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# Abstract

Pulsing circuits generate electromagnetic interference (EMI) that can affect sensitive circuitry and adversely contribute to the spectral signature of equipment. "Flicker noise" concepts, derived from chaos theory, have been employed to efficiently pulse circuitry while generating a virtually undetectable spectral signature. Pure flicker pulsing requires that the components be driven with a set of uncorrelated pulses, with random heights, starting at random times. However, a significant reduction in conspicuous power spectral density (PSD) components can be achieved when imposing practical constraints. We have been able to significantly reduce the dominant components of the power spectrum using fixed pulse durations and magnitudes. We employed flicker pulsing, with a PSD approaching \((1/f)^2\), to drive our components more efficiently, resulting in a 40% increase in battery life. The contribution of the pulses to the spectral signature of the equipment appears only in the background noise of EMI detectors.
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INTRODUCTION

Pulsing circuits generate electromagnetic interference (EMI) that can affect sensitive circuitry and adversely contribute to the spectral signature of the equipment. "Flicker noise" concepts, derived from chaos theory, have been employed to efficiently pulse circuitry while generating a virtually undetectable spectral signature. Pure flicker pulsing requires that the components be driven with a set of uncorrelated pulses, with random heights, starting at random times. Jensen, et al. (ref 1) demonstrated that the power spectral density (PSD) for such a signal generates $\text{(1/f)}^n$ type noise given by

\begin{equation}
S(f) = \frac{v}{(\pi f)^2} \int_0^\infty d\tau G(\tau) \sin^2(\pi f \tau)
\end{equation}

where $v$ is the pulse rate and $G(\tau)$ is the weighted distribution of lifetimes, defined as

\begin{equation}
G(W) = \int_0^\infty dS P(S,W)[S/W]^2
\end{equation}

where $P(S,W)$ is the joint probability for a pulse to have area $S$ and width $W$.

ELECTRONICS APPLICATION

In most circuitry, $S$, $W$, and the pulse spacing, $T$, are normally fixed, with a duty cycle given by $W/(W+T)$.

The PSD for a periodic signal with $W = 122$ usec, $S = nW = 122$ $\mu$v-sec, and $T = 1.83$ msec is given in Figure 1. When $S$, $W$, and $T$ are allowed to vary, the results are dramatically different. Figure 2 shows the PSD for a signal with $W$, $S$, and $T$ given by uniform probability density functions with mean values:

\begin{align*}
\bar{W} &= 122 \text{ usec}, \quad \bar{S} = n\bar{W} = 122 \text{ } \mu\text{volt-sec}, \quad \text{and} \quad \bar{T} = 1.83 \text{ msec}
\end{align*}

The PSD was obtained using Welch’s method of estimating the power spectrum from the time series data signal using a Hanning window of length 256 and using a sampling frequency of $F_s = 820$ kHz.

The analytic solution can be determined from equation (1) assuming fixed pulse amplitudes and a probability density function $P(W)$ of the form

\begin{equation}
P(W) = \int_0^\infty P(S,W)dS = \frac{1}{(W_{\text{max}} - W_{\text{min}})} = \frac{1}{\Delta W}
\end{equation}
Assuming that all pulses are fixed amplitude, $S = nW$, requires

$$P(S) = \int_0^\infty P(S,W)dW = \frac{1}{n\Delta W}$$

Therefore

$$P(S,W) = \frac{1}{n\Delta w} \delta(W - S/n) \text{ and } G(W) = n^2/\Delta W$$

This results in a PSD given by

$$S(f) = \frac{v}{2} \left( \frac{n}{\pi f} \right)^2 \left(1 - \frac{\cos(\pi f(W_{max} + W_{min})\sin(\pi f\Delta W))}{\pi f\Delta W}\right)$$

and for $W_{min} = 0$, $S(f)$ reduces to

$$S(f) = \frac{v}{2} \left( \frac{n}{\pi f} \right)^2 \left(1 - \text{sinc}(2fW_{max})\right)$$

$S(f)$ corresponding to this pulse distribution, with $v = 512$ pulses/sec and $W_{max} = 244$ usec, is given in Figure 3. The analytic solution is in agreement with the PSD obtained from the time series data and also shows the origin of the $(1/f)^2$ dependence.

In our application, we were required to improve the efficiency of LED drivers, without affecting the spectral signature of the system. Pulsing an LED is an effective means of attaining maximum light intensity with minimum power drain. A rule of thumb for most LEDs is that driving the LED in the 1 to 10 mA range is the most optically efficient mode of operation and pulsing LEDs in this range can more than double the light intensity of continuous operation at the same effective current. We were constrained by a low clock speed, so we fixed $W_f$ to the minimum possible pulse width (122 usec) to achieve the lowest possible duty cycle. We also determined that 1.852 msec was the maximum possible pulse spacing, $T$, without introducing noticeable flicker. This resulted in a duty cycle of 6%. We were also forced to separate $T$ into a fixed component, $t_f$, and a random component, $t_r = T - t_f$, to avoid excessive flicker. The random component, $t_r$, was given by a uniform probability density function with a mean value of $\tilde{t_r}$.

The largest ratio $t_r / T$ that provided a uniform light intensity, given these constraints was 46%. The PSD for a signal given these constraints is given in Figure 1 and is shown on an expanded scale in Figure 4. Although both signals in Figure 1 have the same total energy, the PSD of the flicker pulsed signal is reduced to background noise. The analytic solution, $S(f)$, for a signal with $W$ fixed at $W_f$ and $S$ fixed at $S_f$, with $n = 1$ and $v = 512$ pulses/sec is shown in Figure 5. In this case, the joint probability density function becomes

$$P(S,W) = \delta(S - S_f) \delta(W - W_f)$$
so that equations (2) and (1), respectively, reduce to

\[ G(W) = \left[ \frac{S_f}{W} \right]^2 \delta(W - W_f) \]  

(7)

and

\[ S(f) = \nu(nW_f)^2 \text{sinc}^2(fW_f) \]  

(8)

The analytic results agree well with those in Figure 4 and demonstrate that fixing \( W \) and \( S \) and constraining \( T_a \) had a negligible effect on the PSD. The degree to which \( T \) must vary to obtain the desired power at the fundamental frequency, \( f_0 \), is shown in Figure 6. The figure shows the power at the fundamental frequency, \( f_0 \), relative to the power at \( f_0 \) for fixed \( T \), as a function of \( \bar{t}_r / T \). It shows that an 80% reduction in the PSD at \( f_0 \) can be obtained with a \( \bar{t}_r / T \) ratio of only 20%. A \( \bar{t}_r / T \) ratio of 46% reduces the fundamental peak by more than 90%.

CONCLUSION

Flicker pulsing can significantly reduce the spectral signature of the circuitry. Although ideally the pulse durations, magnitudes, and spacings should be random, a significant reduction in conspicuous PSD components can be achieved when imposing practical constraints. We were able to significantly reduce the dominant components of the power spectrum using fixed pulse durations and magnitudes. We employed flicker pulsing, with a PSD approaching \((1/f)^2\), to drive our components more efficiently, resulting in a 40% increase in battery life. The contribution of the pulses to the spectral signature of the equipment appears only in the background noise of EMI detectors.
REFERENCES

Figure 1. Power spectral density for periodic and flicker pulsing (volts$^2$-sec vs. sec$^{-1}$).
Figure 2. Power spectral density for $P(S,W) = \frac{1}{n\Delta w} \delta(W - S/n) \ (dB \ vs. \ log \ (sec^{-1}))$. 
Figure 3. \( S(f) = \frac{v}{2} \left( \frac{n}{\pi f} \right)^2 (1 - \text{sinc}(2fW_{\text{max}})) \) (dB vs. log ( sec' )).
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