OPERATIONAL ANALYSIS OF THE SUSTAINABILITY OF A MOBILE MILITARY PLATFORM

by

James G. Stoneman

September 1998

Thesis Advisor: Donald P. Gaver
Co-Thesis Advisor: Patricia A. Jacobs
Second Reader: Ronald L. Brown

Approved for public release; distribution is unlimited.
OPERATIONAL ANALYSIS OF THE SUSTAINABILITY OF A MOBILE MILITARY PLATFORM

James G. Stoneman

Naval Postgraduate School
Monterey, CA 93943-5000

The views expressed in this thesis are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government.

This thesis illustrates the use of simulation techniques to evaluate the satisfaction of suitability requirements for a mobile platform carrying payload (for example, an Unmanned Aerial Vehicle with sensors) on a military mission (surveillance or reconnaissance). The Institute for Defense Analyses, in support of Director, Operational Test & Evaluation (DOT&E), recently developed a simulation to assist in the analysis of the PREDATOR Unmanned Aerial Vehicle. That simulation has been extended to make it more applicable to a variety of platforms, and the extended simulation has been incorporated into the Military Aircraft Sustainability Simulation (MASS). The primary output from the simulation is Effective Time On Station (ETOS). ETOS is the long-run percentage of time that the region under surveillance is being covered by at least one operating platform. An analytical model for a single platform also has been developed to augment and assist in verifying the MASS. This thesis shows that MASS can be an invaluable tool for evaluating a platform’s suitability for a mission. The simulation can assist during the acquisition process, when the government must decide whether to buy a platform, and the simulation can assist in determining the most effective way to deploy such platforms once they are in use.
Approved for public release; distribution is unlimited.

OPERATIONAL ANALYSIS OF THE SUSTAINABILITY OF A MOBILE MILITARY PLATFORM

James G. Stoneman
Lieutenant, United States Navy
B.S., United States Naval Academy, 1990

Submitted in partial fulfillment
of the requirements for the degree of

MASTER OF SCIENCE IN OPERATIONS RESEARCH

from the

NAVAL POSTGRADUATE SCHOOL
September 1998

Author: 

James G. Stoneman

Approved by:

Donald P. Gaver, Thesis Advisor

Patricia A. Jacobs, Co-Thesis Advisor

Ronald L. Brown, Second Reader

Richard E. Rosenthal, Chairman
Department of Operations Research
ABSTRACT

This thesis illustrates the use of simulation techniques to evaluate the satisfaction of suitability requirements for a mobile platform carrying payload (for example, an Unmanned Aerial Vehicle with sensors) on a military mission (surveillance or reconnaissance). The Institute for Defense Analyses, in support of Director, Operational Test & Evaluation (DOT&E), recently developed a simulation to assist in the analysis of the PREDATOR Unmanned Aerial Vehicle. That simulation has been extended to make it more applicable to a variety of platforms, and the extended simulation has been incorporated into the Military Aircraft Sustainability Simulation (MASS). The primary output from the simulation is an estimate of Effective Time On Station (ETOS), as that depends on platform subsystem reliability and the maintenance resources allocated. ETOS is the long-run percentage of time that the region under surveillance is being covered by at least one operating platform. An analytical model for a single platform also has been developed to augment and assist in verifying the MASS. This thesis shows that MASS can be an invaluable tool for evaluating a platform's suitability for a mission. The simulation can assist during the acquisition process, when the government must decide whether to buy a platform, and the simulation can assist in determining the most effective way to deploy such platforms once they are in use.
THESIS DISCLAIMER

The reader is cautioned that computer programs developed in this research may not have been exercised for all cases of interest. While every effort has been made, within the time available, to ensure that the programs are free of computational and logic errors, they cannot be considered validated. Any application of these programs without additional verification is at the risk of the user.
# TABLE OF CONTENTS

I. INTRODUCTION .................................................................................................................. 1
   A. GENERAL ..................................................................................................................... 1
   B. PROBLEM DESCRIPTION ............................................................................................ 2
   C. SCOPE OF THESIS .................................................................................................... 2
   D. THESIS STRUCTURE .................................................................................................. 3
II. BACKGROUND ................................................................................................................ 5
III. MILITARY AIRCRAFT SUSTAINABILITY SIMULATION (MASS) MODEL ....................... 9
   A. GENERAL ..................................................................................................................... 9
   B. MASS MODEL INPUTS ................................................................................................. 9
   C. MASS MODEL FLOW ................................................................................................ 14
   D. MASS MODEL ASSUMPTIONS ................................................................................. 17
   E. MASS MODEL OUTPUT ............................................................................................ 18
IV. ANALYTIC MODEL .......................................................................................................... 21
V. RESULTS AND ANALYSIS. IMPLICATIONS FOR FIELD TESTING ................................. 25
   A. PRE-POSITIONING OF PLATFORM .......................................................................... 25
   B. TIMES BETWEEN MISSION-AFFECTING FAILURES ................................................. 28
   C. TIME TO REPAIR .................................................................................................... 30
   D. SENSITIVITY TO VARIATION OF TIME-TO-FAILURE DISTRIBUTIONS ............... 34
   E. LAUNCHING WITH NON-MISSION-AFFECTING FAILURES ..................................... 38
   F. REPAIR TIME / DOWN TIME PROCESSES ............................................................ 40
   G. OPERATING IN DEGRADED CONDITIONS: WHEN NOT TO REPAIR ..................... 43
   H. DEPLOYMENT LENGTH ......................................................................................... 47
   I. CONCLUSIONS AND RECOMMENDATIONS ................................................................ 51
LIST OF REFERENCES .......................................................................................................... 53
APPENDIX A: ANALYTIC MODEL CALCULATIONS ............................................................... 55
APPENDIX B: DERIVATION OF TIME-TO-FAILURE DISTRIBUTIONS ............................... 57
INITIAL DISTRIBUTION LIST ........................................................................................... 61
LIST OF FIGURES

FIGURE 1. MASS SEQUENCE OF EVENTS ................................................................. 14
FIGURE 2. ETOS VS. INGRESS TIME ................................................................. 26
FIGURE 3. ETOS VS. MEAN TIME BETWEEN MISSION-AFFECTING FAILURES ........ 29
FIGURE 4. ETOS VS. MEAN TIME TO REPAIR ................................................. 31
FIGURE 5. ETOS VS. MEAN TIME TO REPAIR ................................................. 33
FIGURE 6. HISTOGRAM OF 168 HOUR DEPLOYMENT RESULTS ..................... 50
FIGURE 7. HISTOGRAM OF 672 HOUR DEPLOYMENT RESULTS ..................... 51
LIST OF TABLES

Table 1. Comparison of Analytic Model with MASS ............................................................ 22
Table 2. Description of Positioning Cases ............................................................................ 25
Table 3. ETOS Results from Changes in Positioning ............................................................. 26
Table 4. ETOS Results from Change in Time between Mission-Affecting Failures ............. 29
Table 5. ETOS Results from Change in Time to Repair ....................................................... 31
Table 6. ETOS Results from Change in Time to Repair ....................................................... 33
Table 7. Time-To-Failure Distributions and the Resulting ETOS ......................................... 35
Table 8. Mean Values of the Time-To-Failure Distributions ............................................... 36
Table 9. Sample Failure Times ............................................................................................ 37
Table 10. ETOS Comparison for Change in Maintenance Process ..................................... 39
Table 11. ETOS Comparison for Change in Repair Time Calculations ................................ 42
Table 12. Percentage of Time Platform Spend in Each State .............................................. 45
Table 13. On-Station Percentage of Time Sensor Operates ................................................. 46
Table 14. ETOS Results from Different Deployment Lengths ............................................. 48
Table 15. ETOS Results with Start-Up Costs Removed ....................................................... 49
EXECUTIVE SUMMARY

The suitability of a platform to carry out a mission is a very important factor in determining the success of that mission. Simulations can be used to explore scenarios for testing the suitability of platforms where there is no field test data, and to extrapolate and interpret such data when it becomes available. This thesis illustrates the use of simulation techniques to evaluate the suitability of a mobile platform carrying payload (for example, an Unmanned Aerial Vehicle (UAV) with sensors) for a military surveillance or reconnaissance mission.

Mr. Joseph A. Post and Dr. Catherine W. Warner of The Institute for Defense Analyses, in support of Director, Operational Test & Evaluation (DOT&E), recently developed a simulation to assist in the analysis of the **PREDATOR** Unmanned Aerial Vehicle. That simulation has been extended and generalized to make it applicable to a variety of platforms, and the extended simulation has been incorporated into the **Military Aircraft Sustainability Simulation (MASS)** program. **MASS** is modified and explored extensively in this thesis.

The primary output from the simulation is Effective Time On Station (ETOS). ETOS is the long-run percentage of time that the region under surveillance is being covered by at least one operating platform. Other measures of effectiveness can be computed as well.

An analytical model for a single platform has been developed to augment and assist in verifying the **MASS**. The results from the analytical model confirm that, under certain specific conditions, **MASS** accurately estimates ETOS for a single platform. This provides some assurance that the simulation program is valid.
Many analyses are done using the simulation. The first analysis using MASS confirmed one of the findings from the Predator analysis; it is generally important to base the platform as close to the surveillance region as possible. Such basing minimizes the likelihood of a failure occurring during ingress, and minimizes the time for replacement platform to arrive on-station when needed.

One of the more interesting findings was the simulation’s sensitivity to time-to-failure distributions. The common assumption made in many system reliability models is that the times between failures are exponential. Based on analysis in this thesis, such an assumption can be overly optimistic. This finding suggests the need for actual field tests to better understand the operational suitability of the system.

The human factor in the operation being simulated is discussed at length. There are assumptions made in the simulation that affect the results, but that will not necessarily hold true in actual operations. Maintenance of the platforms is a large factor in these assumptions. Repair time calculations in serial are pessimistic when one considers that more than one repair can be completed at a time. The squadron will also have decisions to make on the level of repair with which a platform is allowed to launch. In some cases it is helpful to launch platforms at less than 100% capability.

Deployment length is also a factor in the suitability analysis. Long deployments are used for analysis to ensure the steady-state ETOS is attained. However, shorter deployments may be more realistic, even though it is demonstrated that the variability of the results is greatly increased for those short deployments. Such variability should be accounted for when planning deployments.
MASS can be an invaluable tool in the evaluation of a platform’s suitability for a mission. The simulation can assist first during the acquisition process, when the government must decide whether to buy a platform, and the simulation can later assist in determining the most effective way to deploy such platforms once they are in use.
I. INTRODUCTION

A. GENERAL

One of the most important questions facing a decision-maker during mission planning is how to maximize the probability of a successful mission. Given that a variety of system designs (e.g. platform and sensor payloads) are possible choices for a mission, the ability to evaluate each system's operational suitability and effectiveness for a particular mission will largely decide which platform is chosen for a mission, and how that platform is configured and used. Computer simulation is often the most cost-effective way to estimate a platform’s suitability. Simulations can be used to explore scenarios for which there is no field test data, and to extrapolate and explore such data when available.

There are two critical times when a platform’s suitability is analyzed. The first occurs during the acquisition process, when the government must decide whether or not to buy the platform and ultimately adopt it in the field. Minimum performance requirements are normally established for a platform based on the anticipated usage, and it is important that these include total mission suitability and effectiveness. It is the job of the government (the Test & Evaluation community) to ensure that the platform it is purchasing satisfies these requirements. The second critical time is when platforms and the support assets are deployed. The unit commander must decide how many platforms to deploy, where they should be based, and what type of support they will need to be effective.

This thesis illustrates the use of simulation techniques to evaluate the satisfaction of suitability requirements for a mobile platform carrying payload (an Unmanned Aerial
Vehicle with sensors) on a military mission (surveillance or reconnaissance). With some modifications the same simulation can be used for evaluating other military systems.

B. PROBLEM DESCRIPTION

There are many factors to consider when analyzing the suitability of a platform. The person evaluating such an asset must be able to look at these factors and determine whether the platform can satisfactorily complete its mission, and also make future contributions.

Among the many questions that arise are the following:

How many platforms are needed for the particular deployment in order to achieve adequate area coverage?

How far away from the target area can the platform be effectively based?

Can the platform be refueled in-flight, perhaps near the surveillance region?

How does the platform’s reliability affect the mission?

What type of maintenance support is needed?

How should the maintenance concept of operations be structured?

What type of logistics support is needed?

The answers to such questions are critical inputs for a decision-maker tasked with assessing the suitability of surveillance platforms for particular deployments. Simulation methodology is useful for providing answers.

C. SCOPE OF THESIS

The purpose of this thesis is to produce a simulation tool to assist in answering such important operational questions by providing information about platform suitability for various missions; suitability is meant in the technical Test & Evaluation sense of
DOD 5000.2. By changing input parameters, trends can be detected which allow those platform characteristics most critical to a platform’s successful mission completion to be identified. Identification of critical platform characteristics in this manner also allows required operations capabilities to be specified for future platforms to ensure that they are capable of successfully performing their intended missions.

Most of this study uses event-step Monte Carlo simulation. Analytic (mathematical) models are used in some cases to assist in validating the simulation program, and sometimes to exercise its options quickly. Plausible alternative distributions for the times to failure and repair are tested for their effect on results, as are alternative maintenance and operational strategies. Changes in the distributions of time to failure can importantly affect overall system performance; these sensitivities can be efficiently discovered by simulation. They suggest the importance of actual field tests to verify predictions.

The simulation of this thesis addresses several important operational issues, but there is definitely room for further investigations; some are on-going. Further considerations are examined, and suggestions made for continuing work on this general important problem.

**D. THESIS STRUCTURE**

This thesis has five chapters. The first chapter is an introduction to the problem and the content of the thesis. The second chapter covers the background of the problem being addressed in order to give the reader a better understanding of the motivation for the thesis work. The third chapter focuses on the simulation itself and explains how the program works so that it can be used by people with different backgrounds. The fourth
chapter examines the analytic models used to supplement the simulation. The fifth chapter summarizes the results and the conclusions that can be drawn from them. It also suggests areas for other applications and for further research.
II. BACKGROUND

The Director, Operational Test & Evaluation (DOT&E) is responsible for "reviewing and analyzing the results of OT&E conducted for each major DoD acquisition program" (See the website: http://www.dote.osd.mil/about.html). These results are used to make recommendations to the Secretary of Defense and Congress regarding acquisition programs and to "confirm operational effectiveness and suitability of the defense system in combat use" (http://www.dote.osd.mil/about.html). The Institute for Defense Analyses (IDA) provides technical assistance to DOT&E in carrying out its mission.

One of the acquisition programs recently studied by simulation was the **PREDATOR** Unmanned Aerial Vehicle (UAV). As part of the assessment of this system, Mr. Joseph A. Post and Dr. Catherine W. Warner (IDA) developed a model written in EXTEND™ to assist with the analysis of the Predator system. The model is a discrete-event simulation designed to help determine the Predator's suitability for a continuous surveillance mission (Post and Warner, 1997). The measure of effectiveness chosen and studied is the long-run percentage of effective time on station (ETOS). The ETOS measure is also used throughout this thesis as the main measure of effectiveness (MOE), and is defined as the (estimated) expected number of hours with at least one operational platform on station divided by the total length of the deployment. Other measures are also possible and instructive.

Post and Warner used their model to perform a sensitivity analysis of various operationally important factors, such as the long-run number of UAVs operationally available, and the squadron's ability to complete maintenance. They showed that the
ETOS is very sensitive to the Predator's range to the target area. They also showed that having to order spare parts instead of keeping them on hand could significantly decrease the ETOS. These results made the model so useful and instructive that DOT&E expressed interest in using similar models on other projects.

Dr. Kenneth D. Pendergast of IDA and LT Stoneman have extended the Post-Warner model to make it more applicable to a variety of platforms, and have incorporated the extended model into the *Military Aircraft Sustainability Simulation (MASS)* program (Pendergast, 1998). This thesis discusses the extended *MASS* model, compares *MASS* results to those from a simple analytical model, and further enhances *MASS* to analyze factors that affect the ability of a flying unit to perform operationally significant surveillance tasks.

**Standard Scenario**

The basic scenario for the model is that a squadron of surveillance platforms, here airframes with a sensor payload, has been assigned to maintain constant presence in a (possibly remote) region. The airframe is susceptible to in-flight failures, which will sometimes necessitate a return to base, and the immediate launch of another platform in response. In some cases, certain malfunctions can be repaired in-flight. The platform can remain airborne for a limited time, although in-flight refueling may be a possible option. The question of interest is: does the squadron have the ability to maintain the required surveillance level, given the distance to the region to be covered, the number of platforms in the squadron, and the available capabilities to provide restoration and repair, namely the sustainment assets and concept of operations? The simulation provides a logical and economical answer to this question, but depends on plausible and convenient
assumptions based on anticipated reality. Field operational tests are recommended to reassure the system acquisition decision makers that simulation results can be trusted: test results may call for model modifications, revealing initial model assumption inaccuracies that require correction. These can then be incorporated into the model for new runs. The interplay of modeling and field testing reveals options and minimizes surprises when the platform system is produced or released for field usage.
III. MILITARY AIRCRAFT SUSTAINABILITY SIMULATION (MASS)
MODEL

A. GENERAL

MASS is a Monte Carlo, object-oriented, discrete event simulation written in C++. For each repetition of the simulation, a fictitious but representative deployment (many flights of the platforms over a time period with intervening maintenance periods) is created. The current implementation of the simulation can accommodate many platforms, but they must all be of the same type (at present). There can be multiple repair stations, but they must all perform the same types of maintenance. Each platform is permitted to have multiple failures during a flight; these can occur in the different subsystems that make up the entire platform system.

B. MASS MODEL INPUTS

The inputs for the simulation and a brief description are as follows:

b-1: Number of Platforms – The number of platforms in the squadron. For this study, all platforms in a squadron are assumed to be identical, and the number remains constant; platform attrition is not currently modeled, but its effect can be included with little difficulty if desired.

b-2: Number of Ground Maintenance Paths – The number of platforms that can have ground maintenance performed on them at the same time. This maintenance capability can be limited by the number of maintenance personnel or available workspace. However, one platform can not occupy more than one path, and no more than one platform can be serviced simultaneously by a path. If there are fewer maintenance paths than platforms, delays to begin service can occur.
b-3: Maximum Number of Platforms Flying – This input denotes the number of platforms actually in the air at one time. The present study emphasizes UAVs, that are limited by their requirement for ground control. However, if a squadron had insufficient pilots or aircrew this input could be used to simulate the effect of that shortage also.

b-4: Ingress/Egress Time (in hours) – The length of flight from home base (hangar) to mission location. Distance must be converted to time in the simulation. Egress time is still assumed equal to ingress time if the platform has a failure and must return home from the refueling orbit. Realistically, these times may differ systematically because of wind, and haphazardly/randomly because of route or altitude changes. The program can be changed to recognize these details.

b-5: Scheduled On-Station Time (in hours) – The maximum length of time the platform is scheduled to remain on station, provided there are no mission-terminating failures. This depends on the platform and aircrew endurance, fuel capacity, and environmental conditions.

b-6: Flight Time (in hours) Before Refuel – The length of platform flying time before there is need to refuel. This is the amount of time until the operation controller wants the Platform to be at the refueling orbit; it may not necessarily depend directly on the Platform’s endurance.

b-7: Transit Time (in hours) to Refueling Orbit – The length of platform flying time from mission orbit to refueling track. The platform is not on-station during this time.
b-8: **Time (in hours) Required to Refuel Platform** – The length of time that the platform is expected to stay at the refueling orbit. This includes the time required to actually find the tanker as well as refuel, and return to coverage mode. The platform is not on-station during this time.

b-9: **Probability That a Failure is In-Flight Repairable** – It is assumed that some failures can be repaired by maintenance personnel while the platform is airborne. The long-run fraction of failures that can be repaired while the platform is airborne is used in the simulation as the probability that the failure is in-flight repairable. Obviously an unmanned platform does not have this capability although automatically switched-in redundant (sub) systems is a design option (the switch itself may be failure-prone). The number is set to 0 for UAVs and any other platforms that do not have this capability.

b-10: **Mean Time (in hours) Between Mission-Affecting Failures** – The expected time between failures, the occurrence of which prevent the platform from performing its mission (in this model such failures do not cause aircraft/platform loss, e.g. crash).

**Note:** A mean may not alone be sufficient to summarize and model random time between system failures, nor may the conventional convenient and simple exponential distribution. Such is used here for illustration. Sensitivity tests of the exponential distribution assumption have been performed in this thesis, and they show that an important sensitivity may be present.

b-11: **Mean Time (in hours) Between Non-Mission-Affecting Failures** – The expected time between failures that do not directly affect the Platform operational status. These failures will, however, extend the amount of repair time needed on the
ground and thus eventually reduce ETOS. They may, for example, be failures of subsystems backed up by redundant copies. The times between failures are assumed to have an exponential distribution. **Note:** See comment b-10.

**b-12: Mean Time (in hours) for Ground Repair** – The expected time for ground maintenance personnel to diagnose and repair each reported failure. **Note:** The entire distribution of repair time, by repair type, is actually required, except when the deployment is very long. This thesis uses the exponential distribution for illustration. Repair times depend upon which subsystems have failed, and on the decision whether to repair them during the present deployment.

**b-13: Mean Time (in hours) for In-Flight Repair** – The expected time for maintenance personnel (if present) to diagnose and repair a failure while in-flight. The repair times are assumed to have an exponential distribution. In some cases, diagnosis and repair in-flight will not be possible before the platform is scheduled to return to base, causing mission termination. The possibility of mis-diagnosis is not considered, but the effect can be simulated and should be the subject of field test. **Note:** See comments b-10, b-12.

**b-14: Flight Time (in hours) Between Scheduled Maintenance Actions** – The flight time duration between mandatory maintenance periods. Initially, a constant decision variable. There is no attempt to relate the occurrence of random failures to the scheduled maintenance actions.

**b-15: Scheduled Maintenance Service Time (in hours)** – The time for ground maintenance personnel to complete a scheduled maintenance action. This value is
represented as a constant, since the required maintenance actions are nearly the same each time.

**b-16: Deployment Duration (in hours)** – The length of the deployment (e.g. part or all of an operation or campaign) that is being simulated. Each platform/aircraft will typically accomplish several/many duty cycles (out, on-station, back, restoration/repair; out, …) during such a deployment.

**b-17: Logistics Delay Time (in hours)** – The time to obtain all parts needed for maintenance actions for one Platform from the maintenance department. This time is added to the repair time for the failure. The delay times are assumed to have an exponential distribution. This input in the present simulation assumes that the parts are present at a forward maintenance level. This may not occur. “Cannibalization” of existing failed systems can occur: this would reduce the number of platforms available; its effect can be studied by simulation but this is not done in this paper.

**b-18: Platform Turn-Around Time (in hours)** – The time required to launch a platform, measured from the moment it returns from its previous cycle. Even if a platform is failure-free there is a minimum required time to refuel and prepare for the next launch. The time is assumed to be a constant.

**b-19: Time (in hours) Between Simulation Results Output** – The time of reporting of model outputs. Often, but not always, this will be the length of the deployment just to record the final results, such as the effective time on station. Such time will vary randomly across individual simulations. The simulated distribution of such outputs, for instance its mean, provide information useful to deployment decision makers.
**b-20: Number of Simulated Deployments (Replications)** - This is the number of model replications to be run under the given conditions. The number depends on the confidence levels, and widths of confidence intervals, required by the analyst for the estimates of the measures of effectiveness (MOEs) obtained by simulation: the more precision required, the more replications are needed.

**C. MASS MODEL FLOW**

The flow chart in Figure 1 is a general description of the sequence of events that each Platform in the model goes through.

- While the platform is in the air, it may have failures that shorten the time until it egresses, but the sequence of steps remain the same
- A platform can refuel in-flight from any stage if scheduled to do so

**Figure 1. MASS Sequence of Events**
The first step in running the simulation is to input the desired parameter values for the appropriate mission and platform. These have been discussed already, but it is important to address assignment of a value for a parameter that is not applicable for a given platform. For example, some platforms do not have an in-flight refueling capability. To keep this from affecting the simulation, a sufficiently large number can be input for the Flight Time Before Refuel so that the Platform must land before it tries to go to the refueling orbit. This method is used for any times between, or times until, event input. There is no default value. However, for the simulation runs in this thesis, 50,000 hours was the standard input for events that were not desired. For inputs that affect the length of an event, such as Transit Time to Refueling Orbit, 0 is used as the input for events not used (if a platform is being refueled on-station).

The simulation begins with an order to launch the first platform. The time of the platform's first failures, mission-affecting and non-mission-affecting, and the scheduled on-station, off-station, refueling, and land times are calculated. The failure can be a mission-affecting or non-mission-affecting failure depending on the user inputs and the independent random number draws to determine the times. There is no first failure if its nominal time exceeds the length of the flight.

The platform immediately goes to the Ingress phase of the flight. From this phase on the platform is susceptible to failures, or need to refuel, at any time according to schedule. If the platform has a mission-affecting failure, it requests that another platform be launched, and it returns to base. If the platform has a non-mission-affecting failure, it continues to its station. Non-mission-affecting failures do not affect the conduct of the platform. They do, however, increase the amount of time needed to repair the platform.
once it returns to base. Also, some types of platforms allow for in-flight repairs. Whenever a failure occurs, the time until the next failure is automatically calculated. The Platform can refuel during ingress. If the return time (from refueling) is after the start of the scheduled on-station time, then the platform assumes On Station status.

The next phase for the platform is On Station. The platform remains there until or if it next experiences a mission-affecting failure, is sent to refuel, or it has reached the original return time, whichever comes first. A replacement, if available, will be launched so as to arrive on station as the first platform is leaving to return to base. A replacement, if available, is also launched when an On Station platform experiences a mission-affecting failure. A replacement platform does not arrive to provide coverage while the first platform is refueling.

Some platforms are sent to the Refueling phase, in which case they must transit to the refueling orbit. Such a platform spends the required amount of time refueling and then transits back to be on-station unless the scheduled on-station time has been reached. Failures can occur during this time. If the platform is required to return to base during the refueling phase because of a failure or flight time endurance limit, the egress time is assumed to be the same as if it were returning directly from on-station position.

When it is time for a platform to return to base, it is said to be in the Egress phase. Platforms are allowed to refuel during egress. Failures that occur during this phase may only affect the maintenance time on deck, since the failure occurred during Egress rather than while on-station. Once again, catastrophic events leading to platform loss are not modeled.
Unless a platform has returned early because of the occurrence of a mission-affecting failure, it enters the Land phase at the scheduled time. If it has any failures, it looks for an open maintenance facility. If maintenance is available, the platform enters the Maintenance phase. If all maintenance stations or paths are busy then the platform must wait. There is currently no prioritization of maintenance; repair is “first-come, first served” at present, although “shortest task first” could be more effective if the shortest task were identifiable. Once the repairs are completed, or if none are needed, the platform enters the Turn phase where it is prepared for its next flight. After it has been turned, the platform is ready for Launch and waits until it is needed.

These basic processes are repeated for all platforms for the length of the deployment.

D. MASS MODEL ASSUMPTIONS

Certain assumptions were made for the original version of MASS to simplify the model. Some of the key assumptions are:

- Planned continuous (24 hour) coverage of region (some unrealized because of failures)
- No attrition or loss of platforms
- No weather-related effects
- No ground aborts
- All platforms in unit have identical failure rates and repair times (or, generally, failure and repair time distributions)
- Failure rates do not change during different mission phases
- Failure process stable (no latent defects to occur, be removed)
- All in-flight repairs are successful (if possible; not for UAVs)
- Platforms may not launch with any uncorrected failures (subject to modifications if failure is unrelated to mission)
- Maintenance performed on a First-come, First-served basis
- No functional check flights
- Scheduled maintenance service time is constant
- No crew limitations (air or maintenance)
- No ground system failures
- Platforms are as good as new when launched (e.g. the random time to failure is redrawn from a fixed distribution)
All of the above can be relaxed or changed. Several are strong candidates for change to provide more realism or to perform sensitivity tests. In chapter V of this thesis, some of these assumptions are modified to analyze their effect on platform suitability.

E. MASS MODEL OUTPUT

Output from the simulation includes the following:

Effective Time On Station (ETOS) – The mean percentage of time that the region under surveillance is being covered by at least one operating platform for a given deployment duration.

Platform availability – The mean percentage of time a platform is mission capable (not necessarily over the region). This is presented as one number that is the average availability of all platforms during the deployment. This will vary with the deployment duration and with initial conditions, unless the deployment is very long. This is related to ETOS.

Sortie generation rate (sorties per day) – The average number of launches during each 24-hour period.

Total sorties during deployment (sorties) - The total number of launches for the entire deployment.

For each of the outputs, a mean, and a confidence interval for the mean, are calculated. All confidence intervals in this thesis are calculated based on the assumption of approximate normality of the single mean. These normal confidence intervals were checked against the Bootstrapping method of determining confidence intervals, and the differences were negligible. It is also worth noting that the input, Time Between Simulation Results Output, allows the user to record the results at any point during the
deployment. This gives information about the variability of the coverage (i.e. how
coverage early in the deployment may compare with that later in the deployment,
depending on initial conditions assumed).
IV. ANALYTIC MODEL

An analytical model for a single platform has been developed, see Gaver, Jacobs, and Stoneman (1998) and Appendix A, that can be used to augment and assist in verifying the results of MASS. Such models can efficiently highlight sensitivities to basic model components (distributions of times to failure and times to repair, operational tactics) as a preliminary to more elaborate modeling and actual field testing.

The first analytic model is a single-platform model. It assumes that mission-affecting failures occur according to a Poisson process with rate \( \lambda \), or, equivalently, that time to failure is exponentially distributed. The platform will return to base when a mission-affecting failure occurs. Additional failures that occur during the return add to the repair time. The repair times are independent and have a mean \( 1/\mu \). It is interesting and important that, for very long deployments, the only dependence on the repair time distribution is through its mean. This feature may change if one maintenance facility serves several platforms, and congestion and delays occur.

The formula for long-run proportion of time on station from the analytical model is:

\[
\pi = \frac{e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right]}{\frac{1}{\mu} \left[ 1 - e^{-T} \right] + \frac{2}{\lambda} \left[ 1 - e^{-\lambda T} \right] + e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right] + D}
\]  

(IV.1)

Where

- \( T \) = Ingress/Egress Time
- \( S \) = On Station Time
- \( D \) = Additional On-Deck Time (Turn, Logistics, etc.)

It is notable that the expression for ETOS depends only on the expectations (means) of the repair times, and is insensitive to any other features of the repair time
distribution. Such insensitivity does not extend to times to failure (here exponential, but not necessarily so in practice); see sensitivity studies conducted in Chapter V, Section B.

The inputs to MASS are, in this case, the following:

**Numerical Illustration**

1 // Number of Platforms
1 // Number of Ground Maintenance Paths
1 // Maximum Number of Platforms Flying
2 // Ingress/Egress Time (in hours)
10 // Scheduled On-Station Time (in hours)
50,000 // Flight Time (in hours) Before Refuel
0 // Transit Time (in hours) to Refueling Orbit
0 // Time (in hours) Required to Refuel Platform
0 // Probability That a Failure is In-Flight Repairable
10 // Mean Time (in hours) Between Mission-Affecting Failures
50,000 // Mean Time (in hours) Between Non-Mission-Affecting Failures
0,2,4 // Mean Time (in hours) For Ground Repair; 3 examples
0 // Mean Time (in hours) For In-Flight Repair
50,000 // Flight Time (in hours) Between Scheduled Maintenance Actions
2,160 // Deployment Duration (in hours)
0 // Logistics Delay Time (in hours)
0 // Scheduled Maintenance Service Time (in hours)
0 // Platform Turn-Around Time (in hours)
2,160 // Time (in hours) Between Simulation Results Output
50 // Number of Simulated Deployments (replications)

The highlighted values are those that are used in the analytic model. Note that three different mean repair times are used. The results below in Table 1 indicate that MASS accurately estimates ETOS as computed analytically, and appears trustworthy for further studies with different parameters.

<table>
<thead>
<tr>
<th>Ground Mean Time to Repair</th>
<th>Analytic Result</th>
<th>MASS ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.59</td>
<td>0.59 – 0.59</td>
</tr>
<tr>
<td>2</td>
<td>0.49</td>
<td>0.49 – 0.50</td>
</tr>
<tr>
<td>4</td>
<td>0.42</td>
<td>0.42 – 0.43</td>
</tr>
</tbody>
</table>

Table 1. Comparison of Analytic Model with MASS
One of the interesting facts not inherently obvious from these results is the effect of how MASS begins the deployment. The analytic model results here do not take into account any start-up factors; it “starts in steady state”. MASS, on the other hand, starts off by having an ETOS of 0 for at least the first ingress time and possibly more if the first platform fails before reaching its assigned station. This difference is negated by using a deployment of sufficient length to make the start-up effects unimportant. 2,160 hours (90 days) proves sufficient for this. MASS and the analytical model will tend to disagree more if deployment time is short, with the (present) analytical model ETOS results being somewhat greater than those from MASS runs starting with commencement of ingress time.
V. RESULTS AND ANALYSIS. IMPLICATIONS FOR FIELD TESTING.

There are currently more than 20 inputs for MASS. Instead of discussing the sensitivity analysis for each, the most important inputs have been selected for examination. Also some of the assumptions made are examined for their effect on the simulation.

A. PRE-POSITIONING OF PLATFORM

For this analysis the platform was assumed to have an endurance of 16 hours. The three cases chosen for comparison are shown in Table 2.

<table>
<thead>
<tr>
<th></th>
<th>Ingress/Egress Time</th>
<th>On-Station Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>0</td>
<td>16</td>
</tr>
<tr>
<td>Case II</td>
<td>4</td>
<td>8</td>
</tr>
<tr>
<td>Case III</td>
<td>6</td>
<td>4</td>
</tr>
</tbody>
</table>

Table 2. Description of Positioning Cases.

The other inputs, which remained constant, were as follows:

Numerical Illustration

4 // Number of Platforms
4 // Number of Ground Maintenance Paths
4 // Maximum Number of Platforms Flying
50,000 // Flight Time (in hours) Before Refuel
0 // Transit Time (in hours) to Refueling Orbit
0 // Time (in hours) Required to Refuel Platform
0 // Probability That a Failure is In-Flight Repairable
40 // Mean Time (in hours) Between Mission-Affecting Failures
50,000 // Mean Time (in hours) Between Non-Mission-Affecting Failures
2 // Mean Time (in hours) For Ground Repair
0 // Mean Time (in hours) For In-Flight Repair
50 // Flight Time (in hours) Between Scheduled Maintenance Actions
2,160 // Deployment Duration (in hours)
0.5 // Logistics Delay Time (in hours)
7 // Scheduled Maintenance Service Time (in hours)
1 // Platform Turn-Around Time (in hours)
2,160 // Time (in hours) Between Simulation Results Output
50 // Number of Simulated Deployments (replications)
Note that 50,000 hours is used as an input for flight time before refueling and time between non-mission affecting failures. As discussed earlier, this ensures that neither of these events will occur. Also the length of deployment is 2,160 hours to negate the effect of initial conditions. Most importantly note that by using 4 platforms and 4 maintenance paths with a mean of 40 hours between mission-affecting failures and only 2 hours for the mean time to repair, there is never a shortage of available platforms. This enables the effect of positioning on ETOS to be seen clearly.

The times between mission-affecting failures are independent and have an exponential distribution with appropriate mean time to failure. The times for repairs also are independent and have an exponential distribution with appropriate mean repair time. The results are in Table 3 and Figure 2.

<table>
<thead>
<tr>
<th></th>
<th>ETOS (95% Confidence Interval)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Case I</td>
<td>1</td>
</tr>
<tr>
<td>Case II</td>
<td>0.89 – 0.91</td>
</tr>
<tr>
<td>Case III</td>
<td>0.67 – 0.68</td>
</tr>
</tbody>
</table>

Table 3. ETOS Results From Changes in Positioning

![ETOS vs. Ingress Time](image)

Figure 2. ETOS vs. Ingress Time
The significant change of the ETOS values for these different cases illustrates the advantage of basing the platform as close to the surveillance region as possible. There are two main reasons for this. The first is that any failures that occur during the ingress phase cause the platform to return to base. Such a platform contributes nothing to the time on station, and a replacement is launched, as soon as the failure occurs, if available; but the replacement is also subject to failure, and so on. The second reason positioning is so important is the time it takes for the replacement to reach the surveillance region. When a platform leaves the surveillance region at its scheduled time, a replacement is planned to be there to take its place. However, when there is a failure on station, the time it takes for the replacement to arrive is lost coverage, unless redundant presence is scheduled. These situations are demonstrated by Case I, where there is no ingress time, and the squadron is able to cover the region essentially 100% of the time, given the failure and repair parameters assumed.

Again, a limitation of this analysis is the assumption that the squadron experiences no loss of platforms. This is not realistic since platforms can be lost due to mechanical failure or pilot error. However, this does not affect the user's ability to detect sensitivities in a platform's capabilities unless the platform has been shown to have an extremely high incidence of losses. Of course if that is the case, then the platform should probably be subject to more testing before fielding. It is also reasonable to assume that, the closer the platforms are based to a surveillance region, the greater the danger of attrition of platforms and maintenance facilities. Attrition could be added to the simulation as a decreasing function of distance, so as to effectively model a trade-off between the two variables. Of course for real world operations, the platform would most
likely be based at currently established sites, where attrition by the enemy is not a strong factor, and it is simply a matter of choosing the closest available location.

B. TIMES BETWEEN MISSION-AFFECTING FAILURES

One area of special interest that would seem to be a logical place to affect a platform's suitability are the random times between failures. During the development phase of a platform, the company manufacturing the platform has the best chance to lengthen the time between failures through design and choice of parts to be used. It will then be a question of cost versus return. During the operational phase, a platform will be somewhat limited in its ability to increase the time between failures, but this may be possible by changing the way in which the platform is used.

The question in both cases mentioned above leads to the comparison of ETOS with various mean times between mission-affecting failures. In this example the times between mission-affecting failures are independent and have an exponential distribution with appropriate mean time to failure. The times for repairs are also independent and have an exponential distribution with appropriate mean time to repair. The inputs to MASS for these runs were as follows with the inputs to be varied in boldface:

Numerical Illustration

4 // Number of Platforms
4 // Number of Ground Maintenance Paths
4 // Maximum Number of Platforms Flying
4 // Ingress/Egress Time (in hours)
8 // Scheduled On-Station Time (in hours)
50,000 // Flight Time (in hours) Before Refuel
0 // Transit Time (in hours) to Refueling Orbit
0 // Time (in hours) Required to Refuel Platform
0 // Probability That a Failure is In-Flight Repairable
20,40,60 // Mean Time (in hours) Between Mission-Affecting Failures
50,000 // Mean Time (in hours) Between Non-Mission-Affecting Failures
2 // Mean Time (in hours) For Ground Repair
0 // Mean Time (in hours) For In-Flight Repair
50 // Flight Time (in hours) Between Scheduled Maintenance Actions
2,160 // Deployment duration (in hours)
0.5 // Logistics Delay Time (in hours)
7 // Scheduled Maintenance Service Time (in hours)
1 // Platform Turn-Around Time (in hours)
2,160 // Time (in hours) Between Simulation Results Output
10 // Number of Simulated Deployments (replications)

Note that when the mean time between mission affecting failures is equal to 40
the inputs are the same as Case II in Chapter V, Section A. The results can be seen in
Table 4 and Figure 3.

<table>
<thead>
<tr>
<th>Mean Time Between Mission Affecting Failures</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.81 – 0.82</td>
</tr>
<tr>
<td>40</td>
<td>0.89 – 0.91</td>
</tr>
<tr>
<td>60</td>
<td>0.93 – 0.94</td>
</tr>
</tbody>
</table>

Table 4. ETOS Results From Change in Time Between Mission-Affecting Failures

![ETOS vs. Mean Time Between Mission-Affecting Failures](image)

Figure 3. ETOS vs. Mean Time Between Mission-Affecting Failures

It is interesting to see how the simulation reacts to the changes. The increase in
ETOS when the mean time was increased from 20 to 40 is a significant improvement.
However the increase of mean time from 40 to 60 has a diminished return rate. It would now be a matter of determining the cost associated with accomplishing the increases to determine if they would be worthwhile. This cost could be the result of improving platform reliability, shortening maintenance times, or both.

C. TIME TO REPAIR

Much attention has been paid to maintenance’s ability to affect the suitability of a platform. One simple way for the maintenance facility to increase a platform’s suitability should be to decrease the amount of time a platform is being served by decreasing the mean time to repair.

The comparison of ETOS with various mean times to repair should illustrate the above point. In this example the times between mission-affecting failures are independent and have an exponential distribution with appropriate mean time to failure. The repair times are also independent and have an exponential distribution with appropriate mean time to repair. The scheduled maintenance service time is a constant. The inputs to MASS for these runs were as follows with the inputs to be varied in boldface:

Numerical Illustration

4 // Number of Platforms
4 // Number of Ground Maintenance Paths
4 // Maximum Number of Platforms Flying
4 // Ingress/Egress Time (in hours)
8 // Scheduled On-Station Time (in hours)
50,000 // Flight Time (in hours) Before Refuel
0 // Transit Time (in hours) to Refueling Orbit
0 // Time (in hours) Required to Refuel Platform
0 // Probability That a Failure is In-Flight Repairable
40 // Mean Time (in hours) Between Mission-Affecting Failures
50,000 // Mean Time (in hours) Between Non-Mission-Affecting Failures
2,4,6 // Mean Time (in hours) For Ground Repair
0 // Mean Time (in hours) For In-Flight Repair
50 // Flight Time (in hours) Between Scheduled Maintenance Actions
2,160 // Deployment Duration (in hours)
0.5 // Logistics Delay Time (in hours)
7 // Scheduled Maintenance Service Time (in hours)
1 // Platform Turn-Around Time (in hours)
2,160 // Time (in hours) Between Simulation Results Output
10 // Number of Simulated Deployments (replications)

Note that when the mean time to repair is equal to 2 the inputs are the same as Case II in Chapter V, Section A. The results can be seen below in Table 5 and Figure 4.

<table>
<thead>
<tr>
<th>Mean Time To Repair</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.89 – 0.91</td>
</tr>
<tr>
<td>4</td>
<td>0.89 – 0.91</td>
</tr>
<tr>
<td>6</td>
<td>0.89 – 0.90</td>
</tr>
</tbody>
</table>

Table 5. ETOS Results From Change in Time to Repair

![Figure 4. ETOS vs. Mean Time to Repair](image)

The simulation does not react much to the changes. There are two likely reasons for this lack of effect. The squadron is equipped with 4 platforms and 4 maintenance paths, and each platform has a mean time between mission-affecting failures of 40 hours. In this case, the probability a platform will have a mission-affecting failure during its
ingress or on-station time is \(1 - e^{-12/40} = 0.26\). So, with 4 platforms, the squadron will almost always have a platform available for launch, and the repair time, unless extreme, does not effect ETOS.

It should be mentioned here that 4 maintenance paths for 4 platforms is an extreme number. It is unlikely that a maintenance facility would be able to handle the entire squadron being “down” at one time. This input for maintenance is used throughout this thesis for results that are not influenced by ground practices. In this case, it is worthwhile to see what happens with a smaller support structure. However, just one run confirms that there is not much effect in changing the maintenance structure. Using the inputs from above with a mean time to repair of 6 hours, and now only 1 maintenance path, the ETOS decreases only slightly to 0.87 (0.86 – 0.88 for the 95% CI). So, for these types of changes in a squadron’s support structure, it can be seen that there is not much “bang for the buck”.

Now it should be confirmed that, with a smaller number of platforms, the ETOS will be affected by changing the mean time to repair. Once again the times between mission-affecting failures are independent and have an exponential distribution with appropriate mean time to failure. The times for repairs are also independent and have an exponential distribution. The inputs to MASS for these runs were as follows with the inputs to be varied in boldface. Note that the first three inputs have changed from the previous analysis.

**Numerical Illustration**

\[
\begin{aligned}
2 & \quad \text{// Number of Platforms} \\
2 & \quad \text{// Number of Ground Maintenance Paths} \\
2 & \quad \text{// Maximum Number of Platforms Flying} \\
4 & \quad \text{// Ingress/Egress Time (in hours)}
\end{aligned}
\]
The results can be seen in Table 6 and Figure 5.

<table>
<thead>
<tr>
<th>Mean Time To Repair</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.67 – 0.68</td>
</tr>
<tr>
<td>4</td>
<td>0.62 – 0.64</td>
</tr>
<tr>
<td>6</td>
<td>0.58 – 0.59</td>
</tr>
</tbody>
</table>

Table 6. ETOS Results From Change in Time to Repair

Figure 5. ETOS vs. Mean Time To Repair
It can be seen that there is now a noticeable decrease in ETOS when the mean
time to repair is increased. The smaller number of platforms increases the impact of the
maintenance facility operations. Simply by increasing the number of platforms to 3, and
using the worst case of 6 hours for the mean time to repair, the ETOS is raised back to
0.85 (0.84 – 0.86 for the 95% CI). The interaction between the number of platforms and
the maintenance support on ETOS is an interesting problem. The costs associated with
increasing the number of platforms, or improving the maintenance support, will influence
a decision concerning the most effective way to improve ETOS.

D. SENSITIVITY TO VARIATION OF TIME-TO-FAILURE DISTRIBUTIONS

Any number of distributions may be suitable for modeling the times between
failures on the platform. In the Predator model by Post and Warner, the times were
assumed to have an exponential distribution. This is a conventional assumption for
reliability models; it is a satisfactory and convenient first assumption but not a universal
natural law. In this section, other distributions for the time between mission-affecting
failures are examined for impact on ETOS. A mission-affecting failure is defined as a
failure that requires an immediate return to base. The repair times are still independent
and have an exponential distribution with appropriate mean time to repair. The
simulation inputs were as follows:

Numerical Illustration

4 // Number of Platforms
2 // Number of Ground Maintenance Paths
4 // Maximum Number of Platforms Flying
6 // Ingress/Egress Time (in hours)
4 // Scheduled On-Station Time (in hours)
50,000 // Flight Time (in hours) Before Refuel
0 // Transit Time (in hours) to Refueling Orbit
0 // Time (in hours) Required to Refuel Platform
These inputs remained constant for all model runs. Only the failure time distribution changed. See Appendix B for the derivation of the formulas used. The results are summarized in Table 7.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Alpha</th>
<th>Beta</th>
<th>Time until next mission affecting failure</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td></td>
<td></td>
<td>(1/\lambda)^*(-\ln U)</td>
<td>0.67 - 0.68</td>
</tr>
<tr>
<td>Weibull</td>
<td>sqrt(2 * \lambda)</td>
<td>0.5</td>
<td>(-1/\alpha)^*(-\ln U)^1/\beta</td>
<td>0.51 - 0.52</td>
</tr>
<tr>
<td>Sculptured</td>
<td>.5 * (1/\lambda - 1)</td>
<td></td>
<td>-\ln U * (1 + (\alpha * -\ln U))</td>
<td>0.51 - 0.52</td>
</tr>
<tr>
<td>(a)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sculptured</td>
<td>(1/24)*(1/\lambda - 1)</td>
<td></td>
<td>-\ln U * (1 + (\alpha * -\ln U^3))</td>
<td>0.28 - 0.30</td>
</tr>
<tr>
<td>(b)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

- \lambda = failure rate = 1/40
- U = random uniform variable between (0,1)

**Table 7. Time-To-Failure Distributions and the Resulting ETOS**

The two sculptured distributions were created using a technique described by Gaver (1983). They start with exponential distributions but rearrange the distributional form in a manner convenient for simulation. The shaping functions tend to leave the distribution's relative shape for smaller values alone, but increase the probability of large values. This creates a distribution that is nearly exponential near the origin, but has an exaggerated long right tail (representing missing data, outliers, etc.). The net effect is to produce relatively more short/small times to failure than the exponential, and
correspondingly relatively fewer, but longer, times, so arranged that the mean is the same as in the exponential submodel (first line in Table 6).

It is significant to note how the Effective Time on Station (ETOS) is affected by the different distributions. 100,000 random uniform numbers in the range (0,1) were created in the statistical package S-Plus and put in the above formulas to confirm that all four distributions have the same mean value. Table 8 shows that they do have the same mean with any small differences due to sampling error. The confidence intervals were calculated using the normal assumption.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Mean</th>
<th>95% CI</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>40.02</td>
<td>39.94 – 40.09</td>
</tr>
<tr>
<td>Weibull</td>
<td>39.98</td>
<td>39.79 – 40.17</td>
</tr>
<tr>
<td>Sculptured (a)</td>
<td>40.09</td>
<td>39.90 – 40.28</td>
</tr>
<tr>
<td>Sculptured (b)</td>
<td>40.06</td>
<td>39.42 – 40.70</td>
</tr>
</tbody>
</table>

Table 8. Mean Values of the Time-To-Failure Distributions

It turns out that the ETOS may be generally dependent on, or sensitive to, the shape of the distribution of the times to failure. The ETOS produced by the Weibull distribution is quite comparable to the ETOS produced by the Sculptured (a) distribution. The Sculptured form (b) yields a significant drop in ETOS.

Why did these different distributions produce the results that they did? In general, we make the reasonable assumption that the expected or mean time between mission affecting failures is greater than the flight time of our platform. Since the time until mission affecting failure is an independent draw each time the Platform launches, only failures that will occur during that flight can affect the ETOS. Therefore, only the part of
the distributions that produce values less than the flight time of 16 hours are a factor in the simulation.

To show how the four distributions compare in producing failure times of less than the flight time of 16 hours, 10,000 additional random uniform numbers from (0,1) were created. They were then used in the formulas given in Table 7, and the number of times the resulting value was less than 16 (the sortie duration) was totaled. It can be seen in Table 9 that the distributions that are more likely to produce a failure in the first 16 hours (during the flight) have a lower ETOS.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>ETOS (95% CI)</th>
<th>Number of Failure Times &lt; 16 (out of 10,000)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>0.66 – 0.68</td>
<td>3341</td>
</tr>
<tr>
<td>Weibull</td>
<td>0.51 – 0.52</td>
<td>5953</td>
</tr>
<tr>
<td>Sculptured (a)</td>
<td>0.51 - 0.52</td>
<td>5893</td>
</tr>
<tr>
<td>Sculptured (b)</td>
<td>0.28 - 0.30</td>
<td>8254</td>
</tr>
</tbody>
</table>

Table 9. Sample Failure Times

These results are interesting because they point out how the overall model depends on basic assumptions: if there is a propensity for failure before the planned cycle time that is relatively high compared to an exponential with the same mean, adoption of the exponential model is overly optimistic. However, the alternative distributions explored in this thesis have not been chosen to actually model any real data. One other possibility that was not tested is a bimodal distribution. It has been theorized that the first hour is a likely time for failures because of a likely high number of take-off-related failures. After that, failures may occur less frequently until the platform returns when the landing process may again induce failures. Then it is likely that there are a high number of landing-related failures. This topic presents an area for further study. Actual
field test data, or the equivalent, on times to failure are essential in order to obtain valid results.

E. LAUNCHING WITH NON-MISSION-AFFECTING FAILURES

A non-mission-affecting failure has already been defined as a failure that does not affect the platform’s ability to complete its mission. For this suitability simulation, such failures only affect (increase) the time a platform spends in maintenance, thereby reducing ETOS. Originally, the simulation forced a platform to have all repairs completed before it was allowed to launch again. However, the assumption can be made that a platform will not enter ground maintenance unless it either has a mission-affecting failure, or it needs scheduled maintenance. So a platform with only non-mission-affecting failures will be sent directly to the queue to be launched when needed; the turnaround time is assumed to be 0. This assumption was made so that the simulation will defer repairing non-mission-affecting failures. The non-mission-affecting failures will not be repaired until the platform enters maintenance for one of the other two cases.

The following inputs were used for model runs comparing the original MASS simulation to the variation that allows the platform to launch with non-mission affecting failures. In this example, the times between mission-affecting and non-mission-affecting failures are independent and have an exponential distribution with appropriate mean times to failure. The repair times are also independent and have an exponential distribution with appropriate mean time to repair. The inputs to be varied are in boldface.

Numerical Illustration

| 1,4 | // Number of Platforms |
| 1,4 | // Number of Ground Maintenance Paths |
| 1   | // Maximum Number of Platforms Flying |
| 3   | // Ingress/Egress Time (in hours) |
14
50000
0
0
25
5
2
0
50
2160
0.5
7
0
2160
10

// Scheduled On-Station Time (in hours)
// Flight Time (in hours) Before Refuel
// Transit Time (in hours) to Refueling Orbit
// Time (in hours) Required to Refuel Platform
// Probability That a Failure is In-Flight Repairable
// Mean Time (in hours) Between Mission-Affecting Failures
// Mean Time (in hours) Between Non-Mission-Affecting Failures
// Mean Time (in hours) For Ground Repair
// Mean Time (in hours) For In-Flight Repair
// Flight Time (in hours) Between Scheduled Maintenance Actions
// Deployment Duration (in hours)
// Logistics Delay Time (in hours)
// Scheduled Maintenance Service Time (in hours)
// Platform Turn-Around time (in hours)
// Time (in hours) Between Simulation Results Output
// Number of Simulated Deployments (replications)

The results are summarized in Table 10.

<table>
<thead>
<tr>
<th>Number of Platforms</th>
<th>Maint. Paths</th>
<th>Platforms Airborne</th>
<th>Ingress Time</th>
<th>On Station Time</th>
<th>MASS ETOS (95% CI)</th>
<th>Variant ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>14</td>
<td>0.84 – 0.85</td>
<td>0.84 – 0.86</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>0.53 – 0.55</td>
<td>0.54 – 0.56</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>0.70 – 0.71</td>
<td>0.69 – 0.70</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>0.19 – 0.21</td>
<td>0.20 – 0.21</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>14</td>
<td>0.38 – 0.39</td>
<td>0.37 – 0.38</td>
</tr>
</tbody>
</table>

Table 10. ETOS Comparison for Change in Maintenance Process

The difference between the simulations is negligible when comparing the long-run effective time on station. This is interesting because at first glance it might be expected that allowing the platform to launch with non-mission-affecting failures unrepaired would increase the effective time on station. However this change in policy does not change the long-run amount of time a platform spends in maintenance. The platform will still have the same number of failures and scheduled maintenance activities as it had before. The different policy merely causes the platform to switch from many, short trips to the maintenance facility to fewer, long trips to the maintenance facility that
occur later in the deployment. In the long run this results in approximately the same effective time on station. However, the different pattern of up and down times is potentially of interest.

This can then lead to more areas to explore. For example, short deployments could use this policy to increase their ETOS. Then, of course, the question will arise as to what constitutes a “short” deployment. Since the object is to keep the platforms away from the maintenance facility, without any computations, a suggestion would be to limit the deployment to avoid scheduled maintenance. If all 4 platforms have just come out of scheduled maintenance and 50 hours is kept as the time until scheduled maintenance, then this would allow for 200 total hours of flight time, a significant deployment even when overlapping of the flights is taken into account. One could also rigidly adhere to the assumption that non-mission-affecting failures do not affect the platform, and never repair them. However this is not realistic unless the situation is extreme: a major effort must be conducted during a short deployment, with catch-up later.

**F. REPAIR TIME / DOWN TIME PROCESSES**

In section C, it was shown that allowing the platform to launch with the non-mission-affecting failures did not increase the long-run effective time on station. The reason for this was that the total required ground maintenance time remained the same. The maintenance time remained the same because of the assumption made in the simulation that the repairs are made serially. Therefore, when a platform lands with two failures and the mean time for repair of each is two hours, the platform spends four hours to repair the two failures on average. If the platform has ten failures, then it will take approximately twenty hours to repair.
The assumption of serial repair is not necessarily an accurate one. There should be more than one person or team assigned to do maintenance, and several maintainers could possibly work on different failures in parallel. For example, platforms are worked on by different types of technicians who repair failures in each of their specialties. This is usually split into the type of system on the platform such as airframe, engine, electronics, and mission equipment, such as sensor packages. Even if a platform had multiple electronic failures, it would still be reasonable to assume that more than one person is available in that specialty.

If the repairs are not completed serially, then another way to model them must be found. One possibility is that the repairs are completed in parallel. A repair time for each failure is computed, and then the maximum (rather than the sum) of those numbers is used as the total repair time. This change to \textit{MASS} is accomplished by examining each platform’s failures when they land. The failures’ repair times are compared and the maximum is determined. The failure with the corresponding maximum repair time remains, and all other failures are given repair times of 0.001 hours (this merely a technical programming device). The platform then continues through the standard maintenance flow.

The following inputs were used for model runs comparing the original \textit{MASS} simulation to the variation that performs repairs in parallel. The times between mission-affecting and non-mission-affecting failures are independent and have an exponential distribution with appropriate mean times to failure. The repair times are also independent and have an exponential distribution with appropriate mean time to repair. The inputs to be varied are in boldface. Note that these are the same inputs used in Section E.
Numerical Illustration

1,4  // Number of Platforms
1,4  // Number of Ground Maintenance Paths
1   // Maximum Number of Platforms Flying
3   // Ingress/Egress Time (in hours)
14  // Scheduled On-Station Time (in hours)
50000 // Flight Time (in hours) Before Refuel
0   // Transit Time (in hours) to Refueling Orbit
0   // Time (in hours) Required to Refuel Platform
0   // Probability That a Failure is In-Flight Repairable
25  // Mean Time (in hours) Between Mission-Affecting Failures
5   // Mean Time (in hours) Between Non-Mission-Affecting Failures
2   // Mean Time (in hours) For Ground Repair
0   // Mean Time (in hours) For In-Flight Repair
50  // Flight Time (in hours) Between Scheduled Maintenance Actions
2160 // Deployment Duration (in hours)
0.5 // Logistics Delay Time (in hours)
7   // Scheduled Maintenance Service Time (in hours)
0   // Platform Turn-Around time (in hours)
2160 // Time (in hours) Between Simulation Results Output
10  // Number of Simulated Deployments (replications)

The results are summarized in Table 11.

<table>
<thead>
<tr>
<th>Number of Platforms</th>
<th>Maint. Paths</th>
<th>Platforms Airborne</th>
<th>Ingress Time</th>
<th>On Station Time</th>
<th>MASS ETOS (95% CI)</th>
<th>Parallel ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>14</td>
<td>0.84 – 0.85</td>
<td>0.88 – 0.90</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>0.53 – 0.55</td>
<td>0.71 – 0.72</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>4</td>
<td>6</td>
<td>8</td>
<td>0.70 – 0.71</td>
<td>0.75 – 0.76</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>6</td>
<td>8</td>
<td>0.19 – 0.21</td>
<td>0.23 – 0.24</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>14</td>
<td>0.38 – 0.39</td>
<td>0.44 – 0.45</td>
</tr>
</tbody>
</table>

Table 11. ETOS Comparison for Change in Repair Time Calculations

As expected, the ETOS increased when the time the platforms spent in repair was decreased. These results may be realistic for a small number of failures, but may be very optimistic for a large number of failures. The “real world” is probably somewhere in between the pure serial and pure parallel repair. So another way to formulate the problem is as a combination of the two: series and parallel. For instance, assume that
maintenance can repair no more than three failures at once. Use the maximum repair
time of each group of three as that group’s total time and then sum the values from each
group of three to get a total repair time for the platform. This will penalize a platform for
having a large number of failures but not as severely as the straight serial repair.

Careful study of actual operational test data that include durations and patterns of
repair of multiple failures is necessary to credibly quantify the random total down time
likely to be incurred in operations. Such data can also be used to devise optimal repair
schedules, and study logistics effects.

G. OPERATING IN DEGRADED CONDITIONS: WHEN NOT TO REPAIR

In previous simulation runs the platform has been treated as one system that can fail
repeatedly during a mission because of its complexity. In this section the platform is
separated into two subsystems: the platform (P), and the sensor (S). Each subsystem is
assumed to have three levels of performance: Good, Medium, and Bad. MASS can then
be used with only a few adjustments to track the amount of time a platform spends in
each of the nine possible states over the course of a deployment. The above is still a
simplified version of an actual and specific system. The analysis points the way to study
the levels of degraded operations of a specific new, or upgraded, system. Sensible
options are possible.

MASS inputs used were the same as before, but the time between mission-affecting
failures is now the time between platform failures, and the time between non-mission-
affecting failures is now the time between sensor failures to various performance levels.
In other words, the first mission-affecting failure causes the platform to go to the Medium
state (PM). If there is a second mission-affecting failure, during the same flight, the
platform goes to the Bad state (PB). The non-mission-affecting failures work in the same manner on the sensor. The first non-mission-affecting failure causes the sensor to go to the Medium state (SM), while the second causes the sensor to go to the Bad state (SB).

Suppose it is decided that the platform will return to base if the platform goes to the Medium state (PM). Also, it could possibly go to the Bad state during the return trip (PB). The state of the sensor does not affect the mission. The sensor is assumed to be reliable enough that the probability of it entering the Bad (SB) state before returning to base is minimal. The system will continue to operate in the Medium (SM) or Bad (SB) state, but its effectiveness (e.g. sweep width) is less in the Medium than in the Good state.

Note that many different system control options can be investigated by the model. For instance, if operation in the sensor Medium (SM) state is sufficiently degraded it may be desirable to dispatch the system for repair when it reaches that state, particularly if the maintenance facility is unloaded (no queue delay).

Data collected from these runs is the same as the original MASS, but the flight hours that a platform spends in each possible state combination is recorded also. All failures, both mission-affecting and non-mission-affecting, are repaired before the platform is launched again. The times between failures are independent and have an exponential distribution with appropriate mean time to failure. Repair times are also independent and have an exponential distribution with appropriate mean time to repair. Repair times do not vary according to the type of failure (platform or sensor). Time on the ground is not considered to be in any relevant state. The following were the inputs for the simulation:

**Numerical Illustration**

```
4 // Number of Platforms
4 // Number of Ground Maintenance Paths
```
The ETOS is 0.89 as it was in section A for case II since changing the meaning of the mean time between non-mission-affecting failures does not affect ETOS. Table 11 shows the average percentage of flight time spent by a platform in each state.

<table>
<thead>
<tr>
<th>Platform State, Sensor State</th>
<th>Average percentage of flight time in state</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good, Good</td>
<td>72.5</td>
</tr>
<tr>
<td>Good, Medium</td>
<td>17.0</td>
</tr>
<tr>
<td>Good, Bad</td>
<td>3.8</td>
</tr>
<tr>
<td>Medium, Good</td>
<td>4.4</td>
</tr>
<tr>
<td>Medium, Medium</td>
<td>1.6</td>
</tr>
<tr>
<td>Medium, Bad</td>
<td>0.3</td>
</tr>
<tr>
<td>Bad, Good</td>
<td>0.2</td>
</tr>
<tr>
<td>Bad, Medium</td>
<td>0</td>
</tr>
<tr>
<td>Bad, Bad</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 12. Percentage of Time Platform Spend in Each State

These results are qualitatively predictable, but they can lead to some further analysis. The platform spends 93% of its flight time with the platform in the Good state (PG). This occurs because the platform has a larger expected time between failures than
the sensor, and because the platform returns to base as soon as the platform goes to a Medium state. This latter reason is obviously why no significant time is seen with the platform in Bad state (PB); it is assumed that maintenance is always beneficial, but this may not be true. The quality-of-maintenance effect should be field tested.

The sensor also spent most of its flight time in a good state although not to the same extent as the platform. Since the platform does not return to base with sensor problems, it did operate with the sensor in a medium state for 19% of the flight time. The simulation also represented the system as having a small percentage of time with a bad sensor. This could mean that it is a bad policy not to return to base as soon as sensor problems arise. However, it should be remembered that even though the percentage was very small it may still have been an overestimation of any sensor problems that affected the mission. The platform could easily have been already returning to base when the sensor went bad which would have counted in these calculations.

The evaluation of the sensor effectiveness is accomplished by breaking down the time when the platform is on-station, or when the platform is in a Good state (PG). The percentages of on-station time that the sensor spends in its' three possible states are shown in Table 12.

<table>
<thead>
<tr>
<th>Sensor State</th>
<th>Average Percentage of On-Station Time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Good</td>
<td>73.0</td>
</tr>
<tr>
<td>Medium</td>
<td>23.1</td>
</tr>
<tr>
<td>Bad</td>
<td>3.9</td>
</tr>
</tbody>
</table>

Table 13. On-Station Percentage of Time Sensor Operates
These results show that the platform is on-station for a small percentage of time with the sensor in a Bad state. This would not be an ideal situation. Depending on what the actual real-world meaning of the sensor states is, it could reasonably be recommended that the platform be recalled before the sensor is allowed to degrade to the Bad state; this could shorten the total down time and improve ETOS and the quality/effectiveness of that time. However further field testing could be done to determine if there appears to be an optimal balance between sensor effectiveness and time on-station.

H. DEPLOYMENT LENGTH

The effect of deployment length has not yet been studied extensively. A 90 day (2160 hr) deployment has been used for all previous analyses to allow the simulation to reach a steady state. This is a lengthy deployment from which to expect continuous coverage, but it was done to eliminate any effects from the startup period of the deployment and facilitate the comparison of the effects of varying the input parameters. However, it is useful and realistic to see how the platform performs for shorter deployments.

The inputs for this analysis are the same as those used in section A, case II. The deployment length is then varied to observe the affect it has on the ETOS. The times between mission-affecting failures are independent and have an exponential distribution with appropriate mean time to failure. The repair times are also independent and have an exponential distribution with appropriate mean time to repair. The inputs are as follows:

Numerical Illustration

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>// Number of Platforms</td>
</tr>
<tr>
<td>4</td>
<td>// Number of Ground Maintenance Paths</td>
</tr>
<tr>
<td>4</td>
<td>// Maximum Number of Platforms Flying</td>
</tr>
<tr>
<td>4</td>
<td>// Ingress/Egress Time (in hours)</td>
</tr>
</tbody>
</table>
The deployment duration inputs are highlighted to indicate that they are varied in the simulation runs. The number of hours correspond with 1, 3, 7, 14, and 28 day deployments. The simulation begins with the launch of the first platform. The resulting ETOS are shown in Table 13 along with the ETOS from the steady state run done earlier.

<table>
<thead>
<tr>
<th>Length of Deployment (hrs)</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.70 - 0.77</td>
</tr>
<tr>
<td>72</td>
<td>0.83 - 0.87</td>
</tr>
<tr>
<td>168</td>
<td>0.87 - 0.90</td>
</tr>
<tr>
<td>336</td>
<td>0.88 - 0.90</td>
</tr>
<tr>
<td>672</td>
<td>0.89 - 0.90</td>
</tr>
<tr>
<td>2160</td>
<td>0.89 - 0.91</td>
</tr>
</tbody>
</table>

**Table 14. ETOS Results From Different Deployment Lengths**

The results are very similar for the longer deployments, while the shorter appear to have significantly lower ETOS. However, close inspection shows that the results are comparable for all the deployments. Each deployment must have the initial ingress time counted against it when calculating ETOS. For these runs, that is four hours that are lost.
Table 14 shows what happens when those four hours are taken out of the denominator of the ETOS calculations; this is approximately equivalent to "starting in steady state."

<table>
<thead>
<tr>
<th>Length of Deployment (hrs)</th>
<th>ETOS (95% CI)</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>0.87 – 0.93</td>
</tr>
<tr>
<td>72</td>
<td>0.89 – 0.93</td>
</tr>
<tr>
<td>168</td>
<td>0.89 – 0.92</td>
</tr>
<tr>
<td>336</td>
<td>0.90 – 0.91</td>
</tr>
<tr>
<td>672</td>
<td>0.89 – 0.91</td>
</tr>
<tr>
<td>2160</td>
<td>0.89 – 0.91</td>
</tr>
</tbody>
</table>

Table 15. ETOS Results With Start-Up Costs Removed

The results are much closer together now. The ingress time for the first Platform had a more significant impact on the shorter deployments than the longer ones, and by taking that out it can be seen that the simulation is reaching the steady state in a much shorter deployment than 90 days.

It should be pointed out that the results from the shorter deployments are probably more realistic than the 90 day results. A single squadron is not expected to provide continuous coverage of a region for 90 days without some relief. However, if a user is simply looking for trends in the data to pinpoint areas to improve performance, the 90-day deployment should be used to decrease variability (see Figures 5 and 6). If a user wishes to use the output to state that a platform can achieve a specific ETOS, then there can be some problems with the simulation of the long deployment. Attrition might be more of a factor for a long deployment, and its effect is not modeled. The platforms could begin to show signs of wear and increase their failure rate, and the people operating the systems might show signs of "wear" (fatigue) themselves, and this would affect operations.
The user should also be wary of the short deployment results: the ETOS is not as good a summary of Actual Time On Station (ATOS); there tends to be substantial relative variability for short deployments. Figure 6 is a histogram of the ETOS results from the 168 hour deployment.

![Histogram of ETOS results](image)

**Figure 6. Histogram of 168 hour deployment results**

Figure 7 is a histogram of the results from the 672 hour deployment.
Figure 7. Histogram of 672 hour deployment results

The mean of the results from both runs were comparable, but there is still a difference. The histograms show that the shorter deployment exhibits greater variability of estimated ETOS. This makes sense, because a few quick failures would have a larger effect on the ETOS from the shorter deployment than they would on the longer deployment.

I. CONCLUSIONS AND RECOMMENDATIONS

MASS, especially when extended as has been done in this thesis, is an invaluable tool for test planning, and for the evaluation of a platform's suitability for different missions. The simulation can assist during the early acquisition process, when the government must decide whether or not to buy a particular system; the simulation can also assist in determining the most effective way to deploy a platform and a particular payload once it is in the field.
While this thesis concentrates on UAV operations, MASS (extended) can be used for operational analysis of a variety of platform types. The inputs to the simulation can be varied to determine which facets of a platform provide the most opportunity for improvement of capabilities. Some of the present assumptions may not apply to certain platforms, but it is still possible to look for trends in the results to aid in decision-making. The simulation outcomes suggest operational sensitivities that may be studied further during field experiments. The simulation should help to focus the experiments.

MASS has room for enhancements. Many of the assumptions can be changed (or removed) to adapt the simulation to answer other questions. In the case of UAVs, allowing for losses of platforms in the course of the deployment is one obvious possible modification. Adapting the desired area coverage to less than 24 hours a day would open the simulation to more platforms. Caution must be used to avoid making MASS less generic, thereby limiting the range of its usefulness. Also it could easily reach a point where the inputs to MASS would become numerous enough to discourage some users.

Versions of MASS are currently being used by IDA to analyze a number of programs, including DARK STAR UAV, GLOBAL HAWK UAV, and JSTARS (manned reconnaissance platform). The versions of MASS in this thesis add to what is being learned there and should provide impetus for further applications and future work.
LIST OF REFERENCES


53
APPENDIX A: ANALYTIC MODEL CALCULATIONS

B is the time between takeoff and return to base. Then

\[
B = \begin{cases} 
2X & \text{if } X < T \text{ (fails on flight out)} \\
X + T & \text{if } T < X < T + S \text{ (fails during surveillance time)} \\
2T + S & \text{if } X > T + S \text{ (fails on flight back or does not fail)}
\end{cases}
\]

Thus,

\[
E[B] = \int_0^T 2s \lambda e^{-\lambda s} ds + e^{-\lambda T} \int_0^T (s + 2T) \lambda e^{-\lambda s} ds + e^{-\lambda (T + S)} (2T + S)
\]

\[
E[B] = \frac{2}{\lambda} \left[ 1 - e^{-\lambda T} \right] + e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right]
\]

Let A be the time on station

\[
A = \begin{cases} 
0 & \text{if } X < T \text{ (fails on flight out)} \\
X - T & \text{if } T < X < T + S \text{ (fails during surveillance time)} \\
S & \text{if } X > T + S \text{ (fails on flight back or does not fail)}
\end{cases}
\]

Thus,

\[
E[A] = 0 \left[ 1 - e^{-\lambda T} \right] + e^{-\lambda T} \int_0^T s \lambda e^{-\lambda s} ds + e^{-\lambda (T + S)} S
\]

\[
E[A] = e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right]
\]

The expected length of a cycle is
\[ \pi = \frac{E[A]}{E[C]} \]

\[ \pi = \frac{e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right]}{\frac{1}{\mu} \left[ 1 - e^{-\left(2T + S\right)} \right] + \frac{2}{\lambda} \left[ 1 - e^{-\lambda T} \right] + e^{-\lambda T} \frac{1}{\lambda} \left[ 1 - e^{-\lambda S} \right] + D} \]

The long-run proportion of time the platform is performing surveillance:
APPENDIX B: DERIVATION OF TIME-TO-FAILURE DISTRIBUTIONS

Weibull

$$P\{Y \geq x\} = e^{-\alpha x^\beta}$$

$$E[Y] = \frac{1}{\lambda} = \int_0^\infty e^{-\alpha x^\beta} dx$$

$$z = \alpha x^\beta \quad dz = \alpha \beta x^{\beta-1} dx$$

$$x = \left(\frac{z}{\alpha}\right)^{\frac{1}{\beta}} \quad dx = \frac{1}{\alpha \beta} \left(\frac{z}{\alpha}\right)^{\frac{1}{\beta}-1} dz$$

$$\frac{1}{\lambda} = \frac{1}{\alpha^\beta \beta} \int_0^\infty e^{-z^{1/\beta}} z^{1/\beta-1} dz = \frac{1}{\alpha^\beta \beta} \Gamma\left(\frac{1}{\beta}\right)$$

Use $\beta = \frac{1}{2}$ for long-tailed (hyper-exponential) data

$$\frac{1}{\lambda} = \frac{2}{\alpha^2} \int_0^\infty e^{-z} z dz = \frac{2}{\alpha^2}$$

$$\alpha = \sqrt{2\lambda}$$

$$U = \text{Uniform}\left(0,1\right)$$

$$U = e^{-\alpha y^\beta}$$
\[ Y = \left( -\frac{1}{\alpha} \ln U \right)^{\frac{1}{\beta}} \]

**Sculptured (a)**

\[ X = \exp(\theta) = -\ln(U) \]

\[ P\{X \geq x\} = e^{-\alpha x} \]

\[ Y = X \left( 1 + AX \right) \quad A > 0 \]

\[ E[Y] = \frac{1}{\lambda} \]

\[ E[Y] = E[X] + AE[X^2] = E[X] + A \left( \text{Var}[X] + \left( E[X] \right)^2 \right) \]

\[ \frac{1}{\lambda} = \frac{1}{\theta} + A \left( \frac{1}{\theta^2} + \frac{1}{\theta^2} \right) \]

Set \( \theta = 1 \)

\[ \frac{1}{\lambda} = 1 + 2A \]

\[ A = \frac{1}{2} \left( \frac{1}{\lambda} - 1 \right) \]

\[ Y = -\ln U \left( 1 + \frac{1}{2} \left( \frac{1}{\lambda} - 1 \right) \left( -\ln U \right) \right) \]
Sculptured (b)

\[ Y = X \left( 1 + AX^3 \right) \]

\[ E[Y] = \frac{1}{\lambda} = E[X] + AE[X^4] \]

\[ E[X^4] = \int_0^\infty x^4 e^{-x} \, dx = 24 \]

\[ \frac{1}{\lambda} = 1 + 24A \]

\[ A = \frac{1}{24} \left( \frac{1}{\lambda} - 1 \right) \]

\[ Y = -\ln U \left( 1 + \frac{1}{24} \left( \frac{1}{\lambda} - 1 \right) \left( -\ln U \right)^3 \right) \]
INITIAL DISTRIBUTION LIST

1. Defense Technical Information Center ........................................2
   8725 John J. Kingman Rd., Ste 0944
   Ft. Belvoir, VA  22060-6218

2. Dudley Knox Library ..........................................................2
   Naval Postgraduate School
   411 Dyer Rd.
   Monterey, CA  93943-5101

3. Dr. Kenneth Pendergast ......................................................1
   Institute For Defense Analyses
   1801 N. Beauregard St
   Alexandria, VA  22311-1772

4. Professor Donald P. Gaver ..................................................1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA  93943-5103

5. Professor Patricia A. Jacobs .................................................1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA  93943-5103

6. CDR Ronald L. Brown ..........................................................1
   Department of Operations Research
   Naval Postgraduate School
   Monterey, CA  93943-5103

7. Dr. Ernest Seglie .............................................................1
   Director, Operational Test & Evaluation
   1700 Defense, Pentagon
   Washington, DC  20301-1700

8. COL W.M. Knarr, Jr. .........................................................1
   TSM-UAV
   Bldg 51005 (Riley Barracks)
   Rm 1082
   Fort Huachuca, AZ  85613-6000

9. LT James G. Stoneman .......................................................2
   13010 Freeland Rd
   Rockville, MD 20853