**ABSTRACT** The main objective of this research has been to advance the state-of-the-art in smart control design methodology for the purpose of regulating or otherwise controlling very poorly modeled nonlinear processes. The objective has been approached by examining and modifying a variety hybrid switching control strategies. An important connection is made between the heuristic idea of certainty equivalence and the detectability of an adaptive control system. This observation together with the development of two new switching strategies has yielded provably correct adaptive control techniques applicable to a wide range of problems of practical importance. Among these are methods for automatically calibrating vision-based control systems, for controlling highly uncertain nonholonomic systems, for robustly suppressing from the output of a control system the effect of a disturbance signal consisting of a finite number of sinusoids of unknown frequencies, and for controlling induction motors with uncertain resistances such as those used on TGV trains.
1 Introduction

This is the final technical report on research carried out with the support of AFOSR under Grant F49620-97-1-0108. The report consists of the following sections:

1. Introduction
2. Work Accomplished
3. Publications
4. Individuals Involved in the Research Effort

2 Work Accomplished

With AFOSR support we have prepared and/or published during the 31 month grant period April 1, 1997 – December 31, 1999, one post workshop volume [1], seven full-length technical papers [2, 3, 4, 5, 6, 7, 8], one full-length magazine review article [9], two book chapters [10, 11], one doctoral thesis [12], and twenty conference papers [13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32]. Two additional full-length papers will be published later in the year [33, 34] and three full-length technical reports [35, 36, 37] are near completion.

Our research has been concerned with the development of provably correct methods for controlling highly uncertain linear and nonlinear dynamical systems. This has led us to study various issues concerned with hybrid systems in general and switched control systems in particular. We have tailored some of our findings to problems which arise in the context of vision-based control and we have made new contributions to this largely unexplored area.

With the aim of transitioning our findings, we have organized and run short courses on logic-based switching control at the 1997 IEEE/CDC and also in Portugal during the summer of 1998. In 1997 we organized and ran a workshop on Vision and Control on Block Island, Rhode Island. All of these meetings were well attended by control scientists and engineers from both the academic and industrial sectors.

The major thrust of our research has been directed toward the study of switched control systems and the extension of the idea of supervisory control to meaningful classes uncertain nonlinear process models. We have made significant progress along both of these lines. To transition ideas, we have written and published in the widely read IEEE Control Systems Magazine, a survey covering many issues and advances in the area of switched control systems [9]. Included here is our recent work in which we characterize the stability of a switched linear system \( \dot{x} = A_x x \) in terms of properties of the Lie algebra generated by the \( A_i \) [5, 28]. Also included in the survey are our recent results which establish the stability of
such systems provided, on the average, the time between switches is sufficiently large and each of the $A_i$ is a stability matrix [29].

One of the main goals of our work has been to develop switching-based adaptive control algorithms which can deal with - at the very least - the same kinds in modeling uncertainties that currently popular nonlinear adaptive control can deal with. We have recently attained this goal [31] and will report on it at the forthcoming CDC in December, 2000. Central to our success has been the devising of the concepts of "scale-independent hysteresis switching" [22] and more recently "hierarchical hysteresis switching" [31]. Scale-independence is a property of certain switching algorithms used in an adaptive context which is key to proving an algorithm's correctness when operating in the face of noise and disturbances inputs [10]. The concept of dwell-time switching, exploited in [10] and elsewhere, has the advantage of being scale-independent. Unfortunately the existence of a prescribed dwell-time makes it impossible to rule out the possibility of finite escape in applications of dwell-time switching to the adaptive control nonlinear systems [3]. On the other hand the popular idea of hysteresis switching does not have this shortcoming. Unfortunately hysteresis switching is not a scale-independent algorithm. Moreover, it is still not known how to analyze system's employing hysteresis switching except in the highly unrealistic {noise-free, exact matching} situation when switching necessarily stops, and the class of admissible process models is finite. We have made quite significant progress in dealing with these matters. In particular, we've devised a new form of chatter-free switching called scale-independent hysteresis switching, which does not employ a prescribed dwell-time and which is scale independent – and we have developed key technical results [12] which will enable us to analyze nonlinear supervisory control system's employing this form of switching, under mild assumptions which do not preclude switching going on indefinitely. We demonstrate the utility of this form of switching {for the noise-free, exact matching case} in [3], [19], [17], and [23].

In the area of nonlinear control we have collaborated with E. Sontag in developing constructive procedures for realizing integral-input-to-state stabilizing controllers [26]. We have successfully applied the ideas of supervisory control to the stabilization of uncertain nonholonomic systems [23] and we have developed underlying concepts needed to achieve integral-input-to-state stability in an adaptive context [24, 36].

We have initiated a project with B. D. O. Anderson aimed at systematically constructing a finite family of candidate \{non-adaptive\} controllers for controlling a very poorly modeled dynamical process [34]. We have explored ways to use such a family in an adaptive context where additional constraints imposed by switching must be taken into account [37].

Below we briefly outline some of the specific results we've obtained over the 31 month grand period.

\textit{A Bound for the Disturbance-to-Output Gain of a Supervised Set-Point Control System:} A long-term aim of our research is to develop a bona fide, coordinate-independent, performance-based theory of adaptive control. Toward this end we have developed an especially simple
analysis of the dynamical behavior of a set-point control system consisting of a poorly modeled process, an integrator and a multi-controller supervised by an estimator-based algorithm employing dwell-time switching [10]. For a slowly switched multi-controller implementation of a finite family of linear controllers, explicit upper bounds are derived for the normed-value of the process's allowable unmodelled dynamics as well as for the system's disturbance-to-tracking error gain. In supporting research [12], it is shown that the induced $\mathcal{L}^2$ gain of a time varying linear system obtained by switching arbitrarily, but slowly enough, between the members of a family of time-invariant, stable, linear systems, is tightly bounded above by the largest among the $H^\infty$ gains of the constituent time-invariant linear systems.

Nonlinear Systems: The aim of this work is to extend the concept of an estimator-based supervisor control to various classes of nonlinear systems. In [3] we proved that any input-to-state stabilizing certainty equivalence control causes the familiar interconnection of a controlled process and associated estimator to be detectable through the estimator's output error $e_p$, for every frozen value of the index or parameter vector $p$ upon which both the estimator and controller dynamics depend. Detectability is key because adaptive controller tuning/switching algorithms are invariably designed to make $e_p$ small – and so with detectability, smallness of $e_p$ ensures smallness of the state of the controlled process and estimator interconnection. In [24] we generalize these findings by showing that detectability is also achieved with integral input-to-state stabilizing certainty equivalence controls.

A decade ago we proved in a linear context, that any input-to-output stabilizing certainty equivalence control causes the familiar interconnection of a minimum phase controlled process and associated output estimator to be detectable through the estimator's output error $e_p$, for every frozen value of the index or parameter vector $p$ upon which both the estimator and controller dynamics depend. This particular fact is implicit in classical model reference adaptive control and can be used to almost trivialize the analysis of such adaptive systems. Efforts to extend this result to nonlinear systems have, until recently, been unsuccessful due to the lack of an appropriate notion of minimum phase for nonlinear systems. In collaboration with Eduardo Sontag, we have have recently introduced a general notion of minimum phase [35] which enables us to extend the aforementioned result to a large class of nonlinear systems. The new definition is of interest in its own right, in that it does not rely explicitly on the idea of zero dynamics. We expect the definition to receive wide acceptance as one nonlinear system theory's basic concepts.

In [4] we studied the problem of stabilizing certain classes of nonholonomic systems with switching and logic. Using these ideas together with those in [3] and the recently developed concept of scale independent switching [22] we have devised a provably correct algorithm for controlling a large class of uncertain nonholonomic systems [7].

Reference [26] studies the problem of designing control laws to achieve disturbance attenuation in the integral-input-to-state stability sense. The paper’s main contribution is a new concept of control Lyapunov function whose existence leads to an explicit construction of such an integral-input-to-state stabilizing control law.
The problem of achieving disturbance attenuation in both the input-to-state and integral-input-to-state senses for nonlinear systems using bounded controls is considered in [27]. Two constructions are given, one resulting in a smooth control law and the other in a switching control law.

**Switched Dynamical Systems:** By a switched dynamical system is meant a hybrid dynamical system consisting of a family of continuous-time subsystems and a rule which governs the switching between them. Reference [9] surveys recent developments in three basic problems regarding stability and design of switched systems.

Let $\mathcal{P}$ be a finite set, $\{A_i : i \in \mathcal{P}\}$ be a family of constant $n \times n$ stability matrices, and $\mathcal{S}$ the set of all piecewise-constant signals $\sigma : [0, \infty) \rightarrow \mathcal{P}$. Using Lie’s representation theorem for solvable Lie algebras, we’ve shown [5] that for each $\sigma \in \mathcal{S}$, the switched linear system $\dot{x} = A_\sigma x$ is exponentially stable provided the Lie algebra generated by the $A_i$ is solvable. In fact solvability implies that the $A_i$ share a common Lyapunov function and from this exponentially stability follows at once. More recently, we’ve extended this result: The new result [28] states that one still has exponential stability for each each $\sigma \in \mathcal{S}$ provided the Lie algebra generated by the $A_i$ is the sum of a solvable ideal and a sub algebra with a compact Lie group.

**Quantized Feedback and Switching:** In [8] is addressed the problem of stabilizing a linear system with saturating quantized measurement constraint. A new, provably correct asymptotically stabilizing control strategy is proposed which consists of changing the sensitivity of the quantizer while the system evolves. In [25] a link is established between this stabilization problem and the problem of finding a stabilizing switching sequence for a switched linear system with unstable individual matrices.

**Covering Problem:** An obvious first step, which is often taken for granted in the development of switching control procedures for dealing with uncertain systems is that of designing the family of controllers among whose members switching is to take place. Prompted by this we have studied the following problem: Given a {not necessarily compact} subset $\mathcal{T}$ of admissible process model transfer functions within a suitably defined metric space $\mathcal{M}$, find, if possible, a finite set of controller transfer functions $\mathcal{K}$ with the property that, for each transfer function $t \in \mathcal{T}$ there is at least one controller transfer function $\kappa \in \mathcal{K}$ which in closed-loop with $t$ endows the closed-loop system with at least stability and possibly other prescribed properties. In collaboration with B. D. O. Anderson [34, 37]we have recently developed a procedure for constructing such a family of controllers.

**Supervision of a Family of Disturbance Suppressing Internal-Model Based Feedback Controllers:** This research [19] develops a provably correct algorithm which is capable of adjusting the gains of a linear, internal-model-based loop controller in feedback with a siso process, so as to eliminate from the output of the process the effect of a persistent disturbance input consisting of the sum of a finite number of sinusoids with unknown amplitudes, phases, and frequencies [19]. Each of the controllers being supervised is designed so as to solve the disturbance rejection problem for a specific set of disturbance frequencies.
Decidable Tasks and Projective Invariant Tasks are One and the Same: A long-term goal of this project has been to develop a camera calibration-free framework for visual servoing. Toward this end we've developed conditions which enable one to decide, on the basis of images of point features observed by an imprecisely modeled two-camera stereo vision system, whether or not a prescribed robot positioning task has been accomplished with precision [6]. We've shown that for a stereo vision system with known epipolar geometry, whether or not such a positioning task has been accomplished can be decided with available data, just in case the task function which specifies the task is a projective invariant.

Control of Induction Motors with Uncertain Resistances: We've demonstrated by application to specific physically meaningful problem, the utility of the concept of supervisory control. Field-oriented control is the de-facto industry standard for high-performance control of induction motors. It is known that for current fed machines, appropriately designed field oriented control achieves exponential stability and thus robust stability in the face of variations in rotor resistance. Such variations are unavoidable in some applications. They arise {for example} in induction motor braking on TGV trains. Although changes in resistance do not affect stability, they are known to degrade performance. To address this problem a form of supervisory control specifically tailored to this problem, has been devised and successfully tested in simulation [17]. The ideas involved are based on concepts developed in [3]. Implementation of the overall algorithm on an experimental test bed at CNRS/ Supelec/France is underway.

Control of Shear-Force Microscopes: Near-field scanning microscopy is used for practicing spectroscopy with spatial resolution exceeding the diffraction limit. This is useful in numerous applications like electronics technology, material science, cell biology, etc. Near-field scanning requires driving the microscope's optical fiber probe above the surface on which the sample lies at a distance of about $10^{-8}$ meters. At this scale single DNA molecules constitute significant obstacles and the probe must adjust its vertical position to maintain constant its height relative to the surface. The microscope's probe is attached to a tuning fork mounted on a piezo. The probe/tuning fork/piezo system can be approximately modeled by equations of the form $\dot{y} + f(h)\dot{y} + g(h)y = \cos(\omega t), \ h = v$ where $y$ denotes the horizontal position of the tuning fork, $h$ the distant between the probe and the sample, $v$ the vertical velocity of the tuning fork, and $f$ and $g$ approximately known functions. The only signal that can be measured is the horizontal position of the tuning fork $y$. To obtain high resolution images with near-field scanning microscopy one must keep the distant $h$ between the probe and the sample constant and approximately equal to $10^{-8}$ meters. Since $h$ cannot be directly measured one uses the amplitude of oscillation of $y$ as an indirect measurement of $h$. To do this effectively we have derived an accurate model to describe how changes in $h$ affect the amplitude of oscillation of $y$. This model is obtained by applying a time-varying transformation to the above differential equations, followed by linearization about the equilibrium
point of the resulting time-invariant system. The control system is designed based on the model obtained in this manner. By integrating the vertical velocity of the tuning fork $v$, as the probe scans the sample, one can reconstruct the shape of the surface being scanned and thus obtain additional information about the structure of the sample. Experimental results show that the control system was sensitive to variations in the height of the sample of around $10^{-9}$ meters. This work will be reported in [33].

3 Publications


4 Individuals Involved in the Research Effort

1. A. S. Morse, Principal Investigator
2. W-C Chang, Graduate Student
3. J. P. Hespanha, Graduate Student
4. L. Fang, Graduate Student
5. J. August, Graduate Student
6. Shogo Fujji, Visiting Graduate Student
7. D. Liberzon, Postdoctoral Fellow
8. Mustafa Unel, Postdoctoral Fellow
9. G. D. Hager, Faculty {not supported}
10. Eduardo Sontag {not supported}
11. Brian Anderson {not supported}