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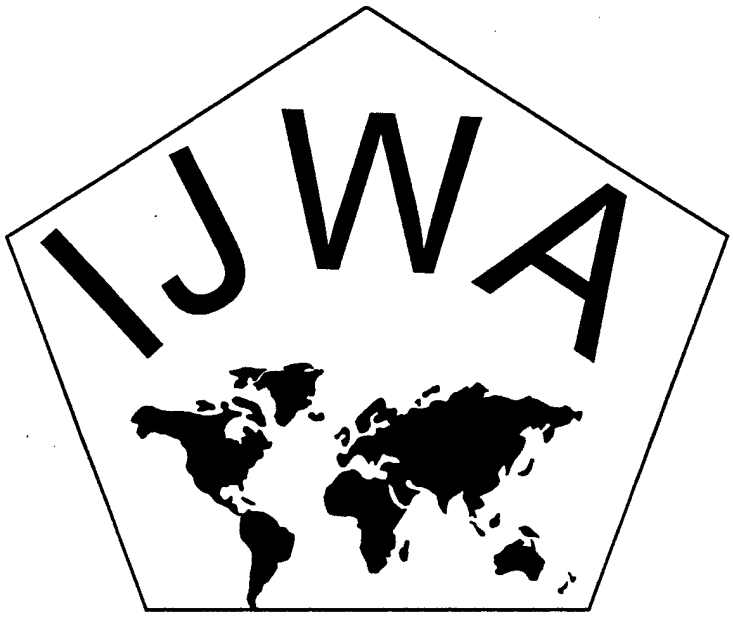
ANALYTICAL MODELS FOR MOBILE SENSOR (UAV) COVERAGE OF A REGION



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Summary

The effective use of mobile sensors (UAVs for example, but not exclusively) to investigate own and opponent force status in geographical regions requires planning to compensate for their limitations. These limitations include *finite endurance*, and realistic *mission unreliability*: the failure propensities of platform and sensing packages, and the consequent need for maintenance and logistics support.

This paper supplies *analytical* (*mathematical*) *models* (AMM) to assist planners, and the acquirers and operational testers of mobile sensor assets. The results are formulas (that can be quickly evaluated numerically) for the *expected time on station* (fraction of time a region has sensor presence) of several cooperatively operating systems supported by several maintenance facilities. These results can be used to estimate the number of sensors of a particular type needed to cover a region adequately.

Limited evidence indicates usefully satisfactory agreement between analytical model estimates and those from more detailed and elaborate Monte Carlo simulations (MCS).

Even with modern fast computers the exercise of AMMs is more rapid than of MCSs, so initial exploratory investigation by AMMs is indicated.

1. A Generic Situation

A platform carrying sensing equipment, e.g. JSTARS, AWACS, or an RPV/UAV is used to carry out surveillance of a region. The platform has finite endurance: it requires a time T to transit outbound from home base (airbase on land, Navy carrier) to the target region, then can remain on-station over that region, performing surveillance, for time S, and finally transit back again for time T. Once back on base it spends a time D for refueling, maintenance, etc.

Often several such platforms must operate cooperatively in order to achieve adequate coverage of a relatively distant region. These may be serviced at home base by several maintenance facilities; in some cases there are fewer maintenance "paths" than platforms, in which case queues can form and the coverage correspondingly degenerates. We provide models that describe and predict the long-run consequences in such cases.

1.1 A Basic Formula: Consider that a *cycle* begins each time the UAV takes off from its home base, and ends when it is again ready to do so. Let $Y(t) = 1$ if the platform is on station and $Y(t) = 0$ otherwise. $\{Y(t);\ t \geq 0\}$ is a degenerate alternating renewal process. The long-run proportion of time the platform is on station if no systems fail is seen to be

$$\lim_{s\to\infty} \frac{1}{s}\int_0^s Y(t)dt = \pi = \frac{S}{2T+S+D}. \tag{1.1}$$

However, it is realistic, and essential, to account for failures during the mission, and their effect on time on station. Failures, and their repair times, make the on-station availability predicted by (1.1) degenerate, and our analytical models efficiently show how this occurs, although approximately. The Monte Carlo simulations of Pendergast and Stoneman (1998) are potentially more versatile and accurate, but require more setup and

computer time. The analytical models (here) are convenient and adequate for explorations.

2. Single System Models

VARIATION 1: Assume mission-affecting failures (MAF) occur according to a Poisson process with rate λ. Once a single MAF occurs, the UAV immediately returns to base (crashes are not modeled here). If additional failures occur when returning to base, these influence the total repair time. Model the repair times as independent and random, with mean $1/\mu$ and let C be the duration of a cycle: total time from one takeoff to the next.

Let X be the time until failure. Let A be a random on-station time, and let B be the time from platform take-off until return to base, then it is shown in Appendix I that

$$\pi = \frac{E[A]}{E[C]} = \frac{e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]}{\frac{2}{\lambda}\left[1-e^{-\lambda T}\right]+e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]+D+\left[1-e^{-(2T+S)}\right]\frac{1}{\mu}} \tag{2.1}$$

VARIATION 2: Suppose the need for unscheduled maintenance activities (UMA) occur according to a Poisson process with rate α_U during the platform flight. These (UMA) do not shorten the flight but each requires a random time to perform with mean $1/\beta_U$ hours. In addition there is one Administrative logistic down time (ALDT) with mean $1/\beta_A$. Assume that scheduled maintenance activities (SMA) occur at a rate of α_S per flight hour. Each scheduled activity requires $1/\beta_S$ hours. Model $E[D]$ as follows

$$E[D] = \underbrace{\alpha_U E[B]}_{\substack{\text{expected number of UMA's to occur in a cycle}}} \frac{1}{\beta_U} + \underbrace{\alpha_S E[B]}_{\substack{\text{approx. number of SMA's to occur in a cycle}}} \frac{1}{\beta_S} + \frac{1}{\beta_A}$$

Numerical Example.

Assume MTBMAF=25 hours. There is scheduled maintenance of 7 hours at 50 hr. intervals. UMAs occur at a rate of $\lambda = 1/5$ per hour and each requires $1/\beta_U = 1.9$ hours of maintenance; the mean ALDT time $1/\beta_A = 0.5$. The mean repair time for a MAF is 1.9 hours +0.5 ALDT. The total time the platform can be in the air is 20 hours. T is the ingress/egress time.

Fraction of Time at Least One UAV is On Station

ETOS [Estimated Time on Station]

T	Analytic ETOS	Simulation ETOS (95% Confidence Interval)
1	0.52	0.5200 – 0.5332
2	0.45	0.4389 – 0.4511
3	0.38	0.3697 – 0.3849
4	0.32	0.3121 – 0.3284
5	0.26	0.2526 – 0.2667
6	0.21	0.1978 – 0.2094

3. Many-System Models: Ample Service

VARIATION 3: Assume there are N platforms. Assume there is ample service capacity, here $N \leq s$, the number of maintenance paths or servers so that no platform needs to wait for maintenance or repair. The platforms are launched independently of one another when they are available; a pessimistic assumption. Then an approximation for the long-run proportion of time the region is under surveillance by at least one platform is $1 - (1 - \pi)^N$. For the parameters of the previous numerical example, wherein $N = 4$, the long-run proportion of time the region is under surveillance by at least one platform is

Fraction of Time at Least One UAV is On Station

T	Analytical Model Fraction of Time	95% Confidence Interval
1	0.95	0.9565 – 0.9642
2	0.91	0.9161 – 0.9247
3	0.85	0.8694 – 0.8946
4	0.79	0.8279 – 0.8451
5	0.70	0.7535 – 0.7949
6	0.60	0.6921 – 0.7093

The simulation results are from the MASS model; Pendergast and Stoneman (1998).

The analytical model gives smaller fractions of time the region is covered. This behavior is because the simulation includes scheduling of the UAVs, which the analytical model does not include.

4. A Model With Limited Service: Markovian Approximation

Suppose there are s maintenance paths: *servers* in queuing theory parlance. Let the times between maintenance/repair for a UAV be iid exponential with mean $1/\gamma$, and assume the times for maintenance/repair are iid exponential with $1/\rho$. Let $X(t)$ be the number of UAVs down (receiving or awaiting maitenance/repair) at time t.

Model: $\{X(t); t > 0\}$ is a continuous time Markov chain with limiting distribution

$$\psi(x) = \lim_{t\to\infty} P\{X(t) = x\} = \prod_{i=0}^{x-1} \frac{(N-i)\gamma}{\min(i+1, s)\rho} \psi(0) \tag{4.1}$$

for $x = 1,\ldots, N$ where N is the number of UAVs. $\psi(0)$ is found by solving

$$1 = \sum_{i=0}^{N} \psi(i) \tag{4.2}$$

Let $\pi_U = \dfrac{E[A]}{E[B]}$ be an approximation for the fraction of time a flying UAV is on station. Model the long-run fraction of time the region is covered by a UAV by

$$\pi_S = \sum_{x=0}^{N} \left[1-(1-\pi_U)^{N-x}\right]\psi(x) \tag{4.3}$$

Numerical Example

Take $1/\gamma = E[B]$ and $1/\rho = E[D] + \left(\frac{1}{\lambda}\Big/\frac{1}{\lambda} + \frac{1}{2T+S}\right)\frac{1}{\mu}$. For the parameter values of the previous numerical example with 4 UAVs, $T = 3$ and a variable number of servers:

Fraction of Time at Least One UAV is On Station

Number of Servers	Model Fraction of Time Area Covered	Simulation 95% Confidence Interval
1	0.62	0.8299 – 0.8601
2	0.80	0.8754 – 0.8947
3	0.85	0.8713 – 0.8951
4	0.85	0.8750 – 0.8891

If there are 4 UAVs , one maintenance path and variable ingress/egress time

Fraction of Time at Least One UAV is On Station

T	Model Fraction of Time Area Covered	Simulation 95% Confidence Interval
1	0.73	0.9529 – 0.9604
2	0.68	0.8977 – 0.9194
3	0.62	0.8299 – 0.8601
4	0.55	0.7678 – 0.7916
5	0.48	0.6540 – 0.6836
6	0.40	0.5230 – 0.5493

5. A Model with Scheduling: Replacement UAVs Always Available

In this section we describe a model that accounts for scheduling the UAVs so that one is always on station. Initially assume that there are always UAVs to replace on-station UAVs. Let T be the ingress/egress time and S be the time on station. We assume $2T < S$.

Assume a UAV has just arrived on station at time 0. Its replacement must take off at time $S - T$ in order to relieve it when the first UAV starts its trip back to base. The egress time is T.

Assume a UAV experiences a mission affecting failure after an exponential length of time having mean $1/\lambda$.

Assume a replacement UAV is launched T time units before the on-station UAV is to start to return to base or upon failure of the on-station UAV whichever comes first. Let L be the length of time a replacement UAV is late to the region; during this time the region is not covered.

$$E[L] = \underbrace{e^{-\lambda T}}_{\substack{\text{replacement} \\ \text{does not fail} \\ \text{on ingress}}} 0 + \underbrace{\int_0^T \lambda e^{-\lambda x}}_{\text{replacement fails}} \left(x + \underbrace{E[L]}_{\substack{\text{new} \\ \text{replacement} \\ \text{launched}}} \right) dx \tag{5.1}$$

$$= \frac{1}{\lambda}\left[1 - e^{-\lambda T} - (\lambda T)e^{-\lambda T}\right] + \left(1 - e^{-\lambda T}\right)E[L].$$

Thus,

$$E[L] = \frac{\frac{1}{\lambda}\left[1 - e^{-\lambda T} - (\lambda T)e^{-\lambda T}\right]}{e^{-\lambda T}} = \frac{1}{\lambda}\left[e^{-\lambda T} - 1 - \lambda T\right]. \tag{5.2}$$

Notice that the expected lateness increases exponentially with the transit time to station, which argues for getting closer to the region.

Consider a *cycle* to start when a new UAV arrives on station. The expected length of a cycle is

$$E[C] = \underbrace{e^{-\lambda S}}_{\substack{\text{on-station} \\ \text{UAV does not} \\ \text{fail on station}}} \left[S + E[L]\right] + \int_0^{S-T} \underbrace{\lambda e^{-\lambda x} dx}_{\substack{\text{on-station UAV} \\ \text{fails before} \\ \text{replacement} \\ \text{is launched}}} \left[x + E[L] + T\right] + \underbrace{\int_{S-T}^{S} \lambda e^{-\lambda x} dx}_{\substack{\text{on-station UAV fails} \\ \text{after replacement} \\ \text{is launched}}} \left[S + E[L]\right] \tag{5.3}$$

$$= E[L] + e^{-\lambda(S-T)}S + \frac{1}{\lambda}\left[1 - e^{-\lambda(S-T)} - (\lambda(S-T))e^{-\lambda(S-T)}\right] + T\left[1 - e^{-\lambda(S-T)}\right] \tag{5.4}$$

$$= E[L] + \frac{1}{\lambda}\left[1 - e^{-\lambda(S-T)} - \lambda(S-T)e^{-\lambda(S-T)}\right] + e^{-\lambda(S-T)}S + T\left[1 - e^{-\lambda(S-T)}\right]. \tag{5.5}$$

Let V be time the region is not covered during a cycle.

$$E[V] = (T + E[L])\underbrace{\left(1 - e^{-\lambda(S-T)}\right)}_{\substack{\text{probability on-station} \\ \text{UAV fails before} \\ \text{replacement takes off}}} + \underbrace{e^{-\lambda(S-T)} \int_0^T \lambda e^{-\lambda x} [T - x + E[L]] dx}_{\substack{\text{on-station UAV fails} \\ \text{after replacement takes off}}}$$

$$= (T + E[L])\left(1 - e^{-\lambda(S-T)}\right) \tag{5.6}$$

$$+ e^{-\lambda(S-T)} \left\{ [T + E[L]]\left[1 - e^{-\lambda T}\right] - \frac{1}{\lambda}\left[1 - e^{-\lambda T} - \lambda T e^{-\lambda T}\right] \right\}.$$

The long run fraction of time the region is not covered is

$$\alpha = \frac{E[V]}{E[C]}. \tag{5.7}$$

6. A Model with Scheduling: Limited Repairs

Let X_I be the number of failures during a cycle for the ingressing UAV. If there is always a replacement UAV available, then

$$E[X_I] = \left(1 - e^{-\lambda T}\right) + \left(1 - e^{-\lambda T}\right) E[X_I] \tag{6.1}$$

$$E[X_I] = \frac{\left(1 - e^{-\lambda T}\right)}{e^{-\lambda T}} = e^{\lambda T} - 1. \tag{6.2}$$

Let X_O be the number of failures for the UAV on station.

$$E[X_O] = \left(1 - e^{-\lambda(S+T)}\right) \tag{6.3}$$

where T is the egress time.

The long run average failure rate is

$$\lambda_0 = \frac{E[X_O] + E[X_I]}{E[C]}. \tag{6.4}$$

Consider a queue with arrival rate λ_0 and a maximum number of customers waiting or being served equal to N, the total number of UAVs and s servers (maintenance paths,

each with service rate ρ). Let $\phi(x)$ be the long run proportion of time there are x UAVs either waiting for or being repaired.

$$\phi(x) = \prod_{i=0}^{x-1} \frac{\lambda_0}{\min(i+1, s)\rho} \phi(0)$$

where $\phi(0)$ is determined so that the sum is one and N is the total number of UAVs.

Approximate the long run fraction of time during which there is at least one UAV on station as

$$\alpha_s(1) = \underbrace{(1-\phi(N))}_{\substack{\text{long-run fraction of} \\ \text{time at least one UAV} \\ \text{is up from queuing model}}} \alpha \quad \leftarrow \text{From (5.7)}$$

or

$$\alpha_s(2) = \underbrace{(1-[\phi(N)+\phi(N-1)])}_{\substack{\text{long-run fraction of time at least 2 UAVs} \\ \text{are not waiting for or being repaired}}} \alpha$$

and α is the long run fraction of time there is a UAV on station for the schedule model of Section 5.

Numerical Example

Take $\frac{1}{\rho} = E[D] + \left(\frac{1}{\lambda} \Big/ \left(\frac{1}{\lambda} + \frac{1}{2T+S}\right)\right)\frac{1}{\mu}$. Suppose there are 4 UAVs, one maintenance path and variable ingress/egress time. For the parameter values of the previous examples

Fraction of Time at Least One UAV is On Station

T	Simulation 95% Confidence Interval	Schedule Model UAV always available	Schedule Model times Probability at least one UAV is available	Schedule Model time Probability at least two UAVs are available
1	0.95 – 0.96	0.96	0.95	0.91
2	0.90 – 0.92	0.91	0.90	0.86
3	0.83 – 0.86	0.87	0.85	0.80
4	0.77 – 0.79	0.82	0.80	0.75
5	0.65 – 0.68	0.76	0.73	0.64
6	0.52 – 0.55	0.71	0.63	0.52

The simulation results are from Stoneman (1998).

REFERENCES

F.P. Kelly, *Reversibility and Stochastic Networks*, John Wiley and Sons, New York, 1979.

K.D. Pendergast and J. Stoneman, "Military Aircraft Suitability Simulation (MASS)," Presentation Slides, Institute for Defense Analysis, 1998.

J.A. Post and C.W. Warner, "A Discrete-Event Simulation of Predator UAV Target Area Presence," Presentation Slides, Institute for Defense Analysis, January 6, 1997.

APPENDIX I

B is the time between takeoff and return to base. Then

$$B=\begin{cases}2X & \text{if } X<T \text{ (fails on flight out)}\\ X+\mathrm{T} & \text{if } T<X<T+S \text{ (fails during surveillance time)}\\ 2T+S & \text{if } X>T+S \text{ (fails on flight back or does not fail)}\end{cases} \tag{AI.1}$$

Thus,

$$\begin{aligned}E[B]&=\int_0^T 2s\lambda e^{-\lambda s}ds\\ &\quad+e^{-\lambda T}\int_0^S (s+2T)\lambda e^{-\lambda s}ds\\ &\quad+e^{-\lambda(T+S)}(2T+S)\\ &=\frac{2}{\lambda}\left[1-e^{-\lambda T}\right]+e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]\end{aligned} \tag{AI.2}$$

Let A be the time on station

$$A=\begin{cases}0 & \text{if } X<T \text{ (Platform fails on ingress)}\\ X-T & \text{if } T<X<T+S \text{ (Platform fails during surveillance)}\\ S & \text{if } X>T+S\end{cases} \tag{AI.3}$$

Thus,

$$\begin{aligned}E[A]&=0\left[1-e^{-\lambda T}\right]+e^{-\lambda T}\int_0^S s\lambda e^{-\lambda s}ds+e^{-\lambda(T+S)}S\\ &=e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]\end{aligned} \tag{AI.4}$$

The expected length of a cycle is

$$E[C]=E[B]+D+\underbrace{\left(1-e^{-\lambda(2T+S)}\right)}_{\substack{\text{prob of failure}\\ \text{during cycle}}}\ \underbrace{\frac{1}{\mu}}_{\substack{\text{expected}\\ \text{repair time}}} \tag{AI.5}$$

The long-run proportion of time the platform is performing surveillance:

$$\pi = \frac{E[A]}{E[C]}$$

$$= \frac{e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]}{\frac{2}{\lambda}\left[1-e^{-\lambda T}\right]+e^{-\lambda T}\frac{1}{\lambda}\left[1-e^{-\lambda S}\right]+D+\left[1-e^{-(2T+S)}\right]\frac{1}{\mu}} \qquad \text{(AI.6)}$$

APPENDIX II

A. Closed Queueing Network Model

Consider a closed queuing network with 4 "service centers" or states labeled as follows:

	Center	Number of Servers
D	Down	s (number of maintenance paths)
I	Ingress	infinite
S	On Station	infinite
E	Egress	infinite

Consider a closed migration process in the sense of Kelly (1979). The state of the process is $\{(X_D(t), X_I(t), X_S(t), X_E(t)); t > 0\}$ where $X_D(t)$ is the number of platforms down at time t; $X_I(t)$ is the number traveling to the region at time t; $X_S(t)$ is the number on station at time t; and $X_E(t)$ is the number traveling from the region at time t. Assume $\{(X_D(t), X_I(t), X_S(t), X_E(t)); t > 0\}$; is a continuous time Markov chain with

$$P\{(X_i(t+h) = x_i - 1, X_j(t+h) = x_j + 1, X_k(t+h) = x_k \text{for } k \neq i, j | X_\ell(t) = x_\ell \text{ for all } \ell\}$$

$$= \lambda_{ij}\varphi_i(x_i)h + o(h)$$

Put

$$\lambda_{DI} = E[D] + \left(1 - e^{-\lambda(2T+S)}\right)\frac{1}{\mu}$$

$$\lambda_{Dj} = 0 \text{ for } j = S,\ E$$

$$\varphi_D(x_D) = \min(x_D, s)$$

$$\lambda_{ID} = \lambda$$

$$\lambda_{IS} = \frac{1}{T}$$

$$\lambda_{Ij} = 0 \text{ for } j = E$$

$$\varphi_I(x_I) = x_I$$

$$\lambda_{SE} = E[A]$$

$$\lambda_{Sj} = 0 \text{ for } j = D,\ I$$

$$\varphi_S(x_S) = x_S$$

$$\lambda_{ED} = \frac{1}{T}$$

$$\lambda_{Ej} = 0 \text{ for } j = I,\ S$$

$$\varphi_E(x_E) = x_E$$

Theorem 2.3 of Kelly (1979) states that

$$\lim_{t\to\infty} P\left\{\left(X_D(t), X_I(t), X_S(t), X_E(t)\right) = (x_D, x_I, x_S, x_E)\right\}$$

$$= K\prod_{j\in Q}\frac{\xi_j^{x_j}}{\prod_{r=1}^{x_j}\varphi_j(r)}$$

where K is a normalizing constant, $Q = \{D, I, S, E\}$ and ξ_j is the unique solution to the system of equations

$$\xi_j \sum_k \lambda_{jk} = \sum_k \xi_k \lambda_{kj} \text{ for } j = D, I, S, E$$

$$\sum_j \xi_j = 1$$

Numerical Example

Suppose there are 4 UAVs; MTBMAF=25; there is one maintenance path; and the other parameters are as before with the ingress/egress time T variable

Long-run proportion of time at least 1 UAV is on station
1 server

T	Limited Service Model	Queuing Network Model	Simulation
1	0.76	0.79	0.93
2	0.70	0.74	0.88
3	0.64	0.68	0.83
4	0.58	0.61	0.75
5	0.50	0.54	0.60
6	0.42	0.45	0.48

If there are 4 servers

Long-run proportion of time at least 1 UAV is on station
4 servers

T	Independent Alternating Renewal Process Model	Limited Service Model	Queuing Network Model	Simulation
1	0.95	0.96	0.96	0.95
2	0.91	0.92	0.93	0.91
3	0.86	0.87	0.88	0.86
4	0.79	0.80	0.82	0.83
5	0.71	0.71	0.74	0.78
6	0.61	0.61	0.64	0.63

In the present case the multi-state queuing model approach appears from the limited tabulation to be *biased low* for a limited number of repair servers (1). This is not conclusive, but in view of the very good results obtained by the simpler Markovian

model of Section 4 it does not now seem desirable to use the present approach. Some further investigation may be worthwhile.

APPENDIX III

Display of Spreadsheet Implementation of Analytic Models

ANALYTICAL MODELS FOR MOBILE SENSOR (UAV) COVERAGE OF A REGION

PARAMETER VALUES	
1/lam:mean time between mission affecting failure: MAF(hours)	25
T: ingress-egress time	3
E: total endurance time	20
alohau: rate of unscheduled maintenance activities (number per hour)	1/5
1/alphas: time between scheduled maintenance activities (hours)	50
1/betau: mean time to repair each unscheduled maintenance (hours)	1.9
1/betas: mean time to repair each scheduled maintenance (hours)	7
1/mu: mean time to repair mission affecting failure	2.4
s: number of maintenance paths	1
N: number of UAVs (max=10)	4
S: time on station if no failures	14
1/betaa:administrative and logistic down time (ALDT)	0.5

MOEs		
Long run proportion of time at least 1 UAV is on station: limited service model	0.617522645	
Long run proportion of time 1 UAV is on station: 1 UAV and alternating renewal process model	0.378147644	
For infinite repair capacity, the fraction of time at least one UAV is on station	0.850462838	

CALCULATED PARAMETERS		
exp model prob of failure during flight	0.444444444	
lam: MAF failure rate	0.04	
prob of MAF during flight	0.550671036	
prob of MAF during ingress	0.113079563	
prob of MAF during surveillance	0.428790936	
expected time of flight	15.16156427	
expected repair time (Alt. Renewal)	9.980959426	9.672902311 (limited service
expected time flying UAV is on station	9.507586109	
proportion of flying time spent on station	0.627084774	
gam: rate of return for repair	0.065956255	

X: number of UAVs down	long run prop of time number of UAV are down
0	0.053634346
1	0.136872366
2	0.261968949
3	0.334266308
4	0.213258031
5	0
6	0
7	0
8	0
9	0
10	0

calc1	calc2	calc3	calc4	calc5	calc6	calc7
1	1	1	18.64476906	4	0.980660713	0.052597096
2.551953647	2.551953647	2.551953647		3	0.948140259	0.1297742
1.913965235	1.913965235	4.884350563		2	0.860934234	0.225538037
1.275976824	1.275976824	6.232318117		1	0.627084774	0.209613312
0.637988412	0.637988412	3.976146737		0	0	0
0	0	0		0	0	0
-0.637988412	0	0		0	0	0
-1.275976824	0	0		0	0	0
-1.913965235	0	0		0	0	0
-2.551953647	0	0		0	0	0
-3.189942059	0	0		0	0	0

E[repair time]:

Model alt. renewal: $E[\text{maintenance time}] + \left(1 - e^{-\lambda E}\right)\frac{1}{\mu}$

Model limited service: $E[\text{maintenance time}] + \frac{\lambda}{\lambda + \frac{1}{E}}\frac{1}{\mu}$