac{16\pi^2 B_2 \pi (b/2)^2}{\kappa_2 B_2 \pi (b/2)^2 + 1} \left( \cos^{-1} \left( \frac{2g}{(b/2)^2} \left( (1 + k + \kappa_2 B_2 \pi (b/2)^2) \frac{\kappa_2 B_2 \pi (b/2)^2}{b/2} \right) + \frac{8\kappa B}{AB(1 + \lambda)} \left( k + \frac{\pi B_2 \pi (b/2)^2}{b/2} \right) \right) + \right)</td>
</tr>
</tbody>
</table>

**Diagram**: Diagram of a wing with a Mach line and a trailing edge.

NACA TN 2643
TABLE III - CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - Continued

<table>
<thead>
<tr>
<th>Range of ( \frac{x}{b/2} )</th>
<th>Expression for circulation along the span: Mach line from wing apex intersecting trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{h}{b/2} )</td>
<td>[ 2 \bar{g}(b/2)^2 \frac{k^2h^2(b \omega - 1)^2}{k^2h^2(b \omega - 1)^2 + k^2h^2(b \omega - 1)^2 + \frac{\bar{g}h^2}{2}} \left( \frac{x}{b/2} \left( 1 + k + \frac{4k}{k^2h^2(b \omega - 1)^2} \right) + \frac{\bar{g}h^2}{2} \left( 1 + k + \frac{4k}{k^2h^2(b \omega - 1)^2} \right) \right) + \frac{\bar{g}h^2}{2} \left( 1 + k + \frac{4k}{k^2h^2(b \omega - 1)^2} \right) ]</td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{x}{b/2} \leq 1 )</td>
<td>[ \frac{\bar{g}h^2}{2} \left( 1 + k + \frac{4k}{k^2h^2(b \omega - 1)^2} \right) \right) + \frac{\bar{g}h^2}{2} \left( 1 + k + \frac{4k}{k^2h^2(b \omega - 1)^2} \right) ]</td>
</tr>
</tbody>
</table>

Range of \( \frac{X}{b/2} \) | Expression for circulation along the span: Mach line from wing apex coincident with leading edge; unswept trailing edge |
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0 \leq \frac{x}{b/2} \leq \frac{h}{b/2} )</td>
<td>[ \frac{\bar{g}(b/2)^2}{3\pi h^2(b \omega - 1)^2} \left( \frac{x}{b/2} \right)^3 + \frac{1}{3\pi h^2(b \omega - 1)^2} ]</td>
</tr>
<tr>
<td>( \frac{h}{b/2} \leq \frac{x}{b/2} \leq 1 )</td>
<td>[ \frac{2\bar{g}h^2(b/2)^2}{3\pi h^2(b \omega - 1)^2} \left( \frac{x}{b/2} \right)^3 + \frac{1}{3\pi h^2(b \omega - 1)^2} ]</td>
</tr>
</tbody>
</table>

\[ \frac{b}{b/2} = \frac{2\left[ h(1 + \lambda) - 1 \right]}{h(1 + \lambda)} \]
### TABLE III. - CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - Continued

Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip; unswept trailing edge

<table>
<thead>
<tr>
<th>Range of ( \frac{y}{b/2} )</th>
<th>Expression</th>
</tr>
</thead>
</table>
| \( \frac{b}{b/2} \leq \frac{y}{b/2} \leq \frac{d}{b/2} \) | \[
\frac{2gB(y/2)^2}{\sqrt{\Delta B^2 + \frac{16}{\Delta B^2(1 + \lambda)^2}} + \frac{1}{\Delta B^2}} \left( \frac{\tan b}{(b/2)^2} + \frac{\lambda}{\Delta B^2(1 + \lambda)^2} \right) - \frac{1}{\Delta B^2} \left( \frac{\tan b}{(b/2)^2} + \frac{\lambda}{\Delta B^2(1 + \lambda)^2} \right)
\]

| \( \frac{d}{b/2} \leq \frac{y}{b/2} \leq \frac{1}{b/2} \) | \[
\frac{2gB(y/2)^2}{\sqrt{\Delta B^2 + \frac{16}{\Delta B^2(1 + \lambda)^2}} + \frac{1}{\Delta B^2}} \left( \frac{\tan b}{(b/2)^2} + \frac{\lambda}{\Delta B^2(1 + \lambda)^2} \right) - \frac{1}{\Delta B^2} \left( \frac{\tan b}{(b/2)^2} + \frac{\lambda}{\Delta B^2(1 + \lambda)^2} \right)
\]

The above equations represent the circulation expressions for a constant rate of pitch, considering the conditions where the Mach line intersects the trailing edge and the tip of the wing, with an unswept trailing edge.
TABLE III.- CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - Continued

Expression for circulation along the span; Mach line from wing apex intersecting trailing edge and intersecting Mach line from tip; unswept trailing edge

\[
\frac{2\lambda b^{3}(v/p)^{2}}{\pi (b/\delta)^{2}} \left[ \frac{4mB}{\pi B(1 + \lambda)} \left( 1 - \frac{y^{2}}{b^{2}} \right) + \frac{16mB(\pi B^{2} - D_{c})}{\pi B^{2}(1 + \lambda)} \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) \right] + \frac{\ln \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right)}{\pi B^{2}(1 + \lambda)} \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) + \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right) \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) \right] + \frac{\ln \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right)}{\pi B^{2}(1 + \lambda)} \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) + \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right) \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) \right] + \frac{\ln \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right)}{\pi B^{2}(1 + \lambda)} \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) + \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right) \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) \right] + \frac{\ln \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right)}{\pi B^{2}(1 + \lambda)} \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) + \left( \frac{\pi B}{\pi B^{2}} + \frac{4mB}{\pi B^{2}(1 + \lambda)} \right) \cos^{-1} \left( \frac{\pi B(1 + \lambda)}{\pi B^{2}(1 + \lambda)} \right) \right]
### TABLE III. - CIRCULATION EXPRESSIONS FOR CONSTANT RATE OF PITCH - Concluded

<table>
<thead>
<tr>
<th>Range of $\frac{x}{b/2}$</th>
<th>Expression for circulation along the span; each line from center intersecting trailing edge; unswept trailing edge</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{2Lb(x/b)^2}{\sinh(\phi b - \lambda)} \left( \frac{\cosh(\phi b - \lambda)}{\sinh(\phi b - \lambda)} \right) \left( \frac{\cosh(\phi b - \lambda)}{\sinh(\phi b - \lambda)} \right)^{-1} \left( \begin{array}{c} \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \left( \begin{array}{c} 1 \frac{2m \cdot y^2}{b/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \left( \begin{array}{c} 1 \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \right)</td>
</tr>
<tr>
<td>$\frac{b}{b/2} \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
<td>$\frac{2Lb(x/b)^2}{\sinh(\phi b - \lambda)} \left( \frac{\cosh(\phi b - \lambda)}{\sinh(\phi b - \lambda)} \right) \left( \frac{\cosh(\phi b - \lambda)}{\sinh(\phi b - \lambda)} \right)^{-1} \left( \begin{array}{c} \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \left( \begin{array}{c} 1 \frac{2m \cdot y^2}{b/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \left( \begin{array}{c} 1 \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \ \frac{2h}{b} \cdot \frac{y}{y/2} \cdot \frac{y}{b/2} \end{array} \right) \right)</td>
</tr>
</tbody>
</table>

### Range of $\frac{x}{b/2}$

<table>
<thead>
<tr>
<th>Expression for circulation along the span; each line from tip does not intersect remote half-wing</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0 \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
</tr>
<tr>
<td>$\frac{b}{b/2} \leq \frac{x}{b/2} \leq \frac{b}{b/2}$</td>
</tr>
</tbody>
</table>

### Remarks

- $\frac{h}{b/2} = \frac{AM(1 + k) - h}{AM(1 + k) - h(1 - k)}$

- $\frac{b}{b/2} = \frac{AM(1 + k) - h}{AM(1 + k) - h(1 - k)}$

- $\frac{h}{b/2} = \frac{AM(1 + k) - h}{AM(1 + k) - h(1 - k)}$
TABLE IV.- INDEX TO CURVES FOR CIRCULATION ALONG THE SPAN FOR CONSTANT ANGLE OF ATTACK, $\lambda$.

<table>
<thead>
<tr>
<th>$\lambda$ = 0.25</th>
<th>$\lambda$ = 0.50</th>
<th>$\lambda$ = 1.00</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>$\Theta$</td>
<td>$\rho$</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>4</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>2.0</td>
<td>8</td>
</tr>
<tr>
<td>12</td>
<td>3.0</td>
<td>12</td>
</tr>
<tr>
<td>20</td>
<td>5.0</td>
<td>20</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$\Theta$</th>
<th>$\rho$</th>
<th>$\Theta$</th>
<th>$\rho$</th>
<th>$\Theta$</th>
<th>$\rho$</th>
<th>$\Theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5(a)</td>
<td>1.0</td>
<td>5(b)</td>
<td>1.0</td>
<td>5(c)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.1</td>
<td>(p. 35)</td>
<td>1.1</td>
<td>(p. 40)</td>
<td>1.1</td>
<td>(p. 41)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>5.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>4</th>
<th>1.0</th>
<th>4</th>
<th>1.0</th>
<th>4</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5(a)</td>
<td>1.0</td>
<td>5(b)</td>
<td>1.0</td>
<td>5(c)</td>
</tr>
<tr>
<td>1.1</td>
<td>(p. 35)</td>
<td>1.1</td>
<td>(p. 40)</td>
<td>1.1</td>
<td>(p. 41)</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>8</th>
<th>1.0</th>
<th>8</th>
<th>1.0</th>
<th>8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5(a)</td>
<td>1.0</td>
<td>5(b)</td>
<td>1.0</td>
<td>5(c)</td>
</tr>
<tr>
<td>1.1</td>
<td>(p. 35)</td>
<td>1.1</td>
<td>(p. 40)</td>
<td>1.1</td>
<td>(p. 41)</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>12</th>
<th>1.0</th>
<th>12</th>
<th>1.0</th>
<th>12</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5(a)</td>
<td>1.0</td>
<td>5(b)</td>
<td>1.0</td>
<td>5(c)</td>
</tr>
<tr>
<td>1.1</td>
<td>(p. 35)</td>
<td>1.1</td>
<td>(p. 40)</td>
<td>1.1</td>
<td>(p. 41)</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>20</th>
<th>1.0</th>
<th>20</th>
<th>1.0</th>
<th>20</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>5(a)</td>
<td>1.0</td>
<td>5(b)</td>
<td>1.0</td>
<td>5(c)</td>
</tr>
<tr>
<td>1.1</td>
<td>(p. 42)</td>
<td>1.1</td>
<td>(p. 43)</td>
<td>1.1</td>
<td>(p. 44)</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.0</td>
<td>5.0</td>
<td>2.0</td>
<td>5.0</td>
</tr>
<tr>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
<td>7.0</td>
<td>25.0</td>
</tr>
</tbody>
</table>

<p>| 25.0 | 25.0 | 25.0 | 25.0 | 25.0 | 25.0 |</p>
<table>
<thead>
<tr>
<th>AB</th>
<th>Bm</th>
<th>Figure</th>
<th>AB</th>
<th>Bm</th>
<th>Figure</th>
<th>AB</th>
<th>Bm</th>
<th>Figure</th>
<th>AB</th>
<th>Bm</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0</td>
<td>(p. 79)</td>
<td>6</td>
<td>1.0</td>
<td>(p. 79)</td>
<td>12</td>
<td>1.0</td>
<td>(p. 87)</td>
<td>18</td>
<td>1.0</td>
<td>(p. 94)</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>10</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 79)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 79)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 94)</td>
</tr>
<tr>
<td>12</td>
<td>3.0</td>
<td></td>
<td>1.2</td>
<td>2.0</td>
<td></td>
<td>1.4</td>
<td>3.0</td>
<td></td>
<td>1.2</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>5.0</td>
<td></td>
<td>1.7</td>
<td>5.0</td>
<td></td>
<td>1.9</td>
<td>5.0</td>
<td></td>
<td>1.7</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11(a)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 71)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 71)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 94)</td>
</tr>
<tr>
<td>8</td>
<td>3.0</td>
<td></td>
<td>1.6</td>
<td>3.0</td>
<td></td>
<td>1.8</td>
<td>7.0</td>
<td></td>
<td>1.6</td>
<td>3.0</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>7.0</td>
<td></td>
<td>1.9</td>
<td>25.0</td>
<td></td>
<td>1.9</td>
<td>25.0</td>
<td></td>
<td>1.9</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>25.0</td>
<td></td>
<td>20</td>
<td>1.0</td>
<td>12(a)</td>
<td>20</td>
<td>1.0</td>
<td>13(e)</td>
<td>20</td>
<td>1.0</td>
<td>15(a)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>11(b)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 72)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 72)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 95)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>2.0</td>
<td></td>
<td>1.3</td>
<td>5.0</td>
<td></td>
<td>1.3</td>
<td>2.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>5.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>12(e)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 85)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 85)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 96)</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>11(c)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 73)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 73)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 96)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.1</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>12(g)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 81)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 81)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 96)</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>11(d)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 74)</td>
<td>1.6</td>
<td>1.6</td>
<td>(p. 74)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 96)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.1</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1.0</td>
<td>13(a)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 89)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 89)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>13(b)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 90)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 90)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>6</td>
<td>1.0</td>
<td>13(c)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 83)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 83)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 98)</td>
</tr>
<tr>
<td>12</td>
<td>1.0</td>
<td>11(e)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 75)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 75)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 98)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.1</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>7.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>25.0</td>
<td></td>
<td>20</td>
<td>1.0</td>
<td>12(f)</td>
<td>20</td>
<td>1.0</td>
<td>13(f)</td>
<td>20</td>
<td>1.0</td>
<td>15(b)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>13(a)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 85)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 85)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>12</td>
<td>1.0</td>
<td>11(f)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 76)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 76)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.1</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>7.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>13(b)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 86)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 86)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>12(b)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 78)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 78)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.3</td>
<td>7.0</td>
<td></td>
<td>1.1</td>
<td>7.0</td>
<td></td>
</tr>
<tr>
<td>2.0</td>
<td>7.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
<td>2.0</td>
<td>25.0</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1.0</td>
<td>13(c)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 86)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 86)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 97)</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>14(e)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 90)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 90)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 99)</td>
</tr>
<tr>
<td>12</td>
<td>1.0</td>
<td>14(g)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 93)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 93)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 99)</td>
</tr>
<tr>
<td>20</td>
<td>1.0</td>
<td>15(g)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 101)</td>
<td>1.3</td>
<td>1.3</td>
<td>(p. 101)</td>
<td>1.1</td>
<td>1.1</td>
<td>(p. 99)</td>
</tr>
<tr>
<td>AB</td>
<td>m</td>
<td>Figure</td>
<td>AB</td>
<td>m</td>
<td>Figure</td>
<td>AB</td>
<td>m</td>
<td>Figure</td>
<td>AB</td>
<td>m</td>
<td>Figure</td>
</tr>
<tr>
<td>----</td>
<td>---</td>
<td>--------</td>
<td>----</td>
<td>---</td>
<td>--------</td>
<td>----</td>
<td>---</td>
<td>--------</td>
<td>----</td>
<td>---</td>
<td>--------</td>
</tr>
<tr>
<td>1</td>
<td>1.0</td>
<td>16</td>
<td>6</td>
<td>1.0</td>
<td>18(c)</td>
<td>12</td>
<td>3.0</td>
<td>19(1)</td>
<td>2</td>
<td>1.0</td>
<td>21(a)</td>
</tr>
<tr>
<td>5</td>
<td>1.85</td>
<td>17</td>
<td>11</td>
<td>3.3</td>
<td>18(b)</td>
<td>20</td>
<td>1.0</td>
<td>19(f)</td>
<td>11</td>
<td>1.0</td>
<td>21(b)</td>
</tr>
<tr>
<td>6</td>
<td>1.5</td>
<td>18(a)</td>
<td>12</td>
<td>3.0</td>
<td>18(c)</td>
<td>13</td>
<td>3.0</td>
<td>19(2)</td>
<td>12</td>
<td>1.0</td>
<td>21(c)</td>
</tr>
<tr>
<td>8</td>
<td>2.0</td>
<td>18(b)</td>
<td>13</td>
<td>3.0</td>
<td>18(d)</td>
<td>14</td>
<td>3.0</td>
<td>19(g)</td>
<td>13</td>
<td>1.0</td>
<td>21(d)</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>18(c)</td>
<td>14</td>
<td>3.0</td>
<td>18(e)</td>
<td>15</td>
<td>3.0</td>
<td>20(1)</td>
<td>14</td>
<td>1.0</td>
<td>21(e)</td>
</tr>
<tr>
<td>20</td>
<td>5.0</td>
<td>18(d)</td>
<td>15</td>
<td>3.0</td>
<td>18(f)</td>
<td>16</td>
<td>3.0</td>
<td>20(2)</td>
<td>15</td>
<td>2.0</td>
<td>21(f)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>17(e)</td>
<td>16</td>
<td>3.0</td>
<td>18(g)</td>
<td>17</td>
<td>3.0</td>
<td>20(3)</td>
<td>16</td>
<td>2.0</td>
<td>21(g)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td>17(f)</td>
<td>17</td>
<td>2.0</td>
<td>18(h)</td>
<td>18</td>
<td>3.0</td>
<td>20(4)</td>
<td>17</td>
<td>2.0</td>
<td>21(h)</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
<td>17(g)</td>
<td>18</td>
<td>2.0</td>
<td>18(i)</td>
<td>19</td>
<td>3.0</td>
<td>20(5)</td>
<td>18</td>
<td>2.0</td>
<td>21(i)</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>17(h)</td>
<td>19</td>
<td>2.0</td>
<td>18(j)</td>
<td>20</td>
<td>3.0</td>
<td>20(6)</td>
<td>19</td>
<td>2.0</td>
<td>21(j)</td>
</tr>
<tr>
<td>5.0</td>
<td>5.0</td>
<td>17(i)</td>
<td>20</td>
<td>3.0</td>
<td>18(k)</td>
<td>21</td>
<td>3.0</td>
<td>20(7)</td>
<td>20</td>
<td>2.0</td>
<td>21(k)</td>
</tr>
<tr>
<td>7.0</td>
<td>7.0</td>
<td>17(j)</td>
<td>21</td>
<td>3.0</td>
<td>18(l)</td>
<td>22</td>
<td>3.0</td>
<td>20(8)</td>
<td>21</td>
<td>2.0</td>
<td>21(l)</td>
</tr>
<tr>
<td>25.0</td>
<td>25.0</td>
<td>17(k)</td>
<td>22</td>
<td>3.0</td>
<td>18(m)</td>
<td>23</td>
<td>3.0</td>
<td>20(9)</td>
<td>22</td>
<td>2.0</td>
<td>21(m)</td>
</tr>
<tr>
<td>3</td>
<td>1.0</td>
<td>18(a)</td>
<td>23</td>
<td>3.0</td>
<td>18(n)</td>
<td>24</td>
<td>3.0</td>
<td>20(10)</td>
<td>23</td>
<td>2.0</td>
<td>21(n)</td>
</tr>
<tr>
<td>1.1</td>
<td>1.1</td>
<td>18(b)</td>
<td>24</td>
<td>3.0</td>
<td>18(o)</td>
<td>25</td>
<td>3.0</td>
<td>20(11)</td>
<td>24</td>
<td>2.0</td>
<td>21(o)</td>
</tr>
<tr>
<td>1.3</td>
<td>1.3</td>
<td>18(c)</td>
<td>25</td>
<td>3.0</td>
<td>18(p)</td>
<td>26</td>
<td>3.0</td>
<td>20(12)</td>
<td>25</td>
<td>2.0</td>
<td>21(p)</td>
</tr>
<tr>
<td>2.0</td>
<td>2.0</td>
<td>18(d)</td>
<td>26</td>
<td>3.0</td>
<td>18(q)</td>
<td>27</td>
<td>3.0</td>
<td>20(13)</td>
<td>26</td>
<td>2.0</td>
<td>21(q)</td>
</tr>
<tr>
<td>7.0</td>
<td>7.0</td>
<td>18(e)</td>
<td>27</td>
<td>3.0</td>
<td>18(r)</td>
<td>28</td>
<td>3.0</td>
<td>20(14)</td>
<td>27</td>
<td>2.0</td>
<td>21(r)</td>
</tr>
<tr>
<td>25.0</td>
<td>25.0</td>
<td>18(f)</td>
<td>28</td>
<td>3.0</td>
<td>18(s)</td>
<td>29</td>
<td>3.0</td>
<td>20(15)</td>
<td>28</td>
<td>2.0</td>
<td>21(s)</td>
</tr>
</tbody>
</table>

TABLE VI.- INDEX TO CURVES FOR CIRCULATION ALONG THE SPAN FOR CONSTANT RATE OF PITCH, \( \frac{r}{b} \)
(a) Positive $\Lambda$ and $\Lambda_{TE}$.  

(b) Positive $\Lambda$ and negative $\Lambda_{TE}$.

Figure 1.- Types of wing configurations treated. Supersonic leading and trailing edges; streamwise tips. Note that the Mach lines from the leading edge of the center section may intersect either tip or trailing edge and also that the Mach line from either tip does not intersect the remote half-wing.
Figure 2. - The area of integration associated with a point \((x, y)\) on the wing trailing edge.

Figure 3. - The limits of integration for equations (9) and (10).
Figure 4. - The distribution of circulation along the span for triangular wings at a constant angle of attack.
Figure 5. - The distribution of circulation along the span for wings at a constant angle of attack with $\lambda = 0$. 

(a) $AB = 3$. 
(b) $AB = 4$. $\lambda = 0.$
(f) \( AB = 20 \).

Figure 5.– Concluded. \( \lambda = 0 \).
Figure 6. The distribution of circulation along the span for wings at a constant angle of attack with \( \alpha = 0.25 \).
Figure 6. Continued. $\lambda = 0.25$. 

(b) $AB = 4$. 

$\frac{y}{b/2}$
(e) $AB = 12$.

Figure 6.—Continued. $\lambda = 0.25$. 
(f) AB = 20.

Figure 6. - Concluded. $\lambda = 0.25$. 
Figure 7.- The distribution of circulation along the span for wings at a constant angle of attack with \( \lambda = 0.50 \).
Figure 7. - Continued. $\lambda = 0.50$.
Figure 7. - Continued. \( \lambda = 0.50 \).

(e) \( AB = 12 \).
Figure 8. - The distribution of circulation along the span for wings at a constant angle of attack with $\lambda = 0.75$. 

(a) $AB = 3$. 
Figure 8.—Continued. $\lambda = 0.75$. 

$\frac{1}{\sqrt{V_{ac}}}$ 

$Bm = \frac{1}{10}$, $\frac{1}{13}$, $\frac{1}{20}$, $\frac{1}{23}$, $\frac{1}{25}$.
(c) \( AB = 6 \).

Figure 8.- Continued. \( \lambda = 0.75 \).
Figure 8. - Continued. $\lambda = 0.75$.

(a) $AB = 8$. 
Figure 9. The distribution of circulation along the span for wings at a constant angle of attack with $\lambda = 1.00$. 

(a) $AB = 2$. 

$\frac{\Gamma}{\rho_0 b^2}$ 

$\frac{V \cdot \gamma}{\rho_0 b^2}$ 

$B_m = 10, 1.3, 2.0, 3.0, 7.0, \infty$
Figure 9. Continued. \( \lambda = 1.00 \).

\( \frac{Y}{b^2/2} \quad \)
(a) $AB = 6$.

Figure 9.- Continued. $\lambda = 1.00$. 
(f) $AB = 12$.

Figure 9.- Continued. $\lambda = 1.00$. 

Figure 10. - The distribution of circulation along the span for triangular wings with a constant rate of roll.
(a) $AB = 3$.

Figure 11.- The distribution of circulation along the span for wings with a constant rate of roll with $\lambda_r = 0$. 

71
(b) $AB = 4$.

Figure 11.- Continued. $\lambda = 0$. 

\[
\frac{\Gamma}{\rho \left( \frac{b^2}{2} \right)} \quad \frac{y}{b/2}
\]
(c) $AB = 6$, $\lambda = 0$.

Figure 11. Continued.
Figure 11. Continued. \( \lambda = 0 \).
Figure 12. The distribution of circulation along the span for wings with a constant rate of roll with $\lambda = 0.25$. 

(a) $AB = 3$. 

$\frac{\Gamma}{\rho U}$
Figure 12. - Continued. \( \lambda = 0.25 \).
(d) $AB = 8$.

Figure 12.- Continued. $\lambda = 0.25$. 
(e) $AB = 12$.

Figure 12.- Continued. $\lambda = 0.25$. 
(f) $AB = 20$.

Figure 12. Concluded. $\lambda = 0.25$. 
(a) $AB = 3$.

Figure 13.- The distribution of circulation along the span for wings with a constant rate of roll with $\lambda = 0.50$. 

Figure 13. - Continued. \( \lambda = 0.50 \).

(b) \( AB = \frac{4}{b^2} \)
Figure 13. - Continued. \( \lambda = 0.50 \).

(c) \( AB = 6 \).
Figure 13. - Continued. \( \lambda = 0.50 \).

(a) \( AB = 8 \).
(e) $AB = 12.$

Figure 13.- Continued. $\lambda = 0.50.$
(a) $AB = 3$.

Figure 14. The distribution of circulation along the span for wings with a constant rate of roll with $\lambda = 0.75$. 
Figure 14.- Continued. \( \lambda = 0.75 \).

(b) \( AB = 4 \).
(d) $AB = 8$.

Figure 14. - Continued. $\lambda = 0.75$. 
Figure 24.- Continued. \( \lambda = 0.75 \).
(r) \( AB \leq 20 \).

Figure 14.- Concluded. \( \lambda = 0.75 \).
Figure 15.- The distribution of circulation along the span for wings with a constant rate of roll with $\Lambda = 1.00$.

(a) $AB = 2$. 

$\frac{\delta}{b}$
Figure 15. Continued, $\lambda = 1.00$.

(d) $AB = 6$. $Bm$ = 1.1, 1.3, 1.5, 1.7, 1.9, 2.0, 2.5, 3.0.
(g) AB = 20.

Figure 15. Concluded. \( \lambda = 1.00 \).
Figure 16.- The distribution of circulation along the span for triangular wings with a constant rate of pitch.
Figure 11. - The distribution of circulation along the span for wings with a constant rate of pitch with $\lambda = 0$. 

(a) $AB = 3$.
Figure 17. – Continued, $\lambda = 0$.

(b) $AB = 4$. 

$\frac{Y}{q_{1/2}}$
(c) AB = 6.

Figure 17.- Continued. \( \lambda = 0 \).
(f) \( AB = 20 \).

Figure 17.- Concluded. \( \lambda = 0 \).
(a) AB = 3.

Figure 18. - The distribution of circulation along the span for wings with a constant rate of pitch with $\lambda = 0.25$. 
(c) $AB = 6$.

Figure 18.- Continued. $\lambda = 0.25$. 

III
Figure 18. - Continued. \( \lambda = 0.25 \)

(d) \( AB = 8 \).
Figure 18. Continued. $\lambda = 0.25$.

(e) AB = 12.
(f) \( AB = 20 \).

Figure 18. - Concluded. \( \lambda = 0.25 \).
Figure 19.- The distribution of circulation along the span for wings with a constant rate of pitch with $\lambda = 0.50$. 

(a) $AB = 3$. 

$\frac{\Gamma}{Bq_i b^{1/2}}$ 

$\frac{y}{b/2}$
(b) $AB = h$.

Figure 19. Continued. $\lambda = 0.50$. 
(e) $AB = 12$.

Figure 19. - Continued. $\lambda = 0.50$. 

\[ \frac{\Gamma}{B q b_2} \]

\[ \frac{y}{b/2} \]
Figure 20. - The distribution of circulation along the span for wings with a constant rate of pitch with $\lambda = 0.75$. 

(a) $AB = 3$. 
(c) $AB = 6$.

Figure 20. - Continued. $\lambda = 0.75$. 
(d) $AB = 8$.

Figure 20. - Continued. $\lambda = 0.75$. 
(e) $AB = 12$.

Figure 20. - Continued. $\lambda = 0.75$. 
(f) $AB = 20$.

Figure 20.- Concluded. $\lambda = 0.75$. 
Figure 21. - The distribution of circulation along the span for wings with a constant rate of pitch with $\lambda = 1.00$. 

(a) $AB = 2$. 
(d) \(AB = 6\).

Figure 21.- Continued. \(\lambda = 1.00\).
(g) $AB = 20$. 

Figure 21. Concluded. $\lambda = 1.00$. 
(a) Variation with Mach number. \( M = 1.53; \Lambda = 30^\circ; \lambda = 0.5 \).

(b) Variation with aspect ratio. \( M = 1.53; \Lambda = 30^\circ; \lambda = 0.5 \).

Figure 22.- Some illustrative variations of the distribution of the circulation along the span with Mach number, aspect ratio, sweepback, and taper ratio for wings at an angle of attack.
(c) Variation with sweepback. $A = \frac{4}{3}; M = 1.8; \lambda = 0.75$.

(d) Variation with taper ratio. $A = \frac{4}{3}; M = 1.53; \Lambda = 30^\circ$.

Figure 22.- Concluded.
(a) Variation with Mach number. $A = 4; \Lambda = 30^\circ; \lambda = 0.5$.

(b) Variation with aspect ratio. $M = 1.53; \Lambda = 30^\circ; \lambda = 0.5$.

Figure 23.- Some illustrative variations of the distribution of the circulation along the span with Mach number, aspect ratio, sweep-back, and taper ratio for wings with a constant rate of roll.
(c) Variation with sweepback. \( A = 4; M = 1.8; \lambda = 0.75 \).

(d) Variation with taper ratio. \( A = 4; M = 1.53; \Lambda = 30^\circ \).

Figure 23.- Concluded.
(a) Variation with Mach number. $A = 4; \Lambda = 30^\circ; \lambda = 0.5$.

(b) Variation with aspect ratio. $M = 1.53; \Lambda = 30^\circ; \lambda = 0.5$.

Figure 24. Some illustrative variations of the distribution of the circulation along the span with Mach number, aspect ratio, sweepback, and taper ratio for wings with a constant rate of pitch. Static margin of 0.05\%. 
(c) Variation with sweepback. $A = 4; M = 1.8; \lambda = 0.75$.

(d) Variation with taper ratio. $A = 4; M = 1.53; \Lambda = 30^\circ$.

Figure 24. - Concluded.
Figure 25.- Some illustrative variations of the distribution of the circulation along span with the position of the axis of pitch for wings with a constant rate of pitch.
(b) \( M = 20; \, A = 4; \, \Lambda = 30^\circ; \, \lambda = 0.5. \)

Figure 25.- Continued.
(c) \( \Lambda = 12^\circ; A = 4; M = 1.8; \lambda = 0.75. \)

Figure 25.- Continued.
(a) \( \lambda = 54^\circ; A = 4; M = 1.8; \lambda = 0.75. \)

Figure 25.– Concluded.
NACA TN 2643
National Advisory Committee for Aeronautics.

On the basis of the linearized supersonic-flow theory, the span load distributions resulting from constant angle of attack, from steady rolling, and from steady pitching were calculated for a series of thin swept-back tapered wings with streamwise tips and with supersonic leading and trailing edges. The results are valid for the Mach number range for which the Mach line from either wing tip does not intersect the remote half-wing. The results of the analysis are presented as a series of design charts. Some illus-

Copies obtainable from NACA, Washington (over)

NACA TN 2643
National Advisory Committee for Aeronautics.

On the basis of the linearized supersonic-flow theory, the span load distributions resulting from constant angle of attack, from steady rolling, and from steady pitching were calculated for a series of thin swept-back tapered wings with streamwise tips and with supersonic leading and trailing edges. The results are valid for the Mach number range for which the Mach line from either wing tip does not intersect the remote half-wing. The results of the analysis are presented as a series of design charts. Some illus-

Copies obtainable from NACA, Washington (over)

1. Flow, Supersonic (1.1.2.3)
2. Wings, Complete - Theory (1.2.2.1)
3. Wings, Complete - Aspect Ratio (1.2.2.2)
4. Wings, Complete - Sweep (1.2.2.3)
5. Wings, Complete - Taper and Twist (1.2.2.4)
6. Mach Number Effects - Complete Wing (1.2.2.6)
7. Loads, Steady - Wings (4.1.1.1.1)
I. Martin, John C.
II. Jeffreys, Isabella
III. NACA TN 2643

NACA TN 2643
National Advisory Committee for Aeronautics.

On the basis of the linearized supersonic-flow theory, the span load distributions resulting from constant angle of attack, from steady rolling, and from steady pitching were calculated for a series of thin swept-back tapered wings with streamwise tips and with supersonic leading and trailing edges. The results are valid for the Mach number range for which the Mach line from either wing tip does not intersect the remote half-wing. The results of the analysis are presented as a series of design charts. Some illus-

Copies obtainable from NACA, Washington (over)

1. Flow, Supersonic (1.1.2.3)
2. Wings, Complete - Theory (1.2.2.1)
3. Wings, Complete - Aspect Ratio (1.2.2.2)
4. Wings, Complete - Sweep (1.2.2.3)
5. Wings, Complete - Taper and Twist (1.2.2.4)
6. Mach Number Effects - Complete Wing (1.2.2.6)
7. Loads, Steady - Wings (4.1.1.1.1)
I. Martin, John C.
II. Jeffreys, Isabella
III. NACA TN 2643

NACA TN 2643
National Advisory Committee for Aeronautics.

On the basis of the linearized supersonic-flow theory, the span load distributions resulting from constant angle of attack, from steady rolling, and from steady pitching were calculated for a series of thin swept-back tapered wings with streamwise tips and with supersonic leading and trailing edges. The results are valid for the Mach number range for which the Mach line from either wing tip does not intersect the remote half-wing. The results of the analysis are presented as a series of design charts. Some illus-

Copies obtainable from NACA, Washington (over)
NACA TN 2643

Captive variations of the spanwise distributions of circulation with the various design parameters are also presented.

Copies obtainable from NACA, Washington

NACA TN 2643

Captive variations of the spanwise distributions of circulation with the various design parameters are also presented.

Copies obtainable from NACA, Washington