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SECTION 1

INTRODUCTION

This study establishes guidelines for determining how accurately aircraft position and velocity should be measured on a Tactical Aircrew Combat Training System (TACTS) range. The method of analysis is based on the sensitivities of Measures of Effectiveness (MOE) computed from TACTS weapon simulations to input aircraft position and velocity at the time of weapon launch. On the basis of desired MOE accuracies, maximum allowable errors (standard deviations) are determined for launch aircraft position and velocity estimates for a variety of weapon/target scenarios.
SECTION 2

APPROACH

TACTS ranges support the training of Navy pilots. One range function is to track the status of each aircraft, including position, speed, and attitude. The accuracy in estimated values for these parameters depends upon many factors, including geography, hardware, telemetry, and software for data processing.

What accuracies are required? In part, the answer lies in subsequent uses of parameter estimates. One such use is an input to weapon simulation programs which simulate weapon performance and calculate appropriate MOEs.

Aircraft position is typically specified by values for three coordinates, X, Y, Z, and speed is specified by three velocity components, Vx, Vy, Vz. The following analysis focuses on the required accuracy of these position and velocity component values.

Suppose there were no errors in the determination of position and velocity. Then, assuming that the weapon simulation programs are completely accurate, a correct MOE value would be computed in every instance. In actuality, there are errors in position and velocity estimates, and consequently weapon simulations do not always compute an accurate MOE value. In general, the larger the errors in input position and velocity values, the larger will be the error in computed MOE. The actual error sizes are not known in advance; if so, they could be eliminated in the processing of data. At best, error sizes can be characterized by using the language of probability (i.e., small errors imply highly accurate MOEs).

To formulate the analytical method for the determination of TACTS range accuracy requirements the following guidelines were established:

- The method should be general enough to apply to various aircraft/weapon/target combinations.

- The method should require only readily available input.

- The method should produce quantitative output which would support trade-off analysis and provide flexibility to the decision-making process.

The goal of this method was to estimate parametrically the amounts by which aircraft position and velocity component values, as determined by TACTS range telemetry and processing, could be in error while requiring that TACTS weapon simulations compute a reasonably accurate MOE value reasonably often.

As a first step in developing the analytical method, a Performance Goal was defined as follows:

**Performance Goal** - The accuracy with which the MOE is to be determined.
Maximum allowable uncertainties in position and velocity component values consistent with achievement of this Performance Goal were determined. For example, for target-point target and weapon-bomb, probability of kill ($P_k$) is an MOE. A Performance Goal might be a relative error of no more than ten percent (i.e., the difference between computed and correct values for $P_k$ should be no larger than ten percent of the correct value for $P_k$). Achievement of this goal places restrictions on the magnitudes of errors in the values of position and velocity components.

The errors in position and velocity components are treated as unbiased, independent, normal random variables. Qualitatively this means that errors are just as likely to be positive as negative with the average error being zero. Further, the error in determining one position or velocity component value is not related to the error in determining a different component.

For errors which are normal, the likelihood of errors is determined by the distribution parameter "standard deviation" or sigma ($\sigma$). The larger the value of $\sigma$, the more likely will be the occurrence of large errors. The magnitudes of position and velocity component errors for achievement of a Performance Goal can be stated in terms of error distribution standard deviations.

A requirement that the Performance Goal must be achieved almost always will require that TACTS range position and velocity estimates be very accurate almost always. On the other hand, if achieving the Performance Goal less frequently is acceptable, then TACTS range estimates must be accurate less frequently (and consequently accuracy requirements would be relaxed). Thus, we define a Performance Level as follows:

Performance Level - The frequency (fraction of launches) that the Performance Goal is achieved.

Performance Levels are varied from .75 to .95 in this analysis. Both GDOP <5 and 5 < GDOP <12 are considered (GDOP stands for Geometric Dilution of Precision, a function of the TACTS range physical layout of position-locating equipment, reference a). Under favorable conditions (GDOP <5), vertical position ($Z$) and velocity ($V_z$) components are assumed to be determined with the same accuracy as horizontal ($X/V_x$ and $Y/V_y$) values. Under less favorable conditions (5 < GDOP <12), standard deviations for errors in vertical component determinations are assumed to be four times as large as the standard deviations in horizontal component determinations.

For each combination of target, weapon, and MOE, a parametric set of TACTS range accuracy requirements is generated by the following general procedure*:

1. Specify the Performance Goal: the accuracy with which the MOE is to be determined (e.g., computed $P_k$ must be within ten percent of actual $P_k$).

2. Identify values for position and velocity components recommended by tactical considerations (these are "intended" values at weapon release).

* Documented more fully in Appendix A.
3. Using the weapon simulation:
   a. Simulate weapon launch with "intended" values and move target to
      weapon impact position.
   b. For each of several variations** (representing "pilot error") in
      position and velocity component values about their "intended"
      values:
         (1) Simulate weapon launch, determining computed MOE (the "cor-
             rect" value).
         (2) In each of the six components (three position and three
             velocity), one at a time, determine the extent to which the
             value can be changed while still producing a computed MOE
             value which is within the accuracy of the "correct" value,
             specified in the Performance Goal (Step 1). This determina-
             tion is done by running the weapon simulation with altered
             input values.
   c. At this stage, several component value intervals have been deter-
      mined which represent allowable variability under the accuracy
      requirement specified in Step 1.
   d. Repeat from Step 2 if there are additional tactical cases of
      importance.

4. Specify the Performance Level (i.e., fraction of time that the computed
   MOE value is to be within the accuracy specified for the Performance
   Goal in Step 1).

5. Using the BASIC program DISCRETE FIT***, compute combinations of
   standard deviations (σ's) for position and velocity component values
   for which errors within the bounds identified in Step 3.c occur with
   the frequency (fraction of time) specified in Step 4. The resulting
   σ's constitute TACTS range parametric accuracy requirements produced
   by this methodology for the selected combination of target, weapon, and
   MOE at the specified Performance Level.

6. Repeat from Step 4 for additional Performance Levels.

   To summarize, the TACTS weapon simulations were used to determine in
   absolute terms how much variability can be tolerated in position and velocity
   component determination while still maintaining acceptable MOE accuracy.
   Treating these absolute quantities as random variables, the statistical
   approach was to compute corresponding standard deviations which assure that
   tolerable errors in position and velocity component determination are achieved
   with specified frequency.

   Presented in the following sections is an extensive collection of computed
   results pertaining to the following weapon/target combinations:

** See Appendix B.
*** See Appendix C.
1. Bombing a point target (three cases)
2. Bombing a runway (one case)
3. Bombing a point target - retarded bomb (one case)
4. Firing a rocket at a point target (three cases)
5. Firing a gun in air-to-air combat (seven cases)

The different cases considered in each scenario represent different intended weapon launch conditions reflecting tactical options. Cases are defined, case data is reported, scenario MOEs and Performance Goals are given, and computed results are presented graphically, parametrized by Performance Level. Illustrations of the interpretation and use of these parametrically presented results are provided.
SECTION 3

TACTS RANGE ACCURACY GUIDELINES
Bombing a Point Target Scenario

The following conditions are assumed for the point target bombing scenario:

Target – Small fighter aircraft on the ground

Weapon – MK-82 Mod 1 (Std) electric fuse bomb

Aircraft – F/A-18

MOE – $P_k$

Performance Goal – No more than ten percent relative error (alternately, no more than five percent relative error).

Performance Levels – .75, .80, .85, .90, .95

Presented in Table 1 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the three cases investigated in this scenario. The bomb drop parameters were chosen from a Tactical Manual (reference b).

In order to simulate pilot error in the attainment of "intended" launch parameter values, the method documented in Appendix B was employed which yielded the values presented in Table 1 for each case investigated.

Figures 1 and 2 present in graphical and parametric form scenario-specific guidance for assessing current TACTS range accuracies or establishing future accuracy requirements. Figures 1 and 2 pertain to GDOP < 5 and to 5 < GDOP < 12, respectively. Figures 3 and 4 present guidance derived under the more stringent Performance Goal that there be no more than five percent relative error in the determination of $P_k$.

To use the figures, first select a value for $V$, the standard deviation for velocity component error, on the horizontal axis, and a value for $L$, the standard deviation for position component error, on the vertical axis. The corresponding point $(V,L)$ defines the Performance Level; for instance, if $(V,L)$ falls within (below) the curve labelled .85, then for TACTS range accuracies $L$ and $V$, the Performance Level will be at least 85 percent. If $(V,L)$ falls outside (above) the curve labelled .85, then for TACTS range accuracies $L$ and $V$, the Performance Level will be less than 85 percent. Further, all $(V,L)$ pairs within the .85 contour will yield Performance Levels of at least 85 percent.

Figures 1 through 4 provide a ready vehicle for establishing required TACTS range accuracies for a single target type under GDOP < 5 or 5 < GDOP < 12 conditions. For instance, consider the following typical conclusions obtainable from Figure 1:
1. If telemetry leads to a standard deviation in aircraft velocity components of $V = 1$ ft/sec and a standard deviation in aircraft position components of $L = 5$ ft then the bomb simulation would produce a value for $P$, whose relative error is no more than 10 percent between 85 percent and 90 percent of the time.

2. If velocity component error standard deviations are $V > 1$ ft/sec, then the Performance Level can never be as large as 87.5 percent.

3. If position component error standard deviations are $L = 1$ ft, then velocity component error standard deviations ($V$) must not exceed approximately 6 ft/sec if a Performance Level of 85 percent is required.
TABLE 1. CASE DATA - SIMULATED BOMB DROPS ON A POINT TARGET

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1)</td>
<td>Intended aircraft position relative to aim point on target at time of bomb drop</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Xpos</td>
<td>Ypos</td>
<td>Zpos</td>
</tr>
<tr>
<td></td>
<td>4129 ft</td>
<td>8346 ft</td>
<td>4790 ft</td>
</tr>
<tr>
<td></td>
<td>0 ft</td>
<td>0 ft</td>
<td>0 ft</td>
</tr>
<tr>
<td></td>
<td>5000 ft</td>
<td>2000 ft</td>
<td>2000 ft</td>
</tr>
<tr>
<td>(2)</td>
<td>Intended dynamic parameter values at time of bomb drop</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Speed</td>
<td>Pitch</td>
<td>Heading</td>
</tr>
<tr>
<td></td>
<td>802° ft/sec</td>
<td>0°</td>
<td>-15°</td>
</tr>
<tr>
<td></td>
<td>760° ft/sec</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>(3)</td>
<td>Velocity components computed from dynamic data</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$v_x$</td>
<td>$v_y$</td>
<td>$v_z$</td>
</tr>
<tr>
<td></td>
<td>567.1 ft/sec</td>
<td>760 ft/sec</td>
<td>724.44 ft/sec</td>
</tr>
<tr>
<td></td>
<td>0 ft/sec</td>
<td>0 ft/sec</td>
<td>0 ft/sec</td>
</tr>
<tr>
<td></td>
<td>-567.1 ft/sec</td>
<td>0 ft/sec</td>
<td>-194.12 ft/sec</td>
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<td>(4)</td>
<td>Sigmas computed for pilot error</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>$\sigma_x$</td>
<td>$\sigma_y$</td>
<td>$\sigma_z$</td>
</tr>
<tr>
<td></td>
<td>17.8 ft</td>
<td>41.85 ft</td>
<td>22.75 ft</td>
</tr>
<tr>
<td></td>
<td>19.8 ft</td>
<td>25.9 ft</td>
<td>15.81 ft</td>
</tr>
<tr>
<td></td>
<td>24.3 ft</td>
<td>20.3 ft</td>
<td>12.83 ft</td>
</tr>
<tr>
<td></td>
<td>$\sigma_v$</td>
<td>$\sigma_{v^x}$</td>
<td>$\sigma_{v^y}$</td>
</tr>
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<td>2.35 ft/sec</td>
<td>3.87 ft/sec</td>
<td>3.48 ft/sec</td>
</tr>
<tr>
<td></td>
<td>2.7 ft/sec</td>
<td>2.35 ft/sec</td>
<td>2.4 ft/sec</td>
</tr>
<tr>
<td></td>
<td>3.3 ft/sec</td>
<td>1.85 ft/sec</td>
<td>1.95 ft/sec</td>
</tr>
</tbody>
</table>
Notes:
1. $L = \sigma_x = \sigma_y = \sigma_z$
2. $V = \sigma_{V_x} = \sigma_{V_y} = \sigma_{V_z}$

$V$ - Velocity Component Error Standard Deviation (ft/sec)

Figure 1. TACTS Range Accuracy - Bomb vs. Point Target
(GDOP $\leq 5$, 10% Relative Error in $P_k$)
Notes:
(1) $\sigma_x = \sigma_y = \sigma_z / 4$
(2) $\sigma_x = \sigma_y = \sigma_z / 4$

$V$ - Velocity Component Error Standard Deviation (ft/sec)

Figure 2. TACTS Range Accuracy - Bomb vs. Point Target
(5 < GDOP < 12, 10% Relative Error in $P_k$)
Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma_{V_x} = \sigma_{V_y} = \sigma_{V_z}$

Figure 3. TACTS Range Accuracy - Bomb vs. Point Target
(GDOP ≤ 5, 5% Relative Error in $P_k$)
Notes:
1. \( L = \sigma_x = \sigma_y = \sigma_z / 4 \)
2. \( V = \sigma V_x = \sigma V_y = \sigma V_z / 4 \)

Figure 4. TACTS Range Accuracy - Bomb vs. Point Target
(5 < GDOP < 12, 5% Relative Error in \( P_k \))
SECTION 4
TACTS RANGE ACCURACY GUIDELINES
Bombing a Runway Scenario

The following conditions are assumed for the runway bombing scenario:

Target - Long runway

Weapon - MK-82 Mod 1 (Std) electric fuse bomb

Aircraft - F/A-18

MOE - Crater Diameter

Performance Goal - Compute correct crater diameter exactly

Performance Levels - .75, .80, .85, .90, .95

Presented in Table 2 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the case investigated in this scenario. The parameters were chosen from a Tactical Manual (reference b). In addition, sigmas computed for pilot error are listed for the position and velocity components.

Figures 5 and 6 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy. Figures 5 and 6 pertain to GDOP ≤5 and to 5 < GDOP ≤12, respectively.
### TABLE 2. CASE DATA - SIMULATED BOMB DROPS ON A RUNWAY

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
</tr>
</thead>
</table>

1. Intended aircraft position relative to aim point on target at time of bomb drop
   - Xpos: 6206 ft
   - Ypos: 0 ft
   - Zpos: 8000 ft

2. Intended dynamic parameter values at time of bomb drop
   - Speed: 844 ft/sec
   - Pitch: $-45^\circ$
   - Heading: 0°

3. Velocity components computed from dynamic data
   - $V_x$: 596.8 ft/sec
   - $V_y$: 0 ft/sec
   - $V_z$: -596.8 ft/sec

4. Sigmas computed for pilot error
   - $\sigma_x$: 25.8 ft
   - $\sigma_y$: 30.9 ft
   - $\sigma_z$: 41.1 ft
   - $\sigma_{Vx}$: 2.4 ft/sec
   - $\sigma_{Vy}$: 2.9 ft/sec
   - $\sigma_{Vz}$: 3.9 ft/sec
Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma_V_x = \sigma_V_y = \sigma_V_z$

Figure 5. TACTS Range Accuracy - Bomb vs. Runway Target
(GDOP $\leq 5$, Correct Crater Diameter)
Figure 6. TACTS Range Accuracy - Bomb vs. Runway Target
(5 < GDOP ≤ 12, Correct Crater Diameter)

Notes:
(1) \( L = \sigma_x = \sigma_y = \sigma_z \)
(2) \( V = \sigma_V_x = \sigma_V_y = \sigma_V_z / 4 \)
SECTION 5

TACTS RANGE ACCURACY GUIDELINES
Bombing a Point Target With Retarded Bomb Scenario

The following conditions are assumed for the bombing of a point target with a retarded bomb scenario:

Target – Small fighter aircraft on the ground

Weapon – MK-82 Snakeye retarded bomb

Aircraft – F/A-18

MOE – $P_k$

Performance Goal – No more than ten percent relative error

Performance Levels – .75, .80, .85, .90, .95

Presented in Table 3 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the one case investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The initial input parameters were chosen from a Tactical Manual (reference b). A retarded bomb was used in this scenario for compatibility with the low-altitude, high-speed "pop-up" delivery technique.

Figures 7 and 8 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 7 and 8 pertain to GDOP $\leq 5$ and to $5 <$ GDOP $\leq 12$, respectively.
TABLE 3. CASE DATA – SIMULATED BOMB DROPS ON A POINT TARGET – RETARDED BOMB

<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
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<tbody>
<tr>
<td>(1)</td>
<td>Intended aircraft position relative to aim point on target at time of bomb drop</td>
</tr>
<tr>
<td>Xpos</td>
<td>1468 ft</td>
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<tr>
<td>Ypos</td>
<td>0 ft</td>
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<tr>
<td>Zpos</td>
<td>500 ft</td>
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<tr>
<td>(2)</td>
<td>Intended dynamic parameter values at time of bomb drop</td>
</tr>
<tr>
<td>Speed</td>
<td>760 ft/sec</td>
</tr>
<tr>
<td>Pitch</td>
<td>-15°</td>
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<tr>
<td>Heading</td>
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<tr>
<td>$V_Z$</td>
<td>-194.2 ft/sec</td>
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<td>$\sigma_X$</td>
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<td>$\sigma_Y$</td>
<td>5.14 ft</td>
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<td>$\sigma_Z$</td>
<td>3.78 ft</td>
</tr>
<tr>
<td>$\sigma_{V_X}$</td>
<td>5.16 ft/sec</td>
</tr>
<tr>
<td>$\sigma_{V_Y}$</td>
<td>2.04 ft/sec</td>
</tr>
<tr>
<td>$\sigma_{V_Z}$</td>
<td>1.5 ft/sec</td>
</tr>
</tbody>
</table>
Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma V_x = \sigma V_y = \sigma V_z$

Figure 7. TACTS Range Accuracy - Retarded Bomb vs. Point Target
(GDOP ≤ 5, 10% Relative Error in $P_k$)
Notes:
1. \( L = \sigma_x = \sigma_y = \sigma_z / 4 \)
2. \( V = \sigma_{V_x} = \sigma_{V_y} = \sigma_{V_z} / 4 \)

Figure 8. TACTS Range Accuracy - Retarded Bomb vs. Point Target
(5 < GDOP < 12, 10% Relative Error in \( P_k \))
SECTION 6

TACTS RANGE ACCURACY GUIDELINES
Rocket Firing at a Point Target Scenario

The following conditions are assumed for the rocket at a point target scenario:

Target - Small fighter aircraft on the ground

Weapon - MK-4/MK-67 WP/M 427 2.75" Folding Fin Aircraft Rocket (FFAR)

Aircraft - F/A-18

MOE - Miss Distance (In current rocket simulations, the post impact scoring routine is not invoked; therefore, $P_k$ is not calculated.)

Performance Goal - No more than ten percent relative error

Performance Levels - .75, .80, .85, .90, .95

Presented in Table 4 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the three cases investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The initial parameters were obtained from Tactical Manuals and represent a varied set of realistic conditions (reference b).

Figures 9 and 10 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 9 and 10 pertain to GDOP $\leq 5$ and to $5 <$ GDOP $\leq 12$, respectively.
<table>
<thead>
<tr>
<th>Case</th>
<th>1</th>
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<th>3</th>
</tr>
</thead>
<tbody>
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<td>(1) Intended aircraft position relative to aim point on target at time of rocket firing</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Xpos</td>
<td>0 ft</td>
<td>0 ft</td>
<td>0 ft</td>
</tr>
<tr>
<td>Ypos</td>
<td>0 ft</td>
<td>0 ft</td>
<td>0 ft</td>
</tr>
<tr>
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<td>1000 ft</td>
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<td>(2) Intended dynamic parameter values at time of rocket firing</td>
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<td></td>
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<td>Speed</td>
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<td>665.2 ft/sec</td>
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<td>Pitch</td>
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<td>Heading</td>
<td>-30°</td>
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<td>-10°</td>
</tr>
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<td>(3) Velocity components computed from dynamic data</td>
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</tr>
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<td>V_x</td>
<td>658.1 ft/sec</td>
<td>477.65 ft/sec</td>
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</tr>
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<td>V_y</td>
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<td>-477.65 ft/sec</td>
<td>-117.2 ft/sec</td>
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<tr>
<td>V_z</td>
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<td>0 ft/sec</td>
<td>0 ft/sec</td>
</tr>
<tr>
<td>(4) Sigmas computed for pilot error</td>
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<td>σ_V_z</td>
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<td>2.17 ft/sec</td>
<td>1.4 ft/sec</td>
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</tbody>
</table>
Notes:
(1) $L = \sigma_x = \sigma_y = \sigma_z$
(2) $V = \sigma_v_x = \sigma_v_y = \sigma_v_z$

Figure 9. TACTS Range Accuracy - Rocket vs. Point Target
(GDOP < 5, 10% Relative Error in Miss Distance)
Figure 10. TACTS Range Accuracy - Rocket vs. Point Target
(5 < GDOP < 12, 10% Relative Error in Miss Distance)

Notes:
(1) \( L = \sigma_x = \sigma_y = \sigma_z / 4 \)
(2) \( V = \sigma V_x = \sigma V_y = \sigma V_z / 4 \)
SECTION 7

TACTS RANGE ACCURACY GUIDELINES
Gunfire In Air-to-Air Combat Scenario

The following conditions are assumed for the gunfire in air-to-air combat scenario:

Target - Small fighter aircraft

Weapon - M61A1 20 mm gun

Aircraft - F-14

MOE - \( P_k \)

Performance Goal - No more than ten percent relative error

Performance Levels - .75, .80, .85, .90, .95

Presented in Table 5 are input values for position, speed, pitch, heading, and the velocity components computed from the latter dynamic parameters for the seven cases investigated in this scenario. Pilot error sigmas for position and velocity components are also shown. The parameters were chosen from a Tactical Manual (reference b). Sigmas for position and velocity parameters are not presented since the analysis indicated total MOE insensitivity to any change in x-direction components.

Figures 11 and 12 present in graphical and parametric form scenario-specific guidance regarding TACTS range accuracy guidelines. Figures 11 and 12 pertain to GDOP \( \leq 5 \) and to \( 5 < GDOP \leq 12 \), respectively. The standard deviations presented pertain to the differences between fighter and target position components, and between fighter target and target velocity components.

For scenarios involving an airborne target, both launcher and target aircraft position and velocity are estimated. Under the assumption that estimation errors are independent and statistically the same for both aircraft, the error standard deviations for both the launcher and target are each \( 1/\sqrt{2} \) times the error standard deviation in the estimate of the difference of like parameter values. For instance, if \( X_L \) and \( X_T \) are the errors in estimates of launcher and target x-component values at time of weapon firing, then \( \sigma_{X_L - X_T}^2 = \sigma_{X_L}^2 + \sigma_{X_T}^2 \). If \( \sigma_{X_L}^2 = \sigma_{X_T}^2 = \sigma_X^2 \), then \( \sigma_{X_L - X_T} = 2 \sigma_X \) and \( \sigma_X = 1/\sqrt{2} \sigma_{X_L - X_T} \).

Guidance for accuracy requirements based upon airborne target scenarios is presented in Figures 11 and 12 in terms of paired values \( (\sigma_{V_L - V_T}, \sigma_{X_L - X_T}) \). If a particular numerical pair \((V,L)\) assures a Performance Level of .90, then the required single aircraft error standard deviation pair is \((V/\sqrt{2}, L/\sqrt{2})\).
<table>
<thead>
<tr>
<th>Case</th>
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<td></td>
</tr>
<tr>
<td>Launcher:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0 ft</td>
</tr>
<tr>
<td>Ypos</td>
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<td>0 ft</td>
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<td>Zpos</td>
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<td>15000 ft</td>
<td>15000 ft</td>
</tr>
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<td>0 ft</td>
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<td>15000 ft</td>
<td>15000 ft</td>
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<td>(3) Velocity components computed from dynamic data</td>
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26
### TABLE 5. CASE DATA - SIMULATED AIR-TO-AIR GUNFIRE (Cont.)

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</tr>
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</tr>
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<td>Launcher:</td>
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<td>0 ft/sec</td>
<td>0 ft/sec</td>
<td>0 ft/sec</td>
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<td>Target:</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
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<td>0 ft/sec</td>
<td>-500 ft/sec</td>
</tr>
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<td>0 ft/sec</td>
<td>0 ft/sec</td>
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<td>(4) Sigmas computed for pilot error</td>
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<td>4.82 ft</td>
<td>1.6 ft</td>
<td>4.17 ft</td>
</tr>
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<td>4.83 ft</td>
<td>1.6 ft</td>
<td>4.17 ft</td>
</tr>
<tr>
<td>$\sigma V_x$</td>
<td>12.08 ft/sec</td>
<td>12.09 ft/sec</td>
<td>12.08 ft/sec</td>
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</tr>
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<td>12.07 ft/sec</td>
<td>12.07 ft/sec</td>
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</tr>
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</table>
Figure 11. TACTS Range Accuracy - Air-to-Air Gunfire
(GDOP ≤ 5, 10% Relative Error in P_k)

Notes:
(1) L = σ_x = σ_y = σ_z
(2) V = σ_V_x = σ_V_y = σ_V_z
Notes:
1. \( L = \sigma_x = \sigma_y = \sigma_z / 4 \)
2. \( V = \sigma V_x = \sigma V_y = \sigma V_z / 4 \)

Figure 12. TACTS Range Accuracy - Air-to-Air Gunfire
\( 5 \leq GDOP \leq 12, 10\% \text{ Relative Error in } P_k \)
SECTION 8
COMPARISONS

TACTS range accuracy guidelines for five different weapon/target scenarios were presented in the previous five sections. For comparison, the 90 percent Performance Level curves under GDOP < 5 conditions for each of these scenarios are presented in Figure 13. Note that the V and L scales are NOT the same for all five curves.

The most demanding scenarios, in terms of TACTS range accuracy guidelines, are firing a gun in air-to-air combat and bombing a point target with a retarded bomb. The least demanding is bombing a runway. In the former scenarios, small errors in position and velocity component values cause large variations in $P_k$. In the latter scenario, the simulation is more tolerant of errors in position and velocity component values. In order of simulated weapon effectiveness sensitivity to TACTS range accuracy, the five scenarios are ranked from most demanding to least demanding as follows:

Firing a gun in air-to-air combat or
Bombing a point target with a retarded bomb
Bombing a point target
Firing a rocket at a point target
Bombing a runway

Generally, the more sensitive weapon effectiveness is in position and velocity component values at weapon launch, the more stringent are the TACTS range accuracy requirements. Thus, the results of this analysis do conform qualitatively with operational realities.

The TACTS range accuracy requirements should NOT be combined by simple averaging. Any requirement consolidation should include, at a minimum, a determination of the relative frequency with which each scenario is to be exercised. If the TACTS range were instrumented to conform with a "consolidated accuracy requirement," then high Performance Levels would be realized for less demanding scenarios and low Performance Levels for the more demanding scenarios. If different scenarios are to be exercised on the same portion of the TACTS range, then requirement consolidation is unavoidable.

The results of this study also can be used to assess how well a particular range instrumentation would perform. For instance, suppose that TACTS range instrumentation allows for determination of position components with error standard deviation of 1 ft and velocity components with error standard deviation of 2.5 ft/sec. Then computed values of $P_k$ (with GDOP < 5) will have a relative error of no more than 10 percent for: $k(a)$ between 75 percent and 80 percent of all retarded bomb drops on point targets (Figure 7); (b) between 85 percent and 90 percent of all standard bomb drops on a point target (Figure 1); and (c) between 75 percent and 80 percent of all gun firings against an airborne target (Figure 11).
Some options for determining specific TACTS range accuracy requirements are as follows:

a. Accommodate the most stringent accuracy requirements to satisfy a desired Performance Level for the most demanding scenario. Then desired Performance Levels for other scenarios will be exceeded.

b. Implement the best accuracy requirements consistent with technological and financial resources. Then there could be a disparity in Performance Levels for certain scenarios.

c. Implement different accuracy requirements for different multi- and single-purpose sites on the TACTS range. Then range-wide compliance with Performance Level requirements might be achievable within the constraints of technological and financial resources.
SECTION 9

CONCLUSIONS

Specification of TACTS range accuracy will depend in part upon available range instrumentation technology. The figures presented provide a vehicle for trading off position and velocity accuracy requirements for areas of the range used for different weapon/target pairs.

Large values for Performance Level (.90 to .95) lead to stringent TACTS range accuracy requirements. Smaller values for Performance Level (.75 to .90) accommodate the realistic possibility that under some circumstances aircraft position and velocity component values cannot be determined accurately.

Because it is not ideal to aggregate the accuracy requirements over all scenarios investigated, the individual scenario results should each be applied to the particular portions of the TACTS range on which the scenario weapon and target are to be exercised.
REFERENCES

Reference a - "System Specification for the Advanced Tactical Aircrew Combat Training System (TACTS)," SP514-1B

Reference b - "Tactical Manual Ballistic Tables," NAVAIR 01-1C-1T

Reference c - "ACEVAL - AIMVAL Test Plan" Volume X, Missile Simulation/Validation Report
APPENDIX A

GENERAL METHOD FOR DETERMINING
TACTS RANGE ACCURACY GUIDELINES

The general method for determining TACTS range accuracy guidelines is presented in this appendix. The method implements the following goal:

Estimate the amounts by which aircraft position and velocity component values, as determined by TACTS range telemetry and processing, could be in error while requiring that TACTS weapon simulations compute a reasonably accurate value for Measure of Effectiveness (MOE), reasonably often.

The method is applicable to both discrete and continuous MOEs.

Figure A-1 depicts, mathematically, the relationship between launch parameter values and computed MOE (Score) as determined by a TACTS weapon simulation.

Figure A-1. Relationship Between Launch Condition and Score
\( \rho : \mathcal{L} \to \mathcal{S} \) is a many-to-one mapping of launch conditions into scores. This function \( \rho \) is evaluated by the TACTS weapon simulations.

Denote \( \lambda \in \mathcal{L} \) and \( \delta \lambda \) the error in \( \lambda \) at the time of weapon launch. The correct score for launch condition \( \lambda \) is given by \( \rho(\lambda) \). The computed score resulting from the TACTS range-measured data \( \lambda + \delta \lambda \) is \( \rho(\lambda + \delta \lambda) \). Conditions that these two scores are close may be written

\[
|\rho(\lambda + \delta \lambda) - \rho(\lambda)| \leq \varepsilon \quad \text{(A-1)}
\]

or

\[
|\rho(\lambda + \delta \lambda) - \rho(\lambda)| \leq \varepsilon \cdot \rho(\lambda) \quad \text{(A-2)}
\]

in which \( \varepsilon (\geq 0) \) is a prescribed tolerance. The first of the conditions places an absolute constraint on accuracy, independent of \( \lambda \); the second specifies a relative accuracy (i.e., \( \rho(\lambda + \delta \lambda) \) is to differ from \( \rho(\lambda) \) by no more than \( 100\varepsilon \) percent). Note that if the scores \( \rho(\lambda + \delta \lambda) \) and \( \rho(\lambda) \) must match exactly, then choose \( \varepsilon = 0 \). Condition (A-1) is probably more appropriate for discrete MOEs (such as crater diameter, number of hits, etc.); the second condition (A-2) is more appropriate for continuous MOEs (such as \( P_k \), miss distance, percent of target destroyed, etc.).

Denote the absolute difference between \( \rho(\lambda + \delta \lambda) \) and \( \rho(\lambda) \) by

\[
D(\lambda, \delta \lambda) = |\rho(\lambda + \delta \lambda) - \rho(\lambda)| \quad \text{(A-3)}
\]

Then the statistical requirement for TACTS range accuracy may be written

\[
\Pr \{D(\lambda, \delta \lambda) \leq \varepsilon\} \geq 0.95 \quad \text{(A-4)}
\]

or

\[
\Pr \{D(\lambda, \delta \lambda) \leq \varepsilon \cdot \rho(\lambda)\} \geq 0.95 \quad \text{(A-5)}
\]

Note that the .95 is chosen arbitrarily for illustration.
Let \( f(l, \delta l) \) be the joint probability density function for launch conditions \( l \) and the TACTS range error in launch conditions \( \delta l \). Requirements (A-4, -5) are equivalent to

\[
\int_{R} f(l, \delta l) \, d\delta l > 0.95
\]  

(A-6)

in which region \( R = \{(l, \delta l) | D(l, \delta l) \leq \epsilon\} \)

or alternately, \( R = \{(l, \delta l) | D(l, \delta l) \leq \epsilon \rho(l)\} \).

The value of the integral in (A-6) depends upon \( f(l, \delta l) \); in particular, if \( \sigma_{\delta l} = 0 \) (and \( \mu_{\delta l} = 0 \) so that TACTS range errors are unbiased), then the value of the integral is 1; therefore, all scores are correctly determined by the weapon simulation because all input is correct.

As \( \sigma_{\delta l} \) increases, the integral in (A-6) decreases in value. There is a maximum value of \( \sigma_{\delta l} \) for which condition (A-6) holds. For the set of launch platform/pilot/target/environmental specifics represented in \( f(\ldots) \), that maximum \( \sigma_{\delta} \) is the TACTS range accuracy required to guarantee condition (A-6).

As a practical matter \( f(\ldots) \) cannot be known, so a reasonable approximation must be employed for the determination of the integral in (A-6) as a function of \( \sigma_{\delta l} \). Or more correctly, a reasonable approximation for the integral in (A-6) as a function of \( \delta l \) must be used, and from that approximation, the value of \( \sigma_{\delta l} \) determined.

One such method of approximation is as follows: let \( l \in L \) be a typical set of actual launch conditions. Such actual launch conditions represent a realistic variation on intended launch conditions recommended by tactical manuals and other sources of tactical guidance. Evaluate \( \rho(l) \). Then in one component of \( l \) at a time find the largest and smallest variations \( \delta l \) for which \( D(l, \delta l) \leq \epsilon \) [or \( D(l, \delta l) \leq \epsilon \rho(l)\)].
For several values of \( l \) representing realistic variations in launch conditions, determine lower and upper bounds on each of the position and velocity parameter values. The lower bound is the amount by which the parameter can be decreased without changing the value of \( p \) substantially – similarly for the upper bound.

For convenience denote the parameters by \( l_i \); \( i = 1, \ldots, 6 \), with \( l_1 = x \), \( l_2 = y \), \( l_3 = z \), \( l_4 = v_x \), \( l_5 = v_y \), \( l_6 = v_z \). Let the lower and upper bounds be \( l_i^- \) and \( l_i^+ \). Let values for the \( r \)th set of launch conditions be denoted \( l_i^-(r) \), \( l_i^+(r) \), \( i = 1, \ldots, 6 \).

Suppose that the TACTS range standard deviations for the determination of these parameter values are \( \sigma_i \); \( i = 1, \ldots, 6 \).

If the errors are unbiased, normal, and independent, then

\[
M(r) = \Pr\{D(l, \delta l) \leq \varepsilon\} \quad \text{(or Pr}\{D(l, \delta l) \leq \rho(l)\})
\]

\[
\approx \prod_{i=1}^{6} \left[ \Phi \left( \frac{l_i^+(r)}{\sigma_i} \right) - \Phi \left( \frac{l_i^-(r)}{\sigma_i} \right) \right]
\]

(A-7)

in which \( \Phi \) is the cumulative normal distribution function.

The factor \( \Phi \left( \frac{l_i^+(r)}{\sigma_i} \right) - \Phi \left( \frac{l_i^-(r)}{\sigma_i} \right) \) is the probability that the TACTS range determines a launch platform x-coordinate value between \( l_i^-(r) \) and \( l_i^+(r) \) and similarly for other position and velocity component values. The product determines the joint probability that all 6 values are determined with sufficient accuracy.
If the computed simulation score is to be accurate 95 percent of the time, then set

\[
PL = \frac{1}{Nr} \sum_{r=1}^{Nr} M(r) = .95 \quad \text{(A-8)}
\]

in which \(Nr\) = number of simulation runs, \(PL\) = Performance Level (.95 is for illustration only), and \(M(r)\) is given in (A-7). The function \(PL\) decreases with increasing \(\sigma_i\). Search (numerically) for combinations of \(\sigma_i\)'s for which (A-8) holds. These combinations of values specify accuracy required on the TACTS range. Appendix C presents the listings of BASIC programs with the numerical procedures for computing desired combinations of \(\sigma_i\)'s from weapon simulation data.
APPENDIX B
ESTIMATION OF VARIANCES IN LAUNCH PARAMETERS
FROM MEASURES OF ACCURACY

In order to simulate actual launch parameter values, estimates for pilot ability to attain intended launch parameters must be available. This Appendix describes a method of estimation employed in the TACTS weapon simulations for purposes of this analysis. The method is described in the context of bombing a fixed target, and later applied in air-to-air gunnery.

Denote:

$$\mathbf{W} = (x, y, z, \theta, \psi, s)$$
$$\mathbf{X} = (x, y, z, V_x, V_y, V_z),$$

each a vector of launch parameters at the time of bomb release. Coordinate $x$ is the horizontal distance between the aircraft and target; $y$ is the aircraft’s horizontal position perpendicular to the $x$-axis; and $z$ is the vertical distance between the aircraft and the target. Dynamic parameters $\theta$, $\psi$, and $s$ are aircraft pitch, heading, and speed respectively, and velocity components $V_x$, $V_y$, and $V_z$ are determined by values for $\theta$, $\psi$, and $s$, and conversely. A pilot’s ability to attain intended launch parameter values is represented by the variances in the differences between actual and intended values.

Let $\Delta X_h$ and $\Delta Y_h$ be the differences between actual and intended hit position coordinates. If intended launch parameter values are attained, then $\Delta X_h$ and $\Delta Y_h$ will each be zero. But in general,

$$\Delta X_h \approx \frac{\partial X_h}{\partial x} \Delta x + \frac{\partial X_h}{\partial y} \Delta y + \frac{\partial X_h}{\partial z} \Delta z + \frac{\partial X_h}{\partial V_x} \Delta V_x + \frac{\partial X_h}{\partial V_y} \Delta V_y + \frac{\partial X_h}{\partial V_z} \Delta V_z,$$
$$\Delta Y_h \approx \frac{\partial Y_h}{\partial x} \Delta x + \frac{\partial Y_h}{\partial y} \Delta y + \frac{\partial Y_h}{\partial z} \Delta z + \frac{\partial Y_h}{\partial V_x} \Delta V_x + \frac{\partial Y_h}{\partial V_y} \Delta V_y + \frac{\partial Y_h}{\partial V_z} \Delta V_z,$$

in which $\Delta \mathbf{X} = (\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)$ are the errors in $\mathbf{X}$ at launch.

The method which follows estimates variances for the errors in $\Delta \mathbf{X}$ subject to the “known” conditions on the hit position errors $\Delta X_h$, $\Delta Y_h$. The “known” conditions are that bombing accuracy is “5 mils” and gunning accuracy is “14 mils”.

As a first step the partial derivatives $\frac{\partial X_h}{\partial x}, \ldots, \frac{\partial Y_h}{\partial V_z}$ are estimated. Let $\mathbf{W}_I$ denote a set of intended launch parameter values as determined by tactical guidance. Run the bomb simulation to obtain bomb impact position - call it $(X_h, Y_h)$ - to be thought of as the intended hit position.

Then for each component in $\mathbf{X}$ (one at a time), increase the parameter by a small amount and rerun the bomb simulation with the perturbed launch parameter value. (Perturbations in $V_x$, $V_y$, $V_z$ must be converted to perturbations in $\theta$, $\psi$, and $s$ before running). Obtain the bomb impact positions under these six variations. (Variations in $x$ and $y$ need not be run because the bomb trajectory is simply translated so that miss distances on the ground are exactly the values in $x$ and $y$ variation.)

Denote the changes in impact positions $(X_h, Y_h)$ resulting from parameter variations by

$$\Delta X_h(\Delta x)[= \Delta x], \Delta Y_h(\Delta x)[= \Delta x], \Delta X_h(\Delta y)[= \Delta y], \Delta Y_h(\Delta y)[= \Delta y],$$
$$\Delta X_h(\Delta z), \Delta Y_h(\Delta z)[= \Delta z], \Delta X_h(\Delta V_x), \Delta Y_h(\Delta V_x)[= \Delta V_x],$$
$$\Delta X_h(\Delta V_y)[\approx 0], \Delta Y_h(\Delta V_y), \Delta X_h(\Delta V_z), \Delta Y_h(\Delta V_z)[= \Delta V_z].$$

(B-1)

Values known in advance are given in “[- ]".

B-1
Then as a first-order approximation,

\[
\begin{align*}
    \Delta X_h &\approx \Delta z + \left( \frac{\Delta X_h(\Delta z)}{\Delta z} \right) \cdot \Delta z + \left( \frac{\Delta X_h(\Delta V_z)}{\Delta V_z} \right) \cdot \Delta V_z + \left( \frac{\Delta X_h(\Delta V_z)}{\Delta V_z} \right) \cdot \Delta V_z \\
    \Delta Y_h &\approx \Delta y + \left( \frac{\Delta Y_h(\Delta y)}{\Delta V_y} \right) \cdot \Delta V_y
\end{align*}
\]  

(B-2)

in which the terms in \( \{ \ldots \} \) are the numerical estimates of partial derivatives. Alternative numerical estimation formulas for partial derivatives may be substituted for those used to obtain more accurate estimates.

To simplify notation, abbreviate

\[
\left\{ \begin{align*}
    \frac{\Delta X_h(\Delta z)}{\Delta z} &= D_z, \\
    \frac{\Delta X_h(\Delta V_z)}{\Delta V_z} &= D V_z, \\
    \frac{\Delta X_h(\Delta V_z)}{\Delta V_z} &= D V_z, \text{ and} \\
    \frac{\Delta Y_h(\Delta V_y)}{\Delta V_y} &= D V_y,
\end{align*} \right. \]

If it is assumed that the errors \( \Delta X = (\Delta z, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z) \) in \( X \) at launch are independent normally distributed random variables with 0 means, then \( \sigma_{\Delta z}^2 = E[\Delta z^2], \sigma_{\Delta y}^2 = E[\Delta y^2], \sigma_{\Delta z \Delta y} = E[\Delta z \Delta y](= 0) \), etc., in which \( E[\cdot] \) is the "expectation" function. Consequently, \( \Delta X_h, \Delta Y_h \) in equation (B-2) are normal and independent,

\[
E[\Delta X_h] = E[\Delta Y_h] = 0,
\]

and

\[
\begin{align*}
    \sigma_{\Delta X_h}^2 &\approx \sigma_{\Delta z}^2 + D z^2 \cdot \sigma_{\Delta z}^2 + D V_z^2 \cdot \sigma_{\Delta V_z}^2 + D V_z^2 \cdot \sigma_{\Delta V_z}^2 \\
    \sigma_{\Delta Y_h}^2 &\approx \sigma_{\Delta y}^2 + D V_y^2 \cdot \sigma_{\Delta V_y}^2.
\end{align*}
\]  

(B-3)

Suppose that bombing accuracy is 5 mils. Then the median miss distance in the plane perpendicular to the weapon trajectory at the point of impact is 5L feet as shown in the following Figure, where \( L = \) trajectory length in thousands of feet (e.g., \( L = 2 \) for a 2000 ft trajectory).

![Diagram](image)

The elliptical area on the ground plane which contains 50 percent of the bomb hits has axes

\[
A_y = 5L \quad \text{and} \quad A_x = \frac{5L}{\sin \Theta_f}
\]

in which \( \Theta_f = \) weapon impact angle. Thus

\[
\frac{\sigma_{\Delta X_h}}{\sigma_{\Delta Y_h}} = \frac{A_x}{A_y} = \frac{1}{\sin \Theta_f}.
\]  

(B-4)

B-2
The probability within a circle of radius \( r \) centered at the mean for a circular normal random variable \((U, V)\) with \( \sigma_U = \sigma_V = \sigma \) is given by \( 1 - \exp(-r^2/2\sigma^2) \). If this has value .5, then the corresponding \( r = \sigma \sqrt{2 \cdot \ln 2} \) is the median miss distance, where \( \sqrt{2 \cdot \ln 2} \approx 1.177 \).

Thus for \( CEP = 5L \), the corresponding standard deviation in the plane perpendicular to the trajectory at impact is \( \sigma = 5L/1.177 \). Thus

\[
\sigma_{\Delta Y_h} = \frac{5L}{1.177} \quad \text{and} \quad \sigma_{\Delta x_h} = \frac{5L}{1.177} \cdot \frac{1}{\sin \theta_I}.
\]

Now abbreviate:

\[
C_x = \frac{5L}{1.177} \cdot \frac{1}{\sin \theta_I}
\]
\[
C_y = \frac{5L}{1.177}.
\]

Then equations (B-3) and (B-5) together imply the constraints

\[
\begin{align*}
\sigma_{\Delta x}^2 + Dz^2 \cdot \sigma_{\Delta z}^2 + D V_x^2 \cdot \sigma_{\Delta V_x}^2 + D V_y^2 \cdot \sigma_{\Delta V_y}^2 &= C_x^2 \\
\sigma_{\Delta y}^2 + D V_y^2 \cdot \sigma_{\Delta V_y}^2 &= C_y^2
\end{align*}
\]

(B-6)

on the six error standard deviations.

Under the normality and independence assumptions on the components if \( \Delta X \), the joint probability density function for the six error components is proportional to \( Q \):

\[
Q = \exp \left[ -\frac{1}{2} \left( \frac{(\Delta x)^2}{\sigma_{\Delta x}} + \frac{(\Delta y)^2}{\sigma_{\Delta y}} + \frac{(\Delta z)^2}{\sigma_{\Delta z}} + \frac{(\Delta V_x)^2}{\sigma_{\Delta V_x}} + \frac{(\Delta V_y)^2}{\sigma_{\Delta V_y}} + \frac{(\Delta V_z)^2}{\sigma_{\Delta V_z}} \right) \right]
\]

(B-7)

Among all volumes which contain the six error values \((\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)\) 50 percent of the time (so that the resulting bombing accuracy is 5 mils), the smallest volume is defined by \( Q \geq K_1 \) for some constant \( K_1 \) (whose actual value is irrelevant).

To confirm that \( Q \geq K_1 \) defines the smallest volume, note that \( f(\mathbf{x}) \equiv f(x_1, \ldots, x_n) \) is a probability density function for random variable \( \mathbf{X} \) if and only if \( f(\mathbf{x}) \geq 0 \) for all \( \mathbf{x} \) and \( \int_{E^n} f(z) dV = 1 \). \( E^n \) is all of \( n \)-dimensional space \( = \{(x_1, \ldots, x_n) | -\infty < x_i < +\infty, i = 1, \ldots, n\} \) and \( dV \) is the volume differential (\( = dz_1 dz_2 \cdots dz_n \)).

A subset \( B \subset E^n \) is a confidence volume for \( \mathbf{X} \) at confidence level \( C \), \( 0 \leq C \leq 1 \), if \( \int_B f(\mathbf{z}) dV = C \). Define

\[
A(k) = \{ \mathbf{z} | f(\mathbf{z}) \geq k \}.
\]

Then \( A(0) = E^n \) and \( A(k + \epsilon) \subseteq A(k) \) for all \( \epsilon \geq 0 \). As \( k \) increases, \( A(k) \) gets smaller; that is \( \lim_{k \to \infty} A(k) = \emptyset \), the empty set: further,

\[
\Pr(A(k)) = \int_{A(k)} f(\mathbf{z}) dV \geq \int_{A(k)} k dV = k \int_{A(k)} dV = k \cdot \text{Vol}(A(k))
\]

and because \( \Pr(A(k)) \leq 1 \), then \( \lim_{k \to \infty} k \cdot \text{Vol}(A(k)) \leq 1 \) and therefore \( \lim_{k \to \infty} \text{Vol}(A(k)) = 0 \). Thus \( \lim_{k \to \infty} \Pr(A(k)) = 0 \). Consequently, for fixed \( C \), \( 0 < C < 1 \), there is a constant \( k = k(C) \) such that \( \int_{A(k(C))} f(\mathbf{z}) dV = C \), provided that \( f \) is continuous with non-zero partial derivatives.

The set \( A(k(C)) \) (abbreviated \( A \)) has smallest volume of all confidence volumes at level \( C \), as shown by the following (\( B \) is any other confidence volume at level \( C \)):

\[
\int_A f(\mathbf{z}) dV = \int_{A \cap B} f(\mathbf{z}) dV + \int_{A \setminus B} f(\mathbf{z}) dV = C,
\]

B-3
\[ \int_B f(z) dV = \int_{A \cap B} f(z) dV + \int_{\bar{A} \cap B} f(z) dV = C, \]

so that by subtraction,
\[ \int_{A \cap B} f(z) dV = \int_{\bar{A} \cap B} f(z) dV. \]

But
\[ \int_{A \cap B} f(z) dV \geq \int_{A \cap B} k \, dV = k \, Vol(A \cap \bar{B}) \]
\[ \int_{\bar{A} \cap B} f(z) dV \leq \int_{\bar{A} \cap B} k \, dV = k \, Vol(\bar{A} \cap B). \]

Thus
\[ Vol(A \cap \bar{B}) \leq Vol(\bar{A} \cap B), \]

and
\[ Vol(A \cap B) = Vol(A \cap \bar{B}), \]

so adding,
\[ Vol(A) \leq Vol(B). \]

The conclusion is that of all confidence volumes at confidence level \( C \), the one of smallest volume is \( A(k(C)) \).

The shape of that smallest volume is an ellipsoid. Set \( Q \geq K_1 \) (\( Q \) is defined in equation (B-7)) and take the natural logarithm, obtaining the volume defined by
\[ \left( \frac{\Delta x}{\sigma_{\Delta x}} \right)^2 + \left( \frac{\Delta y}{\sigma_{\Delta y}} \right)^2 + \left( \frac{\Delta z}{\sigma_{\Delta z}} \right)^2 + \left( \frac{\Delta V_x}{\sigma_{\Delta V_x}} \right)^2 + \left( \frac{\Delta V_y}{\sigma_{\Delta V_y}} \right)^2 + \left( \frac{\Delta V_z}{\sigma_{\Delta V_z}} \right)^2 \leq K_2^2 \]

for some constant \( K_2 \). This volume is an ellipsoid in the 6-dimensional space of errors (whose points are labeled \((\Delta x, \Delta y, \Delta z, \Delta V_x, \Delta V_y, \Delta V_z)\)), with semi-axes proportional to \( \sigma_{\Delta x}, \sigma_{\Delta y}, \sigma_{\Delta z}, \sigma_{\Delta V_x}, \sigma_{\Delta V_y}, \) and \( \sigma_{\Delta V_z} \).

The volume of \( A(k(C)) \) is proportional to the product of the ellipsoid semi-axes, namely;
\[ V = K_3 \cdot \sigma_{\Delta x} \sigma_{\Delta y} \sigma_{\Delta z} \sigma_{\Delta V_x} \sigma_{\Delta V_y} \sigma_{\Delta V_z} \]

(B-8)

for a constant \( K_3 \) (whose value is irrelevant).

The two constraints of equation (B-6) do not provide sufficient information for the determination of the six error standard deviations. Further, specific information regarding relative sizes of the six error standard deviations is not available. Consequently, a mathematical optimization is employed in which values for \((\sigma_{\Delta x}, \sigma_{\Delta y}, \sigma_{\Delta z}, \sigma_{\Delta V_x}, \sigma_{\Delta V_y}, \sigma_{\Delta V_z})\) are chosen to maximize the volume in equation (B-8) subject to the constraints of equation (B-6). The optimization is performed analytically using the method of Lagrange multipliers. Notice that the optimization
\[ \max_{\{\sigma_i \}} V = \max_{\{\sigma_i \}} K_3 \cdot \sigma_{\Delta x} \sigma_{\Delta y} \sigma_{\Delta z} \sigma_{\Delta V_x} \sigma_{\Delta V_y} \sigma_{\Delta V_z} \]

is equivalent to the two optimizations
\[ \max_{\{\sigma_{\Delta x}, \sigma_{\Delta y}, \sigma_{\Delta z}, \sigma_{\Delta V_x}, \sigma_{\Delta V_y}, \sigma_{\Delta V_z} \}} \sigma_{\Delta x} \sigma_{\Delta z} \sigma_{\Delta V_x} \sigma_{\Delta V_y} \sigma_{\Delta V_z} \]

S.T. \[ \sigma_{\Delta x}^2 + Dx^2 \cdot \sigma_{\Delta z}^2 + DV_x^2 \cdot \sigma_{\Delta V_x}^2 + DV_y^2 \cdot \sigma_{\Delta V_y}^2 = C_z^2 \]

B-4
and

$$\max_{\{\sigma, \sigma\Delta V\}} \sigma_{\Delta V} \sigma \Delta V$$

S.T. \hspace{1em} \sigma_{\Delta V}^2 + D V_{\Delta V}^2 \cdot \sigma \Delta V = C^2_y$$

because the objective function \( V \) and the constraints separate into two independent sets.

Each of these optimizations has the form

$$\max_{\{X_i\}} \prod_{i=1}^{n} X_i$$

S.T. \hspace{1em} \sum_{i=1}^{n} a_i^2 X_i^2 = b^2$$

which is a constrained optimization problem. This can be solved by maximizing the Lagrangian function

$$H(X_1, \ldots, X_n, \lambda) = \prod_{i=1}^{n} X_i - \lambda\left(\sum_{i=1}^{n} a_i^2 X_i^2 - b^2\right).$$

Differentiation yields

$$\frac{\partial H}{\partial X_i} = \prod_{j=1,j\neq i}^{n} X_j - 2\lambda a_i^2 X_i = 0, \quad i = 1, \ldots, n$$

$$\frac{\partial H}{\partial \lambda} = \sum_{i=1}^{n} a_i^2 X_i^2 - b^2 = 0.$$

The first yields (multiplying by \( X_i \))

$$\prod_{j=1}^{n} X_j = 2\lambda a_i^2 X_i^2, \quad i = 1, \ldots, n$$

so that all terms \( a_i^2 X_i^2 \) are equal. Let their common value be \( u^2 \); then from the condition \( \frac{\partial H}{\partial \lambda} = 0 \) above,

$$\sum_{i=1}^{n} u^2 - b^2 = 0, \quad nu^2 = b^2, \quad u = \frac{|b|}{\sqrt{n}},$$

and consequently

$$\hat{X}_i = \frac{|b|}{|a_i|\sqrt{n}}, \quad i = 1, \ldots, n.$$

Apply this general result to the first optimization above by letting \( X_1 = \sigma_{\Delta x}, X_2 = \sigma_{\Delta z}, X_3 = \sigma_{\Delta V_x}, X_4 = \sigma_{\Delta V_z}, a_1^2 = 1, a_2^2 = D z^2, a_3^2 = D V_x^2, a_4^2 = D V_z^2, b^2 = C^2_y, \) and \( n = 4 \). Similarly, apply this general result to the second optimization above by letting \( X_1 = \sigma_{\Delta y}, X_2 = \sigma_{\Delta V}, a_1^2 = 1, a_2^2 = D V_y^2, b^2 = C^2_y, \) and \( n = 2 \). As a result, the following estimates are obtained:

$$\hat{\sigma}_{\Delta x} = \frac{C_x}{2}; \quad \hat{\sigma}_{\Delta y} = \frac{C_y}{\sqrt{2}}; \quad \hat{\sigma}_{\Delta z} = \frac{C_z}{2|Dz|};$$

(B-9a)

$$\hat{\sigma}_{\Delta V_x} = \frac{C_x}{2|DV_x|}; \quad \hat{\sigma}_{\Delta V_y} = \frac{C_y}{\sqrt{2}|DV_y|}; \quad \hat{\sigma}_{\Delta V_z} = \frac{C_z}{2|DV_z|}$$

(B-9b)

B-5
These error standard deviation estimates in equations (B-9) are used to simulate actual launch conditions with respect to stationary targets.

A similar approach is used to determine variances in launch parameter values for air-to-air gunnery.

Assume that the Closest Point of Approach (CPA) occurs in the r-s plane perpendicular to the bullet's path. At CPA the target is at position \((0, r, s)\) as shown in the following figure, where CPA = \(\ell\), and \(\ell^2 = r^2 + s^2\).

Assume at CPA that \(r, s\) are values of independent, normal random variables, each with means \(\mu = 0\) and equal variances \(\sigma^2\). Then \((\ell/\sigma)^2\) is a random variable with Chi-square distribution with 2 degrees of freedom (abbreviated \(\chi^2, 2df\)). The density function for this random variable is

\[
f_{\chi^2}(x) = \frac{1}{2} \exp(-\frac{x}{2}), \quad x \geq 0.
\]

If \(m\) is the median miss distance, then \(Pr(\ell \leq m) = .5\); that is, \(Pr(\ell^2 \leq m^2) = .5\) or \(Pr((\ell/\sigma)^2 \leq (m/\sigma)^2) = .5\). Thus \((m/\sigma)^2\) is the median \(M\) for the \(\chi^2\) random variable with 2\(df\). The numerical value \(M\) is determined as follows; if

\[
\int_0^M \frac{1}{2} \exp(-\frac{x}{2}) dx = .5,
\]

then

\[
1 - \exp(-\frac{M}{2}) = .5 \quad \text{and} \quad M = \ln 4 \approx 1.386.
\]

The *Air Combat/ Missile Evaluation Study*, from 1976 (reference c) suggests that \(m = 14L\) (a "14-mil error") so that

\[
\left(\frac{14L}{\sigma}\right)^2 = \ln 4 \quad \text{and} \quad \sigma^2 = \frac{(14L)^2}{\ln 4}, \quad \text{(B-10)}
\]

where \(\sigma^2\) is the common variance for \(r\) and \(s\).

Next, determine the variance for \(\ell\);

\[
\sigma_\ell^2 = E[(\ell - \mu_\ell)^2] = E[\ell^2] - \mu_\ell^2. \quad \text{(B-11)}
\]

From \(E[(\ell/\sigma)^2] = 2\) (the expected value of a \(\chi^2\) random variable = \(df\)) there follows

\[
E[\ell^2] = 2\sigma^2 \quad \text{(using linearity of the expectation function).} \quad \text{(B-12)}
\]
Further, \( \mu_t = E[\ell] \). Let \( w = (\ell/\sigma)^2 \); then \( w \) is a \( \chi^2 \) random variable with \( 2df \); \( \ell = \sigma \sqrt{w} \); and
\[
\mu_t = \int_0^\infty \sigma^2 \sqrt{w} f_{\chi^2}(w)dw \text{ which can be simplified to yield }
\]
\[
\mu_t = \frac{\sigma}{2} \sqrt{2\pi}.
\]  
(B-13)

Putting results in equations (B-10) through (B-13) together yields
\[
\sigma_t^2 = 2\sigma^2 - \frac{2\pi \sigma^2}{4} = \frac{4 - \pi}{2} \sigma^2,
\]
and
\[
\sigma_t^2 \approx C_t^2 = \frac{4 - \pi (14L)^2}{\ln 4},
\]  
(B-14)
in which \( C_t^2 \) is an abbreviation for the constant given. Standard deviation \( \sigma_t \) in miss distance \( \ell \) is a function of deviations in parameters \( x, y, z, V_x, V_y, V_z \) at the time of gun firing; the actual miss distance \( \Delta \ell \) is given by
\[
\Delta \ell \approx \frac{\partial \ell}{\partial x} \Delta x + \frac{\partial \ell}{\partial y} \Delta y + \frac{\partial \ell}{\partial z} \Delta z + \frac{\partial \ell}{\partial V_x} \Delta V_x + \frac{\partial \ell}{\partial V_y} \Delta V_y + \frac{\partial \ell}{\partial V_z} \Delta V_z.
\]

Under the assumption that the above parameter deviations \( \Delta \lambda \) are unbiased and independent,
\[
\sigma_t^2 \approx E[\Delta \ell^2],
\]
or
\[
\left( \frac{\partial \ell}{\partial x} \right)^2 \sigma_x^2 + \left( \frac{\partial \ell}{\partial y} \right)^2 \sigma_y^2 + \left( \frac{\partial \ell}{\partial z} \right)^2 \sigma_z^2 + \left( \frac{\partial \ell}{\partial V_x} \right)^2 \sigma_{V_x}^2 + \left( \frac{\partial \ell}{\partial V_y} \right)^2 \sigma_{V_y}^2 + \left( \frac{\partial \ell}{\partial V_z} \right)^2 \sigma_{V_z}^2 = C_t^2.
\]  
(B-15)

As in the treatment of stationary targets, abbreviate the estimate for \( \frac{\partial \ell}{\partial x} \) as \( Dx \), ... , and for \( \frac{\partial \ell}{\partial V_z} \) as \( DV_z \). Then perform the following optimization:
\[
\max_{\{ \sigma \}} \sigma_{\Delta x} \sigma_{\Delta y} \sigma_{\Delta z} \sigma_{\Delta V_x} \sigma_{\Delta V_y} \sigma_{\Delta V_z}
\]
S.T. \( Dx^2 \cdot \sigma_{\Delta x}^2 + Dy^2 \cdot \sigma_{\Delta y}^2 + Dz^2 \cdot \sigma_{\Delta z}^2 + DV_x^2 \cdot \sigma_{\Delta V_x}^2 + DV_y^2 \cdot \sigma_{\Delta V_y}^2 + DV_z^2 \cdot \sigma_{\Delta V_z}^2 = C_t^2 \),

obtaining the estimates
\[
\hat{\sigma}_{\Delta x} = \frac{|C_t|}{|Dx|\sqrt{6}}; \quad \hat{\sigma}_{\Delta y} = \frac{|C_t|}{|Dy|\sqrt{6}}; \quad \hat{\sigma}_{\Delta z} = \frac{|C_t|}{|Dz|\sqrt{6}},
\]  
(B-16a)
\[
\hat{\sigma}_{\Delta V_x} = \frac{|C_t|}{|DV_x|\sqrt{6}}; \quad \hat{\sigma}_{\Delta V_y} = \frac{|C_t|}{|DV_y|\sqrt{6}}; \quad \hat{\sigma}_{\Delta V_z} = \frac{|C_t|}{|DV_z|\sqrt{6}}.
\]  
(B-16b)

If an estimate \( |Dp| \) for a partial derivative \( \frac{\partial \ell}{\partial p} \) for a parameter \( p \) is nearly 0 (as is the case for parameters \( x \) and \( V_x \) ), then \( \ell \) is insensitive to \( p \). In this instance, we may take \( \hat{\sigma}_p = 0 \), because the \( \hat{\sigma}_p \) 's are employed in simulating errors in \( p \) at the time of gun firing, and the error in \( p \) would be inconsequential insofar as simulating bullet CPA and measures of effectiveness depending upon CPA.

These error standard deviation estimates in equations (B-16) are used to simulate actual launch conditions with respect to airborne targets.

B-7
APPENDIX C

PROGRAM LISTINGS

Presented in this appendix are listings of two BASIC programs used in the determination of TACTS range accuracy requirements.

Program DISCRETE FIT implements the methodology of Appendix A for the computations of position and velocity component standard deviations. Equations (A-7) and (A-8) are involved, together with data produced by TACTS weapon simulation runs.

The other program, DFIT BAT PLOT performs the computations of DISCRETE FIT automatically for a variety of Performance Levels and data from one or several TACTS weapon simulation runs. Analyzed results are stored in data files for subsequent graphics presentations.
DISCRETE - FIT

100 REM DISCRETE_FIT
110 !
120 REM WRITTEN: AUGUST 14, 1986
130 REM LAST MODIFIED: SEPTEMBER 4, 1986
140 !
150 COM /Consts/ P1,B(5)
160 COM /Ratios/ Ratio(1:6)
170 COM /Bounds/ Lower,Upper
180 COM /Output/ Sig(1:6)
190 COM /Intervals/ Nruns,X(1,6,100)
200 DIM Sig$(1:6),Nin$(80)
210 !
220 PRINT PAGE
230 GOSUB 350 ! INITIALIZATION
240 GOSUB 660 ! READ SIMULATION DATA
250 GOSUB 860 ! INPUT PRECISION
260 GOSUB 950 ! PRINT HEADER
270 WHILE FNPlvel(0,Sigv)>Desol
280 GOSUB 1140 ! BISECTION SEARCH
290 PRINT USING "6(2X,4D.2D,X)";Sigv(*)
300 Sigv=Sigv+Delsigv
310 END WHILE
320 PRINTER IS CRT
330 END
340 !
350 REM INITIALIZATION
360 !
370 DATA .2316419
380 DATA .31938153,-.356563782,1.781477937,-1.821255978,1.330274429
390 READ P1
400 FOR Param=0 TO 5
410 READ B(Param)
420 NEXT Param
430 !
440 Lower=0
450 Upper=1
460 !
470 Epsilon=.00001
480 Sigv=0
490 Delsigv=.01
500 !
510 Sig$(1)="SIGMA-X"
520 Sig$(2)="SIGMA-Y"
530 Sig$(3)="SIGMA-Z"
540 Sig$(4)="SIGMA-Ux"
550 Sig$(5)="SIGMA-Uy"
560 Sig$(6)="SIGMA-Uz"
570 !
580 Ratio(1)=1
590 Ratio(2)=1
600 Ratio(3)=4
610 Ratio(4)=1
620 Ratio(5)=1
630 Ratio(6)=4

C-2
RETURN

REM READ SIMULATION DATA

REM X(1,6,100) ! 1 = Lower,Upper
REM ! 6 = X,Y,Z,Vx,Vy,Vz
REM ! 100 = Runs

PRINT "INPUT NAME OF DATAFILE: "
INPUT ",Nin$
PRINT Nin$
PRINT
ASSIGN $1 TO Nin$
READ $1;Nruns
FOR R=1 TO Nruns
FOR Param=1 TO 6
READ $1;X(0,Param,R),X(1,Param,R)
NEXT Param
NEXT R
ASSIGN $1 TO *
RETURN

REM INPUT PRECISION

PRINT
PRINT "INPUT DESIRED PERFORMANCE LEVEL (Between 0 & 1) = "
INPUT ",Despl
PRINT Despl
PRINT
PRINT Despl
PRINT
PRINT NAME OF DATAFILE: ";Nin$
PRINT
PRINT "DESIRABLE PERFORMANCE LEVEL = ";Despl
PRINT
PRINT "POSITION ERROR RATIO - X:Y:Z = ";
PRINT USING "D,A,D,A,D":Ratio(1);":Ratio(2);":Ratio(3)
PRINT "VELOCITY ERROR RATIO - Vx:Vy:Vz = ";
PRINT USING "D,A,D,A,D":Ratio(4);":Ratio(5);":Ratio(6)
PRINT
PRINT USING "2X,6(10A)";Sig$(*)
PRINT
RETURN

REM BISECTION SEARCH
REM FNLEVEL(...) IS A DECREASING FUNCTION IN BOTH ARGUMENTS

Sigilo=.01
WHILE FNlevel(Sigilo,Sigv)<Despl
Sigilo=Sigilo/2

C-3
1200 END WHILE
1210 Siglh=1
1220 WHILE FNLevel(Siglh,Sigv)>Despl
1230 Siglh=2*Siglh
1240 END WHILE
1250 Siglmid=.5*(Sigllo+Siglh)
1260 Plmid=FNLevel(Siglmid,Sigv)
1270 WHILE ABS(Plmid-Despl)>Epsilon
1280 IF Plmid>Despl THEN
1290 Sigllo=Siglmid
1300 ELSE
1310 Siglh=Siglmid
1320 END IF
1330 Siglmid=.5*(Sigllo+Siglh)
1340 Plmid=FNLevel(Siglmid,Sigv)
1350 END WHILE
1360 RETURN
1370 !
1380 END!
1390 !
1400 DEF FNLevel(Sx,Sv)
1410 COM /Bounds/ Lower,Upper
1420 COM /Ratios/ Ratio(*)
1430 COM /Output/ Sig(*)
1440 COM /Intervals/ Nruns,X(*)
1450 DIM S(1:6)
1460 Prob_sum=0
1470 S(1)=Sx*Ratio(1)
1480 S(2)=Sx*Ratio(2)
1490 S(3)=Sx*Ratio(3)
1500 S(4)=Sv*Ratio(4)
1510 S(5)=Sv*Ratio(5)
1520 S(6)=Sv*Ratio(6)
1530 FOR Run=1 TO Nruns
1540 Prob=1
1550 FOR Param=1 TO 6
1560 IF S(Param)=0 THEN
1570 Factor=1
1580 ELSE
1590 Factor=FN(P(X(Upper,Param,Run)/S(Param)))
1600 Factor=Factor*FN(P(X(Lower,Param,Run)/S(Param)))
1610 END IF
1620 Prob=Prob*Factor
1630 NEXT Param
1640 Prob_sum=Prob_sum+Prob
1650 NEXT Run
1660 Plevel=Prob_sum/Nruns
1670 MAT Sig=S
1680 RETURN Plevel
1690 FNEND
1700 !
1710 DEF FNZ(X)
1720 IF ABS(X)>5 THEN
1730 Z=0
1740 ELSE
1750 Z=(1/SQR(2*PI))*EXP(-X*X/2)
1760 END IF
1770 RETURN Z
1780 FNEND
1790
1800 DEF FNP(Xin)
1810 COM /Consts/ P1,B(*)
1820 IF Xin>0 THEN
1830 X=Xin
1840 ELSE
1850 X=-Xin
1860 END IF
1870 T=1/(1+P1*X)
1880 Temp=0
1890 FOR I=5 TO 0 STEP -1
1900 Temp=Temp+B(I)
1910 NEXT I
1920 P=1-FNZ(X)*Temp
1930 IF Xin<0 THEN P=1-P
1940 RETURN P
1950 FNEND
DFIT - BAT - PLOT

29 Jan 1987  10:33:02

100  REM DFIT_BAT_PLOT
110  !
120  REM  WRITTEN:  SEPTEMBER 11, 1986
130  REM  LAST MODIFIED:  OCTOBER 2, 1986
140  !
150  COM /Consts/  Pi,B(5)
160  COM /Ratios/  Ratio(1:6)
170  COM /Bounds/  Lower,Upper
180  COM /Output/  Sig(1:6)
190  COM /Intervals/  Nrns,X(1,6,100)
200  DIM Sig$(1:6),Indexfile$(30),Nin$(80)
210  DIM Xout(0:50),Yout(0:50)
220  !
230  PRINT PAGE
240  GOSUB 710  !  INITIALIZATION
250  GOSUB 1010!  DETERMINE CASES TO BE RUN
260  FOR Case=1 TO Ncases
270  GOSUB 1210!  READ SIMULATION DATA
280  FOR Despl=.99 TO .7 STEP -.05
290  GOSUB 1870!  BISECTION SEARCH FOR MaxSigv
300  Neval=25 ! NUMBER OF EVALUATIONS EQUALLY SPACED
310  Extra=4 ! EXTRA EVALUATIONS TO SMOOTH UP GRAPH
320  Delsigv=Maxsigv/Neval
330  GOSUB 1390!  PRINT HEADER
340  FOR Eval=0 TO Neval+Extra
350    SELECT Eval
360      CASE =Neval+Extra
370        GOSUB 1790
380      CASE ELSE
390        IF Eval<Neval THEN
400          Sigv=Eval*Delsigv
410        ELSE
420          Sigv=(Neval-1)*Delsigv+(Eval+1-Neval)*Delsigv/(Extra+1)
430        END IF
440        GOSUB 1550!  BISECTION SEARCH
450    END SELECT
460    PRINT USING "6(2X,4D.2D,X)";Sig(*)
470    Xout(Eval)=Sig(4)
480    Yout(Eval)=Sig(1)
490  NEXT Eval
500  GOSUB Putoutplotdata
510  IF NOT Hc THEN WAIT 1
520  NEXT Despl
530  NEXT Case
540  PRINT PAGE
550  PRINTER IS CRT
560  END
570  !
580  Putoutplotdata:  !ESTABLISH PLOTDATASEARCH
590  !
600  Pname$=Casename$(Case)""&VAL$(Ratio(3))""&VAL$(100*Despl)
610  CREATE DATA Pname$10
620  ASSIGN #1 TO Pname$
630  PRINT $1;Neval+Extra+1

C-6
FOR Eval = 0 TO Nmax + Extra
   PRINT $1; Xout(Eval), Yout(Eval)
   NEXT Eval
   ASSIGN $1 TO *
   PRINT $1
   RETURN

REM INITIALIZATION
DATA .2316419
DATA 0., .31938153, - .356563782, 1.781477937, -1.821255978, 1.330274429
READ P1
FOR Param = 0 TO 5
   READ B(Param)
NEXT Param
DATA Lower = 0
DATA Upper = 1
DATA Epsilon = .00001
DATA Delsigv = .05
DATA Sig$(1) = " SIGMA-X ", Sig$(2) = " SIGMA-Y ", Sig$(3) = " SIGMA-Z ",
   Sig$(4) = " SIGMA-Vx ", Sig$(5) = " SIGMA-Vy ", Sig$(6) = " SIGMA-Vz ",
   Ratio(1) = 1
   Ratio(2) = 1
   Ratio(3) = 1
   Ratio(4) = 1
   Ratio(5) = 1
   Ratio(6) = 1
RETURN

REM DETERMINE CASES TO BE RUN
PRINT "INPUT INDEX FILE NAME OF CASES TO BE RUN: ";
INPUT "", Indexfile$
PRINT Indexfile$
ASSIGN $1 TO Indexfile$
READ $1; Ncases
FOR Case = 1 TO Ncases
   READ $1; Casename$(Case)
   READ $1; Case$
NEXT Case
ASSIGN $1 TO *
PRINT "HARDCOPY (Y or N): ";
INPUT "", Hc$
PRINT Hc$
Hc = (Hc$ = "Y")
IF Hc THEN PRINTER IS 401
RETURN
1200 !
1210 REM READ SIMULATION DATA
1220 !
1230 REM X(1,6,100): 1 = Lower, Upper
1240 ! 6 = X, Y, Z, Vx, Vy, Vz
1250 ! 100 = Runs
1260 !
1270 N;n$=Casename$(Case)
1280 PRINT
1290 ASSIGN $1 TO N;n$
1300 READ $1;Nruns
1310 FOR R=1 TO Nruns
1320 FOR Param=1 TO 6
1330 READ $1;X(0,Param,R),X(1,Param,R)
1340 NEXT Param
1350 NEXT R
1360 ASSIGN $1 TO *
1370 RETURN
1380 !
1390 REM PRINT HEADER
1400 !
1410 PRINT PAGE
1420 PRINT "NAME OF DATAFILE: ";N;n$
1430 PRINT "DESIRED PERFORMANCE LEVEL = ";Despl
1440 PRINT " POSITION ERROR RATIO - X:Y:Z = ";
1450 PRINT USING "D,A,D,A,D";Ratio(1);":";Ratio(2);":";Ratio(3)
1460 PRINT " VELOCITY ERROR RATIO - Vx:Vy:Vz = ";
1470 PRINT USING "D,A,D,A,D";Ratio(4);":";Ratio(5);":";Ratio(6)
1480 PRINT
1490 PRINT USING "2X,6(10A)";Sig$(*)
1500 PRINT
1510 RETURN
1520 !
1530 REM BISECTION SEARCH
1540 REM FNPLEVEL(,,,) IS A DECREASING FUNCTION IN BOTH ARGUMENTS
1550 !
1560 SIGilo=.01
1570 WHILE FNPLEVEL(Siglio,Sigv)<Despl
1580 Siglio=Siglio/2
1590 END WHILE
1600 Siglhi=1
1610 WHILE FNPLEVEL(Siglhi,Sigv)>Despl
1620 Siglhi=2*Siglhi
1630 END WHILE
1640 SIGlmid=.5*(Siglio+Siglhi)
1650 Plmid=FNPLEVEL(Siglmid,Sigv)
1660 WHILE ABS(Plmid-Despl)<Epsilon
1670 IF Plmid>Despl THEN
1680 Siglio=Siglmid
1690 ELSE
1700 Siglhi=Siglmid
1710 END IF
1720 SIGlmid=.5*(Siglio+Siglhi)
1730 Plmid=FNPLEVEL(Siglmid,Sigv)
1740 !
1760 END WHILE
1770 RETURN
1780 !
1790 REM SIG(NEVAL) VALUES
1800!
1810 MAT Sig=(0)
1820 FOR I=4 TO 6
1830 Sig(I)=Maxsig*Ratio(I)
1840 NEXT I
1850 RETURN
1860!
1870 REM BISECTION SEARCH
1880 REM FNPLEVEL(S,)
1890!
1900 Sigllo=.01
1910 WHILE FNPLEVEL(0,)
1920 Sigllo=Sigllo/2
1930 END WHILE
1940 Siglhi=1
1950 WHILE FNPLEVEL(0,)
1960 Siglhi=2*Siglhi
1970 END WHILE
1980 Siglmid=.5*(Sigllo
1990 Plmid=FNPLEVEL(0,)
2000 WHILE ABS(Plmid-Dem)
2010 IF Plmid>Despl Th
2020 Sigllo=Siglmid
2030 ELSE
2040 Siglhi=Siglmid
2050 END IF
2060 Siglmid=.5*(Sigllo
2070 Plmid=FNPLEVEL(0,)
2080 END WHILE
2090 Maxsig=Siglmid-Ep
2100 RETURN
2110 END
2120!
2130 DEF FNPLEVEL(Sx,Sv
2140 COM /Bounds/ Lows
2150 COM /Ratios/ Rati
2160 COM /Output/ Sig(2170 COM /Intervals/ N
2180 DIM S(1:6)
2190 Prob_sum=0
2200 S(1)=Sx*Ratio(1)
2210 S(2)=Sx*Ratio(2)
2220 S(3)=Sx*Ratio(3)
2230 S(4)=Sv*Ratio(4)
2240 S(5)=Sv*Ratio(5)
2250 S(6)=Sv*Ratio(6)
2260 FOR Run=1 TO Nruns
2270 Prob=1
2280 FOR Param=1 TO 6
2290 IF S(Param)=0 THEN
2300 Factor=1
2310 ELSE
2320
Factor=FNP(X(Upper,Param,Run)/S(Param))
Factor=Factor-FNP(X(Lower,Param,Run)/S(Param))
END IF
Prob=Prob*Factor
NEXT Param
Prob_sum=Prob_sum+Prob
NEXT Run
Plevel=Prob_sum/Nruns
MAT Sig=S
RETURN Plevel
FNEND

DEF FNZ(X)
IF ABS(X)>5 THEN
Z=0
ELSE
Z=(1/SQR(2*PI))*EXP(-X*X/2)
END IF
RETURN Z
FNEND

DEF FNP(Xin):
COM /Consts/ P1,B(*)
IF Xin>0 THEN
X=Xin
ELSE
X=-Xin
END IF
T=1/(1+P1*X)
Temp=0
FOR I=5 TO 0 STEP -1
Temp=Temp*T*B(I)
NEXT I
P=1-FNZ(X)*Temp
IF Xin<0 THEN P=1-P
RETURN P
FNEND