

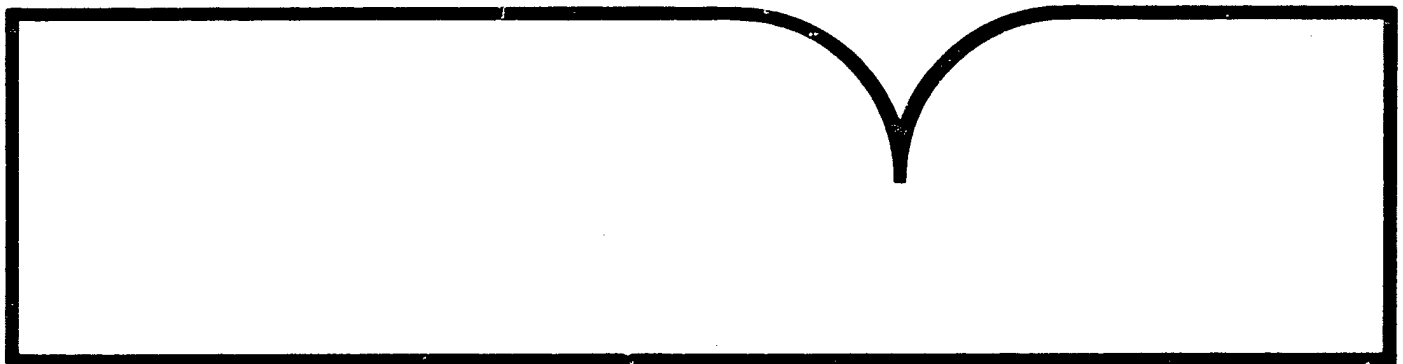
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Extended Range Modified Gradient Technique for Profile Inversion

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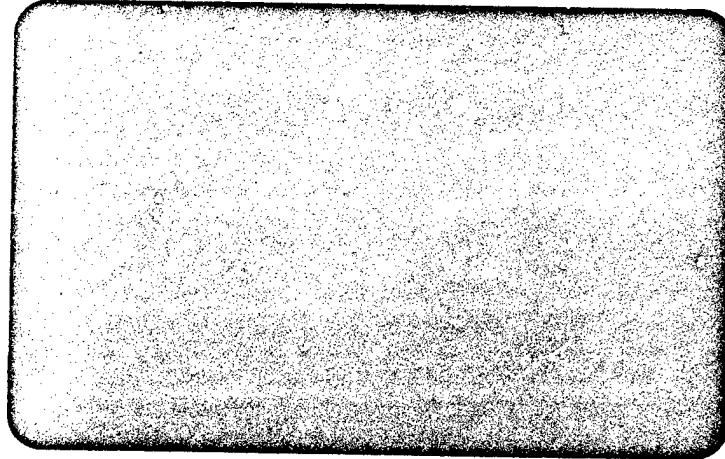
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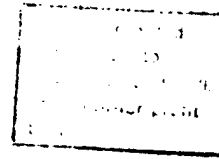
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# AN EXTENDED RANGE MODIFIED GRADIENT TECHNIQUE FOR PROFILE INVERSION

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**ABSTRACT.** A method for reconstructing the complex index of refraction of a bounded inhomogeneous object from measured scattered field data is presented. Some numerical examples are given indicating the limits on the contrasts which can be reconstructed.

## 1. INTRODUCTION

Assume that an inhomogeneous obstacle  $D$  is irradiated successively by a number of known incident fields  $u_i^{inc}$ ,  $i = 1, \dots, I$ . For each excitation, the direct scattering problem may be reformulated as the domain integral equation

$$L(\chi)u_i(p) = u_i(p) - G_D\chi u_i(p), \quad p \in D, \quad (1)$$

where

$$G_D\chi u_i(p) = \int_D G(p, q)\chi(q)u_i(q)dvq, \quad p \in D. \quad (2)$$

Here,  $u_i$  is the total field,  $k$  is the wavenumber,  $\chi$  is the complex contrast ( $\chi = n^2 - 1$ , where  $n$  is index of refraction),  $G(p, q)$  is the free-space Green's function and  $p$  and  $q$  are position vectors.  $G_D$  is an operator mapping  $L^2(D)$  (square integrable functions in  $D$ ) into itself. If  $S$  is a surface enclosing  $D$  then the scattered field on  $S$ ,  $u_i^{sc}$ , is given by  $G_S\chi u_i$  where  $G_S$  is the same operator defined in Eq. (2), except the field point  $p$  now lies on  $S$ . Hence  $G_S$  is an operator mapping  $L^2(D)$  into  $L^2(S)$ . We assume that  $u_i^{sc}$  is measured on  $S$  and denote by  $f_i(p)$ ,  $p \in S$ , the measured data for each excitation  $i$ ,  $i = 1, \dots, I$ . The profile inversion problem is that of finding  $\chi$  for given  $f_i$ , or solving the equation

$$G_S\chi u_i(p) = f_i(p), \quad p \in S, \quad (3)$$

for  $\chi$  subject to the additional condition that  $u_i$  and  $\chi$  satisfy Eq. (1) in  $D$ . We will seek  $u_i$  and  $\chi$  simultaneously to minimize the  $L^2$  error on  $D$  in satisfying Eq. (1) and the  $L^2$  error on  $S$  in satisfying Eq. (3).

## 2. THE INVERSION ALGORITHM

Here we propose an iterative inversion algorithm which incorporates the ideas of successive over-relaxation as well as the conjugate gradient method. Specifically we propose the iterative construction of sequences  $\{u_{i,n}\}$  and  $\{\chi_n\}$  as follows:

$$\begin{aligned} u_{i,0} &= u_i^{initial}, & \chi_0 &= \chi^{initial}, \\ u_{i,n} &= u_{i,n-1} + \alpha_n v_{i,n}, & \chi_n &= \chi_{n-1} + \beta_n d_n, \\ r_{i,n} &= u_i^{inc} - L(\chi_n)u_{i,n}, & \rho_{i,n} &= f_i - G_S\chi_n u_{i,n}, \end{aligned} \quad (4)$$

where  $\alpha_n$  and  $\beta_n$  are in general complex constants which are chosen at each step to minimize

$$F_n = w_D \sum_{i=1}^I \|r_{i,n}\|_D^2 + w_S \sum_{i=1}^I \|\rho_{i,n}\|_S^2, \quad w_D = \left( \sum_{i=1}^I \|u_i^{inc}\|_D^2 \right)^{-1}, \quad w_S = \left( \sum_{i=1}^I \|f_i\|_S^2 \right)^{-1}, \quad (5)$$

and the subscripts  $D$  and  $S$  on the norm  $\|\cdot\|$  and inner product  $(\cdot, \cdot)$  in  $L^2$  indicate the domain of integration. The minimization of the quantity  $F_n$  of Eq. (5) leads to a nonlinear problem for the coefficients  $\alpha_n$  and  $\beta_n$  at each step, which we solve using a conjugate gradient method. The starting value for  $\alpha_n$  is obtained by taking  $\beta_n = 0$  and minimizing  $F_n$ , while the starting value for  $\beta_n$  is found by setting  $\alpha_n = 0$  and again minimizing  $F_n$ .

### 3. INITIAL GUESS AND CORRECTION DIRECTIONS

In our previous treatment of this problem [1] we chose  $\chi_0 = 0$ , while the update direction for the field was directly adapted from the successive over-relaxation method for solving the direct problem with known contrast to be  $v_{i,n} = r_{i,n-1}$  and the update direction for the contrast was chosen to be the gradient of the error in the measured data at the previous,  $(n-1)$ st, step. In the present work we refine these choices considerably.

We obtain the initial guess by finding the constant contrast  $\chi^{initial}$  and associated fields  $u_i^{initial}$ , which minimize the functional  $F_n$ . Specifically, we proceed as follows. Define the normalized change in the field by

$$\varepsilon_n = \left( \sum_{i=1}^I \|u_{i,n} - u_{i,n-1}\|_D^2 \right)^{\frac{1}{2}} \left( \sum_{i=1}^I \|u_{i,n-1}\|_D^2 \right)^{-\frac{1}{2}}. \quad (6)$$

We set an arbitrary switching criterion,  $\epsilon$ , and run the algorithm of Eq. (4) with  $\chi_0^{initial} = 0$ ,  $u_{i,0}^{initial} = u_i^{inc}$ ,  $d_n = 1$  and  $v_{i,n} = r_{i,n-1}$  until  $\varepsilon_{n-1} < \epsilon$ , then switch the definition of  $v_{i,n}$  to

$$v_{i,n} = g_{i,n}^v + \gamma_n^v v_{i,n-1}, \quad \gamma_n^v = \left( \sum_{i=1}^I \langle g_{i,n}^v, g_{i,n}^v - g_{i,n-1}^v \rangle_D \right) / \left( \sum_{i=1}^I \|g_{i,n-1}^v\|_D^2 \right), \quad (7)$$

with the gradient

$$g_{i,n}^v = w_D (r_{i,n-1} - \bar{\chi}_{n-1} \bar{G}_D r_{i,n-1}) + w_S \bar{\chi}_{n-1} \bar{G}_S \rho_{i,n-1}, \quad (8)$$

where the overbar denotes complex conjugate and  $\bar{G}_S$  is a map from  $L^2(S)$  to  $L^2(D)$ . The choice of the direction  $v_{i,n}$  in Eqs. (7) - (8) is the Polak-Ribière conjugate gradient direction assuming the contrast does not change. Continue this algorithm until we again achieve  $\varepsilon_n < \epsilon$ . The resulting values are taken as  $u_i^{initial}$  and  $\chi^{initial}$ .

With these initial choices we run the algorithm of Eq. (4) with  $v_{i,n}$  as in Eqs. (7) - (8) and  $d_n$  is taken in one of the two ways: if  $\varepsilon_{n-1} \geq \epsilon$  then  $d_n$  is taken to be the gradient direction

$$g_n^d = -w_D \sum_{i=1}^I \bar{u}_{i,n-1} \bar{G}_D r_{i,n-1} + w_S \sum_{i=1}^I \bar{u}_{i,n-1} \bar{G}_S \rho_{i,n-1}, \quad (9)$$

whereas if  $\varepsilon_{n-1} < \epsilon$ , we use the Polak-Ribière conjugate gradient direction

$$d_n = g_n^d + \gamma_n^d d_{n-1}, \quad \gamma_n^d = \left( \langle g_n^d, g_n^d - g_{n-1}^d \rangle_D \right) / \left( \|g_{n-1}^d\|_D^2 \right). \quad (10)$$

Continue the iteration until either  $F_n$  meets a preset error criterion or ceases to change.

### 4. NUMERICAL EXAMPLES

The inversion method is illustrated in a particular case in 2-D scattering by a square cylinder of dimension  $d \times d$  with sinusoidal varying profile,  $\chi = \sin(\pi x/\lambda) \sin(\pi y/\lambda)$  for  $0 < x, y < 3\lambda$ , so that  $kd|\chi_{max}| = 6\pi$ . Results are shown for twenty and thirty equally spaced measurement stations distributed on a circle of radius  $3\lambda$  containing the cylinder with each station serving successively as the source and all stations serving as receivers,  $I = 20, 30$ . The cylinder is discretized into  $29 \times 29$  subsquares. The original profile is illustrated in Fig. 1a. Using  $\epsilon = 0.01$  the reconstructed profile is shown in Figs. 1b and 1c, employing twenty and thirty stations, respectively.

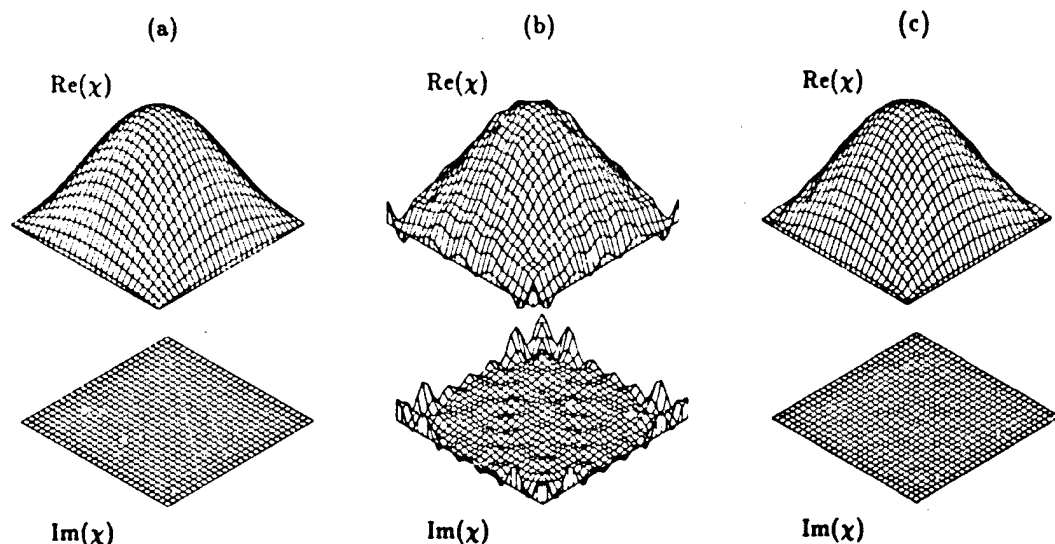


Figure 1. The original profile (a) and the reconstructed profiles for 20 measurement stations (b) and for 30 measurement stations (c).

The reconstructions shown in Figs. 1b and 1c are the results after 128 iterations of which 56 were required to obtain the initial guess for 20 stations. For the case of 30 stations only 38 of the 128 iterations were needed to obtain the initial guess. The values of the functional, Eq. (5), which was to be minimized were  $F_{128} = 0.013$  for  $I = 20$  and  $F_{128} = 0.002$  for  $I = 30$ .

## 5. CONCLUSIONS

An iterative method for complex profile construction has been described and tested. The method combines the features of successive over-relaxation, gradient and conjugate gradient methods to minimize a functional consisting of normalized errors in satisfying the field equation and the error in matching the measured data. The field equation serves as the regularizer for the ill-posed problem finding a function in  $D$  to minimize the error in solving Eq. (3). The nonlinear optimization problem is not linearized, however, the two components of the functional in Eq. (5) are treated somewhat separately. The algorithm was constructed to delay large changes in the contrast until the field was somewhat stable. This was the motivation for the separate treatment of the initial guess as well as the subsequent switching in the algorithm based on the size of  $\epsilon_n$ . The numerical results presented here as well as additional experiments indicate that the algorithm successfully reconstructs complex contrasts for  $kd|\chi_{max}| \leq 6\pi$ . To achieve reconstructions for large values of  $\chi_{max}$  low frequency measurements will not suffice to give reasonable resolution. Future work is directed toward extending the method to include measurements at more than one frequency to accommodate larger contrasts.

## 6. REFERENCES

- [1] R.E. Kleinman and P.M. van den Berg, A modified gradient method for two-dimensional problems in tomography, to appear in: *Journal of Computational and Applied Mathematics*, 1992.

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