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Buckling of Ship Grillages - Part II

by

D. A. Danielson
D. P. Kihl

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The report was prepared by:

Donald Danielson
Professor
Department of Mathematics

Reviewed and Released by:

W. Max Woods
Chairman
Mathematics Department

David W. Netzer
Associate Provost and Dean of Research
**Title and Subtitle**
BUCKLING OF SHIP GRILLAGES - PART II

**Authors**
- D. A. Danielson
- D. P. Kihl

**Performing Organization Name(s) and Address(es)**
- Naval Postgraduate School
  - Code MA/DI
  - Monterey, CA 93943
- Naval Surface Warfare Center
  - Code 653
  - Carderock Division
  - Bethesda, MD 20084-5000

**Sponsoring/Monitoring Agency Name(s) and Address(es)**
- Naval Surface Warfare Center
  - Carderock Division
  - Washington, D.C. 20084-5000
- Naval Postgraduate School
  - Monterey, CA 93943

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**Abstract**
The subject of this report is the mechanical behavior of stiffened plates, basic structural components of ships and submarines. The buckling loads of grillages subjected to axial compression with and without lateral pressure are calculated using a finite element based eigenvalue analysis.
BUCKLING OF SHIP GRILLAGES - PART II

D. A. Danielson
Code MA/Dd
Naval Postgraduate School
Monterey, CA 93943

D. P. Kihl
Code 653
Naval Surface Warfare Center, Carderock Division
Bethesda, MD 20084-5000

August 1997

ABSTRACT

The subject of this report is the mechanical behavior of stiffened plates, basic structural components of ships and submarines. The buckling loads and modes of grillages subjected to axial compression without and with lateral pressure are calculated using finite element based eigenvalue analyses.
INTRODUCTION

Stiffened plates are basic structural components of ships and submarines. In our earlier report*, we have calculated the axial buckling loads of grillages with the use of a well-known finite element code. In the present work, we continue the calculation of buckling loads for new grillages.

The grillages now modeled consist of 3 base plates, 2 longitudinal edge plates, 4 longitudinal stiffeners, and 2 transverse stiffeners. Two sets of grillages (I and II) having different plating thicknesses and stiffener dimensions are considered (Figures 1, 2 and 12, 13). We define the following grillage dimensions, of which the first 4 are common to all grillages:

\[\begin{align*}
a_1 &= \text{length between transverse stiffeners} = 96'' \\
a_2 &= \text{length between transverse stiffener and grillage end} = 96'' \\
b_1 &= \text{width between longitudinal stiffeners} = 27'' \\
b_2 &= \text{width between outer longitudinal stiffener and grillage side} = 7.5'' \\
b_3 &= \text{width of longitudinal edge plates} \\
t_1 &= \text{thickness of inner base plate and longitudinal edge plates} \\
t_2 &= \text{thickness of outer base plates} \\
dw_1 &= \text{depth of longitudinal webs} \\
tw_1 &= \text{thickness of longitudinal webs} \\
df_1 &= \text{depth of longitudinal flanges} \\
tf_1 &= \text{thickness of longitudinal flanges} \\
dw_2 &= \text{depth of transverse webs} \\
tw_2 &= \text{thickness of transverse webs} \\
df_2 &= \text{depth of transverse flanges} \\
tf_2 &= \text{thickness of transverse flanges}
\end{align*}\]

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In these grillages, the outer base plates also have thickness t1 in regions of width 6" adjacent to the transverse stiffeners. The material is isotropic steel with Young’s modulus $E = 3 \times 10^7$ psi and Poisson’s ratio $\nu = .3$. For grillages I, the base plates and longitudinal edge plates are HSLA steel with yield stress 85 ksi, whereas the longitudinal and transverse stiffeners are HS steel with yield stress 60 ksi. For grillages II, the steel types are interchanged: the plates are HS steel, whereas the stiffeners are HSLA steel.

All grillages have the same imposed boundary conditions:

1. One end of a grillage has all 3 displacement components zero and all 3 rotation components zero.
2. The other end of the grillage where the force is applied has uniform axial displacement with the other 2 displacement components zero and all 3 rotation components zero.
3. The ends of the transverse stiffeners have vertical displacement zero but the other 2 displacement components and all 3 rotation components are free.
4. For the pressure cases, the longitudinal edge plates are also restrained vertically at 10.5" intervals along the outer sides.
5. All other nodes on the plates edges are completely free.

To each of these grillages we apply compressive axial loading together with one of 3 normal loads: no pressure, an initial pressure loading of 10 psi (simulating fluid on the plating side), or an initial pressure loading of -10 psi (simulating fluid on the stiffener side). Furthermore, to assess the effects of the longitudinal edge plates, we consider both uncut and cut edge plates. This makes a total of 12 different cases to study.

The finite element code used is MSC Nastran/Patran. The pre-processing and post-processing is done with Patran (version 6.2). The buckling analysis (solution 105) is done with Nastran (version 69). The meshing is done entirely with Quad 4 plate elements. Note that this is a full finite element model of the entire grillage and does not assume symmetry. We use linear geometry and linear material properties for all cases. Computations are performed on a Silicon Graphics Indy Workstation.
GRILLAGES I

The first set of grillages has the following dimensions:

\[ b_3 = 6'' \]
\[ t_1 = .4375'' \]
\[ t_2 = .5'' \]
\[ dw_1 = 6'' \]
\[ tw_1 = .17'' \]
\[ df_1 = 4'' \]
\[ tf_1 = .215'' \]
\[ dw_2 = 14'' \]
\[ tw_2 = .255'' \]
\[ df_2 = 5'' \]
\[ tf_2 = .42'' \]

The mesh size used is:

- Mesh length of all elements, mesh width of base plate elements = 1.5''
- Mesh width of web and edge plate elements = 1''
- Mesh width of flange elements = .5''

*Subsequent to the completion of the grillages I cases, it was discovered that the longitudinal edge plates should have extended 4'' below the base plate (instead of 4'' on the stiffeners' side). For the unpressurized uncut case, the buckling force is then 1860 kips (as opposed to 1865 kips). Since the difference between the two results is so small, we did not correct the grillages I cases. (In the grillages II cases, the longitudinal edge plates are exactly centered along the base plate.) In addition, in the positively pressurized grillages I cases, the longitudinal edge plates should have been constrained vertically along the base plate's side rather than the stiffeners' side. For the positively pressurized cut case, the buckling force is then 1350 kips (as opposed to 1352 kips). Since the difference between the two results is so small, and to ensure the end displacement under pressure -10 psi is the negative of the end displacement under pressure 10 psi, we did not change the side along which the constraints are applied. (In the pressurized grillages II cases, the longitudinal edge plates are constrained vertically along the base plate's side.)
NO PRESSURE

First we consider the linear response under an axial compressive force of 1865 kips. The prebuckling state is one of uniform axial compression with little bending. The magnitude of the maximum stress occurs in the thinner regions of width 6" in the outer base plates (Figure 3). Note that the magnitude of the maximum stress is about 50 ksi, considerably less than the yield stress.

The bifurcation buckling force is 1865 kips. The buckling mode is a deformation involving primarily bending with little stretching (Figure 4). The inner base plate buckles into a square quilt of half-wavelength 24". The adjacent webs also buckle with half-wavelength 24". The flanges mainly just rotate about their center line. The edge plates also just rotate about their line of attachment to the base plate. In our earlier report we called this buckling mode TRIPPING.

To assess the effects of the longitudinal edge plates, we next cut the element sides lying along vertical lines at intervals 10.5". The force required to buckle this grillage with 48 edge cuts is 1721 kips (8% less than the uncut case). The buckling mode is similar but with relatively larger deformations near the cut edges (Figure 5).

POSITIVE PRESSURE (Fluid on Plating Side)

Next we consider the linear response under a normal pressure loading of 10 psi and an axial compressive force of 1436 kips. The prebuckling state now involves axial compression and bowing out of the entire grillage with lateral torsional bending of the stiffeners (Figure 6). However, the magnitude of the stress is still less than the yield stress, except at the ends of the central longitudinal stiffeners.

In the presence of positive pressure, the bifurcation buckling force is reduced to 1436 kips (23% less than the unpressurized case). The buckling mode is similar to the unpressurized case (Figure 7).

To again assess the effects of the longitudinal edges, we again make 48 vertical cuts in
the edge plates. This cut grillage buckles at an axial compressive force of 1352 kips (6% less than the uncut pressurized case). Since the edge plates are now restrained vertically along their upper edges at each of the cuts, the buckling mode does not show as much deformation near the cut edges as the cut unpressurized case (Figure 8).

NEGATIVE PRESSURE (Fluid on Stiffener Side)

Finally we consider the linear response under a normal pressure loading of -10 psi and an axial compressive force of 1516 kips. Note that the prebuckling compressive stress in the center of the longitudinal stiffeners is now larger than in the base plates (Figure 9). The magnitude of the stress is now less than the yield stress everywhere.

In the presence of negative pressure, the bifurcation buckling force is 1516 kips (19% less than the unpressurized case). The buckling mode now has half-wavelength 48" (Figure 10).

The cut grillage under negative pressure buckles at an axial compressive force of 1427 kips (Figure 11) (again 6% less than the uncut pressurized case).
ship4: Buckling force 1865 kips - Magnitude of displacement

Figure 4
ship6: Buckling force 1721 kips - Magnitude of displacement

Figure 5
ship12: Pressure 10 psi, Force 1436 kips - Von Mises stress
ship11: Pressure 10 psi, Buckling force 1436 kips - Magnitude of displacement

Figure 7
ship15: Pressure 10 psi, Buckling force 1352 kips

Magnitude of displacement
ship17: Pressure -10 psi, Force 1516 kips - Von Mises stress
ship16: Pressure -10 psi, Buckling force 1516 kips - Magnitude of displacement

Figure 10
ship18: Pressure -10 psi, Buckling force 1427 kips - Magnitude of displacement

Figure 11
GRILLAGES II

The second set of grillages has the following dimensions:

\[ b_3 = 8.5'' \]
\[ t_1 = .375'' \]
\[ t_2 = .4375'' \]
\[ dw_1 = 5.5'' \]
\[ tw_1 = .3125'' \]
\[ df_1 = 4.25'' \]
\[ tf_1 = .3125'' \]
\[ dw_2 = 13.5'' \]
\[ tw_2 = .25'' \]
\[ df_2 = 5'' \]
\[ tf_2 = .4375'' \]

The mesh size now used is:

mesh length of all elements, mesh width of base plate elements = 1.5''

mesh width of edge plate elements = .85''.

mesh width of webs = 5''.

mesh width of longitudinal flanges = .53125''

mesh width of transverse flanges = .625''
NO PRESSURE

The magnitude of the maximum stress in the prebuckling state under an axial force of 1552 kips is about 35 ksi, much less than the yield stress.

The bifurcation buckling force is 1552 kips for the uncut grillage and 1406 kips for the cut grillage. The buckling modes are similar to the corresponding grillages I cases (Figures 14 and 15).

POSITIVE PRESSURE (Fluid on Plating Side)

The magnitude of the maximum stress in the prebuckling state under a normal pressure of 10 psi and an axial compressive force of 1403 kips is about 46 ksi, still considerably less than the yield stress.

In the presence of positive pressure, the bifurcation buckling force is 1403 kips for the uncut grillage and 1293 kips for the cut grillage. The buckling modes are similar to the corresponding grillages I cases (Figures 16 and 17).

NEGATIVE PRESSURE (Fluid on Stiffener Side)

In the presence of negative pressure, the bifurcation buckling force is 1685 kips for the uncut grillage and 1555 kips for the cut grillage. The buckling mode now has half-wavelength of about 20” (Figures 18 and 19).
ship21: Buckling force 1552 kips - Magnitude of displacement

Figure 14
Ship23: Pressure 10 psi, Buckling force 1403 kips

Magnitude of displacement

Figure 16
ship24: Pressure 10 psi, Buckling force 1293 kips

Magnitude of displacement

Figure 17
Ship23: Pressure -10 psi, Buckling force 1685 kips
Magnitude of displacement

Figure 18
ship24: Pressure -10 psi, Buckling force 1555 kips
Magnitude of displacement

Figure 19
CONCLUSIONS

GRILLAGES I

<table>
<thead>
<tr>
<th>Pressure 0</th>
<th>Continuous Edges</th>
<th>Cut Edges</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-.3047&quot;, 1865 kips</td>
<td>-.2963&quot;, 1721 kips</td>
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<tr>
<td>Force 0</td>
<td>.001643&quot;, 10 psi</td>
<td>.001758&quot;, 10 psi</td>
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TABLE 1: END DISPLACEMENT, FORCE OR PRESSURE

<table>
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<tr>
<th>Pressure 0</th>
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<tr>
<td></td>
<td>1865 kips , 1</td>
<td>1721 kips , .923</td>
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<tr>
<td>Pressure 10 psi</td>
<td>1436 kips , .770</td>
<td>1352 kips , .725</td>
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<tr>
<td>Pressure -10 psi</td>
<td>1516 kips , .813</td>
<td>1427 kips , .765</td>
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TABLE 2: COMPRESSIVE BUCKING FORCE, NORMALIZED BUCKLING FORCE

GRILLAGES II

<table>
<thead>
<tr>
<th>Pressure 0</th>
<th>Continuous Edges</th>
<th>Cut Edges</th>
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<tbody>
<tr>
<td></td>
<td>-.2542&quot;, 1552 kips</td>
<td>-.2477&quot;, 1406 kips</td>
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<tr>
<td>Force 0</td>
<td>.001938&quot;, 10 psi</td>
<td>.002130&quot;, 10 psi</td>
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TABLE 3: END DISPLACEMENT, FORCE OR PRESSURE

<table>
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<tr>
<td></td>
<td>1552 kips , 1</td>
<td>1406 kips , .906</td>
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<tr>
<td>Pressure 10 psi</td>
<td>1403 kips , .904</td>
<td>1293 kips , .833</td>
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<tr>
<td>Pressure -10 psi</td>
<td>1685 kips , 1.086</td>
<td>1555 kips , 1.002</td>
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TABLE 4: COMPRESSIVE BUCKING FORCE, NORMALIZED BUCKLING FORCE
The axial displacement at the end of a grillage under the compressive buckling force or the positive pressure is given in Tables 1 or 3. Since the prebuckling state is linear, the axial displacement under both loads is the linear superposition of the two displacements in a column. To get the displacement at any other load, we simply multiply by the displacement/load ratio obtained from Tables 1 or 3.

The predicted buckling loads for the 12 cases are summarized in Tables 2 and 4. Dividing each force by the upper left entry in the table, we obtain the normalized buckling force, from which we can immediately see the reduction in buckling force due to applying pressure or/and cutting the edge plates.

Judging from our earlier work, the buckling loads we have now calculated are likely to be greater than collapse loads to be measured in future experiments. Unmodeled factors which would tend to decrease the buckling loads include geometric imperfections, residual stress, and relaxed boundary conditions. On the other hand, we have calculated only the forces required to initiate buckling, but the grillages are likely to have significant postbuckling strength before ultimate collapse.

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