**Title:** Modeling Of Pulsed Thermography In Anisotropic Media

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MODELING OF PULSED THERMOGRAPHY IN ANISOTROPIC MEDIA

Ignacio Perez, Rachel Santos, Paul Kulowitch and Steven Shepard*
Naval Air Warfare Center, Aircraft Division
Materials Division, Patuxent River MD, 20670

Thermal Wave Imaging, Inc.
18899 W. 12 Mile Rd.
Lathrup Village, MI 48076

A simple thermographic model has been developed that accurately describes the surface temperature response of an aluminum panel with flat bottom holes of different depths and diameters to a short heat pulse. This model assumed that a thin layer of material at the surface is instantaneously heated by the pulse, and that subsequent cooling of the surface is due to diffusion of the deposited energy into the bulk of the material. The model accounts for sample thickness, density, specific heat, in-plane and out-of-plane thermal conductivity and defect size and depth. However, heat pulse parameters such as pulse duration and intensity were not included. In this talk we will present experimental and modeling results on graphite epoxy composites with flat bottom holes of different radii and depth. The experimental results were collected with standard pulse thermographic equipment. The experimental data was analyzed with our model. The effects of anisotropy in the thermal conductivity will be presented and discussed.

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MODELING OF PULSED THERMOGRAPHY IN ANISOTROPIC MEDIA

By:

Dr. Ignacio Perez
Paul Kulowitch
Rachel Santos
Steven Shepard
OUTLINE

- EXPERIMENTAL
- DATA ANALYSIS
- SIMPLE CALORIMETRIC MODEL
- SIMPLE FINITE ELEMENT MODEL
- EXPERIMENTAL RESULTS
- SUMMARY AND CONCLUSION
THERMOGRAPHIC SYSTEM

CAMERA SPECIFICATIONS
Amber Engineering Model AE-4128
128X128 InSb FPA
207 frames/s (max)
Sensitive to 0.01°C

FLASH LAMP SPECIFICATIONS
Speedtron Model 4803CX Capacitors
Speedtron Model 206VF Lamps
Delivers 5KJ per lamp (2) in 5 ms
1/8" Thick Al-7075 panel

3%
21%
43%
62%
72%
80%

1" Diameter Holes
CONTRAST vs DEPTH

Contrast Temp. (a.u.) vs Depth d (Inches)
NO LATERAL HEAT CONDUCTIVITY APPROXIMATION

\[ q = m \cdot c \cdot \Delta T \]

\[ q_2 = \rho \cdot A_2 \cdot t \cdot c \cdot T_2 \]

\[ q_1 = \rho \cdot A_1 \cdot d \cdot c \cdot T_1 \]

\[ \Delta T = \frac{Q}{\rho \cdot c} \left( \frac{1}{d} - \frac{1}{t} \right) \]

\[ \Delta T = T_1 - T_2 \]

\[ Q = q / A \]
\[
\Delta T = \frac{Q}{\rho \cdot c} \left( \frac{1}{d} - \frac{1}{t} \right)
\]

1. THE CONTRAST (\(\Delta T\)) INCREASES LINEARLY WITH THE AMOUNT OF DEPOSITED ENERGY PER UNIT AREA (Q).

2. THE HIGHER THE SPECIFIC HEAT-DENSITY OF A MATERIAL (\(\rho c \uparrow\)) THE SMALLER THE PEAK CONTRAST (\(\Delta T \downarrow\)).

3. THE CLOSER THE DEFECT TO THE SURFACE (d \(\to\) 0) THE HIGHER THE PEAK CONTRAST (\(\Delta T \to \infty\)).

4. AS THE DEFECT DEPTH APPROACHES THE PANEL THICKNESS (d \(\to\) t) THE CONTRAST VANISHES (\(\Delta T \to 0\)).

5. FOR A GIVEN DEFECT DEPTH D, THE THICKER THE PANEL (t \(\to\) \(\infty\)) THE LARGER THE CONTRAST (\(\Delta T \to Q/\rho cd\)).
SIMPLE MODEL CORRELATION
(no lateral heat flow)

**CONTRAST vs DEPTH**

\[ \Delta T = \frac{Q}{\rho \cdot c \cdot \left( \frac{1}{d} - \frac{1}{t} \right)} \]

**DEPTH OF RESOLUTION vs ENERGY**

- 80% Mass Loss
- 52% Mass Loss
- 35% Mass Loss
- Camera Resolution

Contrast (a.u.) vs Defect Depth (mil)

Peak Contrast (a.u.) vs Lamp Energy (a.u.)
\[
\rho \cdot A_1 \cdot P \cdot c \cdot \frac{dT_1}{dt} = k \cdot A_1 (T_1' - T_1) + k_L \cdot A_p (T_2 - T_1)
\]

\[
\rho \cdot A_2 \cdot P \cdot c \cdot \frac{dT_2}{dt} = k \cdot A_2 (T_2' - T_2) + k_L \cdot A_p (T_1 - T_2)
\]

\[
\rho \cdot A_2 \cdot h \cdot c \cdot \frac{dT_2'}{dt} = k \cdot A_2 (T_2' - T_2'')
\]

\[k = \text{Effective Contact Normal Thermal Conductivity}\]

\[k_L = \text{Effective Contact Lateral Thermal Conductivity}\]
MODEL ASSUMPTIONS

- The energy "Q" is absorbed by a thin layer of thickness "p". The expressions derived in this work are derived in the limit when "p → 0"

- No energy is dissipated radiatively or convectively to the surrounding environment.

- The conductance "K" between elements can has been expressed as "K = k A/l". The lateral and normal conductivities are assumed to be different.
LATERAL HEAT FLOW EFFECTS
(effective contact conductivity model)

\[ \Delta T(t) = \frac{Q}{\rho c \cdot d \cdot (1 - a + r)} \left( e^{-\frac{a \cdot k}{d \cdot \rho c} t} - e^{-\frac{1 + r \cdot k}{d \cdot \rho c} t} \right) \]

\[ t_{\text{peak}} = \frac{\rho c \cdot d}{k \cdot (1 - a + r)} \ln \frac{1 + r}{a} \]

\[ \Delta T_{\text{peak}} = \frac{Q \left( \frac{1}{d} - \frac{1}{t_o} \right)}{\rho c \left( \frac{1}{d} - \frac{1}{t_o} \right)} \cdot \left\{ \frac{t_o}{a \cdot h \left[ \frac{a \cdot h}{t_o} \right]^\frac{1}{1 - \frac{a \cdot h}{t_o}}} \right\} \]

\[ a = \frac{k_L \cdot A_L}{k_n \cdot A_n} \]

\[ h = t - d \]

\[ r = \frac{d}{t - d} \]

LATERAL HEAT FACTOR
Fit of Contrasts Curves

\begin{align*}
\Delta T(t) &= \frac{Q}{\rho c \cdot d \cdot (1 - a + r)} \left( e^{\frac{a \cdot k}{d \cdot pc}} - e^{\frac{1+r \cdot k}{d \cdot pc}} \right) \\
\Delta T_{\text{peak}} &= \frac{Q}{\rho c \left( \frac{1}{d} - \frac{1}{t_o} \right)} \left\{ \frac{t_o}{a \cdot h} \left[ \frac{a \cdot h}{t_o} \right] \left( \frac{1}{1 - \frac{a \cdot h}{t_o}} \right) \right\}
\end{align*}
LATERAL HEAT FACTOR
(effective contact conductivity model)

Lateral Heat Factor

\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c} \left( \frac{1}{d} - \frac{1}{t_o} \right) \left\{ \frac{t_o}{a \cdot h} \left[ \frac{a \cdot h}{t_o} \right]^{\frac{1}{1 - \frac{a \cdot h}{t_o}}} \right\} \]
\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c} \left( \frac{1}{d} - \frac{1}{t_o} \right) \left\{ \frac{t_o}{a \cdot h} \left[ \frac{a \cdot h}{t_o} \right]^{1 - \frac{a \cdot h}{t_o}} \right\} \]

\[
a = \frac{k_L \cdot A_L}{k_n \cdot A_n}
\]

\[
h = t - d
\]

1. THE CONTRAST (\(\Delta T\)) INCREASES LINEARLY WITH THE AMOUNT OF DEPOSITED ENERGY PER UNIT AREA (Q).

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LATERAL HEAT FLOW MODEL
(specific thermal conductivity)

\[ \rho \cdot A_1 \cdot p \cdot c \cdot \frac{dT_1}{dt} = k \cdot \frac{A_1}{p + d} (T'_1 - T_1) + k_L \cdot \frac{A_p}{R} (T_2 - T_1) \]

\[ \rho \cdot A_2 \cdot p \cdot c \cdot \frac{dT_2}{dt} = k \cdot \frac{A_2}{p + d} (T'_2 - T_2) + k_L \cdot \frac{A_p}{R} (T_1 - T_2) \]

\[ \vdots \]

\[ \rho \cdot A_2 \cdot h \cdot c \cdot \frac{dT_2''}{dt} = k \cdot \frac{A_2}{h + d} (T'_2 - T_2'') \]

\[ k = \text{Thermal Conductivity} \]

\[ k_L = \text{Lateral Thermal Conductivity} \]
SPECIFIC THERMAL CONDUCTIVITY

\[ K = \frac{k \cdot A}{l} \]

\[ \Delta T(t) = \frac{Q}{\rho c \cdot t_0 (d - a \cdot h)} \left( e^{-\frac{a \cdot k}{\rho c d^2 t}} - e^{-\frac{d \cdot k}{\rho c d^2 t}} \right) \]

\[ t_{\text{peak}} = \frac{\rho c}{k} d^2 \frac{h}{a \cdot h - d} \ln \frac{a \cdot h}{d} \]

\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c \left( \frac{1}{d} - \frac{1}{t_0} \right)} \left\{ \frac{d}{a \cdot h} \left[ \frac{a \cdot h}{d} \right]^{-1} \frac{1}{1 - \frac{a \cdot h}{d}} \right\} \]

\[ a = \frac{k_L \cdot A_L \cdot d}{k_n \cdot A_n \cdot R} \]

EFFECTIVE CONTACT CONDUCTIVITY

\[ K = k \cdot A \]

\[ \Delta T(t) = \frac{Q}{\rho c \cdot d \cdot (1 - a + r)} \left( e^{-\frac{a \cdot k}{d \rho c t}} - e^{-\frac{1 + r \cdot k}{d \rho c t}} \right) \]

\[ t_{\text{peak}} = \frac{\rho c}{k} \frac{d}{1 - a + r} \ln \frac{1 + r}{a} \]

\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c \left( \frac{1}{d} - \frac{1}{t_0} \right)} \left\{ \frac{t_0}{a \cdot h} \left[ \frac{a \cdot h}{t_0} \right]^{-1} \frac{1}{1 - \frac{a \cdot h}{t_0}} \right\} \]

\[ a = \frac{k_L \cdot A_L}{k_n \cdot A_n} \]
EXPERIMENTAL DATA
(80% mass removal)

1/2" → 80%
80% 60% 40% 20%
1" →
3/4" →
\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c} \left( \frac{1}{d} - \frac{1}{t_o} \right) \left\{ \frac{d}{a \cdot h} \left[ \frac{a \cdot h}{d} \right]^{1-a \cdot h/d} \right\} \]

Effects of Radii

- Fit 1.00 inch Dia. Hole
- Fit 0.75 inch Dia. Hole
- Fit 0.50 inch Dia. Hole
- Data 1.00 inch Dia Hole
- Data 0.75 inch Dia Hole
- Data 0.50 inch Dia Hole

Contrast Temp. (a.u.) vs. Flaw Depth (mil)
MODEL TIME-RESPONSE PREDICTIONS
(varying defect sizes and locations)

Dia = 1.00"

Dia = 0.75"

Dia = 0.50"
\[ \Delta T_{\text{peak}} = \frac{Q}{\rho c \left( \frac{1}{d} - \frac{1}{t_o} \right)} \cdot \left\{ \frac{d}{a \cdot h} \left[ \frac{a \cdot h}{d} \right] \cdot \frac{1}{1 - \frac{a \cdot h}{d}} \right\} \]
SUMMARY AND CONCLUSIONS

- Calorimetric model was developed to predict thermal contrast.

- Model accounts for defect size, location, and lateral conductivity effects.

- Calorimetric model correlates well with experimental results.

- Anisotropic thermal conductivity can be modeled.

- Model accuracy should improve as the element mesh is refined.