A COMPLETE MODEL FOR SEISMIC SOURCE BEHAVIOR OF PEACEFUL NUCLEAR EXPLOSIONS

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The four most widely accepted forms used to represent the source time history of a nuclear explosion have been unified into a single simple time domain expression. A unique feature of this expression is that the Mueller-Murphy representation is given in terms of two characteristic frequency parameters (k's) and two independent source shaping parameters (B's). The Haskell, Helmburger-Hadley and von Seggner-Blandford representations have always been specified by one of each. There are a total of ten free parameters in the generalized fitting function which are too many for fitting any one type of seismic observation. Yet, they may be useful for unifying different types of data. In particular the Mueller-Murphy parameters are useful for modelling cavity scaling observations and the Helmburger-Hadley are best for studies of near field seismograms. However, there is a clear need to relate the investigations of both types of information. No unique mathematical relationship between the ten free parameters exists, but the differences between the pulses they predict can be measured in a least squares sense. To accomplish this, we have assumed that the Mueller-Murphy source is in fact correct and have found those source functions of the other three representations which approximate it most closely. An important result is that each of the four realizations of the unified source representation can be related to the others in a meaningful way, and the differences are negligible in a least squares sense. Future work needs to be directed at unifying the scaling laws of each representation which is where the real physics lies.
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Unified Source Representations for Nuclear Explosions

A wide variety of investigations in seismology call for a nuclear source representations specified by a time history for an isotropic point force acting in an elastic medium. Because of the diverse nature of these studies, a number of alternate representations have emerged. Unfortunately, more candidate formulations, along with their associated free parameters, have been proposed than required to fit the observable quantities. The purpose of this report is to attempt to present a unified view of the most commonly used representations, to tabulate their mathematical representations in a uniform way and to interrelate their free parameters. This last step serves to reduce the number of unknowns to those which can be meaningfully constrained by the observable quantities.

The four source representations we attempt to unify are those proposed in chronological order by Haskell (1967), Mueller and Murphy (1971), von Seegern and Blandford (1972) and Helmberger and Hadley (1981). Haskell (1967) attempted to cast his formulation in a mode compatible with his well-known representation for a double couple. The time-reduced displacement potential is given by

\[ \Psi(t) = \Psi_\infty f(t - \tau) \]  

(1)

The function \( f \) is arbitrarily chosen in this instance to be a damped fourth order polynomial (See Table 1). He argued that the high order of the polynomial was required to keep the predicted acceleration function continuous; a restriction abandoned in later models. The far field displacement history is then just the time derivative of the potential. For the purposes of differentiating between the four basic source representations, it is desirable to consider three key factors; the number of free parameters, whether any additional physical model is required and the data which the source function was tailored to fit. In the Haskell (1967) source, there are three free parameters. The first is a corner frequency which we designate as \( k_4 \), the second a source shape parameter, \( B_4 \), and the third is a source strength parameter,
\( \Psi_{\infty} \). It is comparable to earthquake moment and is assumed to be related to the final cavity volume. The choice of three free parameters is comparable to the widely adopted triangle function used to represent earthquake sources (Burdick and Mellman, 1976). It has a corner frequency (inverse to total duration), a shaping parameter (the ratio of rise to fall time) and a strength parameter (earthquake moment). In contrast, the Brune (1970) earthquake source has only the two free parameters corner frequency and moment. No additional physical model is involved for the Haskell (1967) representation other than it being the time history of an isotropic source. The data it used to fit were a relatively few direct observations of the very close-in observations of small events reported on by Werth and Herbst (1963). It achieved its purpose of fitting these observations though von Seegern and Blandford (1972) concluded that significant improvement could be made through the use of their representation.

The Mueller and Murphy (1971) source model is unique among the four in that is is based on a more complex physical model than the others. It requires the specification of a pressure time history at the surface of a sphere with elastic radius, \( r_{el} \). Nonlinear source processes are assumed to occur inside the radius and medium behavior is assumed to be elastic outside of it. Mueller and Murphy (1971) chose the arbitrary form for the pressure function of

\[
p(t) = p_{\infty}[1 + (p_{0r}/p_{\infty}e^{-k_0t})]
\]

where \( p_{\infty} + p_{0r} \) is the initial and \( p_{\infty} \) is the residual pressure at large \( t \). The parameter \( k_0 \) is the first of two corner frequencies introduced in the model. Note that at this point there are already three free parameters in the model; the total number in the previous case. The simple introduction of the more complex physical model may seem innocuous, but it leads inevitably to the requirement of several additional free parameters. The first is a new corner frequency given by \( v_p/r_{el} \). This inherently involves one of three source medium parameters, and in general, \( v_s \) and density, \( \rho \), are required also. The source medium is generally assumed to be a Poisson solid which eliminates one medium parameter, but the Mueller and Murphy (1971) source does require four source and two medium parameters. As given in Table 1,
the source requires two corner frequency \( (k) \), two source shape \( (B) \) and one source strength \( (\Psi_\infty) \) parameters. The explicit relationships between them along with event yield \( (Y) \) and burial depth \( (h) \) dependence are given in Table 2. A significant feature of the representation is that the simple introduction of the more detailed physical model introduces an additional time history into the problem. The response of the medium to a step in pressure is not a delta function but a damped sinusoid.

It is important to emphasize that the slight drawbacks involved in adding additional free parameters in the Mueller and Murphy (1971) representation is more than offset by the advantages gained in being able to unify a wide variety of observations. These most significantly include a variety of non-dynamic medium and cavity properties which can be measured before and after the shot along with the usual teleseismic and regional spectral ratios. After being expanded upon by Murphy (1977), the source representation has been successfully tested and used in more separate investigations than any of the others. A small sample of these would include Murphy et al. (1989), Bache (1982), Burger et al. (1987) and Saikia and Burdick (1991).

The von Seggern and Blandford (1972) source was introduced in a study which had much in common with this one. It represents an attempt to unify the two previously discussed source representations. The order of the polynomial in the basic Haskell (1967) formulation was reduced to two. The predicted far field velocity pulse is discontinuous and the acceleration pulse singular, just as it is for the Mueller-Murphy. There are three free parameters as listed in Table 1. The additional physical considerations in the Mueller-Murphy representation are embodied in the scaling laws as discussed in the following section. Since the basic form of the von Seggern-Blandford source was chosen to mimic the Mueller-Murphy, it is not surprising that it has been utilized just as successfully. A partial list of the investigations in which it played an important role includes Der et al. (1985) and Burdick and Helmberger (1979).

The final representation to be included in the discussion is that of Helmberger and Hadley
### TABLE 1: SOURCE REPRESENTATION FUNCTIONS

**GENERAL FORMULATION**

\[
\frac{\Psi(t)}{\Psi_\infty} = 1 + P(B_n, k_1, t)e^{kt} - B_0e^{-k_1t}
\]

Haskell, von Seggern-Blandford, Helmberger-Hadley

\[
P = \sum_{n=1}^{4} B_n k^n
\]

Mueller-Murphy

\[
P = -B_5 \sin(k_1 t) - B_6 \cos(k_1 t)
\]

<table>
<thead>
<tr>
<th>Source Function</th>
<th>$B_0$</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
<th>$k$</th>
<th>$k_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Haskell</td>
<td>0</td>
<td>1</td>
<td>.5</td>
<td>1/6</td>
<td>$B_4$</td>
<td>0</td>
<td>$k_4$</td>
<td>0</td>
</tr>
<tr>
<td>von Seggern-Blandford</td>
<td>0</td>
<td>1</td>
<td>$B_2$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>.5</td>
<td>$B_3$</td>
<td>0</td>
<td>0</td>
<td>$k_3$</td>
<td>0</td>
</tr>
<tr>
<td>Mueller-Murphy</td>
<td>$B_0$</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>$B_5$</td>
<td>$k_3$</td>
<td>$k_5$</td>
</tr>
</tbody>
</table>


TABLE 2: THE MUELLER-MURPHY PHYSICAL MODEL

Basic Physical Parameters:
- Elastic Radius - \( r_{el} \)
- Cavity Radius - \( r_c \)
- Model Corner Frequency - \( \omega_0 = r_{el}/v_p \)
- Pressure Corner Frequency - \( k_0 \)
- Medium parameters; P velocity, S velocity and density - \( v_p, v_s, \text{and} \rho \)
- Pressure Function - \( P(t) = P_0 e^{-k_0 t} + P_{0c} \)
  with \( P_0 \) and \( P_{0c} \) being functions of \( r_{el}, r_c \), the medium parameters and source depth.

Combining Equation (1) from Haskell (1967) and Equation (1) from Murphy (1977) transferred into the time domain and integrating produces the convolution operation:

\[
\Psi(t) = \left( \frac{r_{el}}{4\rho b} \right) P(t) * e^{-k_s t} \sin(bt)
\]

with the definitions:
\[
k_s = \frac{2\omega v_s^2}{v_p^2}
\]
\[
b = k_s (v_p^2/v_s^2 - 1)^{1/2}
\]
\[
c = (k_s - k_0)
\]

Carrying out the convolution gives:

\[
\Psi(t)/\Psi_\infty = 1 - B_0 e^{-k_0 t} - B_5 e^{-k_s t} \sin(bt) - B_6 e^{-k_s t} \cos(bt)
\]

with
\[
\Psi_\infty = r_{el}^3 P_{0c}/4\rho v_p^2
\]
\[
B_0 = -P_0(k_s^2 + b^2)/[P_{0c}(c^2 + b^2)]
\]
\[
B_5 = -(k_s + cB_0)/b
\]
\[
B_6 = 1 - B_0
\]
(1981). Its form was chosen to be intermediate between that of Haskell and von Seegern and Blandford. The damped polynomial was chosen to be of third order, so that teleseismic velocity is continuous and acceleration discontinuous (Table 1). There are three free parameters as in the two related cases. There is no additional physical model involved, but there are a unique and significant variety of data that it was designed to accommodate. These are near field (<25 km) observations of ground velocity from broadband velocity meters. The prediction for the Mueller-Murphy or von Seegern-Blandford source would be that the observations would be discontinuous in an elastic half space, and of course, they are not. For this very valid reason, the Helmberger-Hadley source was introduced. The actual near field studies in which it was applied were Helmberger and Hadley (1981) and Barker et al. (1991) for NTS and Burdick et al. (1984) for Amchitka. Since then, it has been used in many other investigations including Lay (1985), Burger et al. (1987) and Lay et al. (1984).

To summarize, each of the four representations makes use of a subset of the seven possible damped terms in the general formulation. The Haskell includes four of the polynomial terms, but only $k_4$ and $B_4$ are treated as unknowns. It is notable for being chronologically first, but it has not been used to model as extensive a data base as the later versions. The Mueller-Murphy source involves three damping terms with $B_6$, $B_7$, $B_8$, $k_6$ and $k_7$ being variable. It requires that the medium constants be specified, but through its more involved physical model it can unify the widest variety of observations. It has probably been successfully used in the widest number of cases. The von Seegern-Blandford uses two damping terms with $k_2$ and $B_2$ being variable. It has been used to explain source variations in a wide variety of teleseismic and regional signals. The Helmberger-Hadley source employs three damping terms with $k_3$ and $B_3$ variable. It has proved most useful in modeling near field observations for which it predicts a continuous velocity pulse. Of the seven possible corner frequency and seven source shaping parameters, five of each are used in the four possible source representations. If each representation were used to fit an observation which was not exactly any of them, ten independent values would be determined. As we show in
the following, practical considerations can be used to effectively reduce this number very substantially.

**Yield Scaling Relations**

In the above, we have treated the four source representations as ansatz fitting functions with the corner frequencies and shaping parameters being the free variables. In practice, a seismologist is generally confronted with observations from an event for which a number of standard parameters are known. In an ideal situation, these include event location, magnitude ($m_b$), burial depth and emplacement medium parameters. The information has been historically used in both an inverse and forward modelling sense. That is, the free parameters ($B$'s and $k$'s) discussed in the previous section have been used in an inverse sense to measure their yield dependence and in a forward sense to estimate explosion source functions for other types of studies. In fact, in most cases today, a known or suspected nuclear explosion has only associated with it an $m_b$ and a location. Other source parameters are then predicted from this information and used to predict a source time history. These are then finally used in earth structure (Saikia and Burdick, 1991) or event yield and discrimination studies (Murphy, 1989).

The inverse type of studies have been directed toward establishing the dependence of the fitting parameters on yield, depth and emplacement medium and expressing them in terms of what are termed yield scaling laws. In practice, the scaling laws, as given for each source type in Table 3 should be thought of as an integral part of the source representations given in Table 1. Perhaps they are the more important part since they embody the actual measured results. As above, the work of Haskell (1967) was seminal and mapped the course for the later studies. Using simple arguments based on dimensionality, he inferred that $\Psi_{\infty}$ should be proportional to yield, $k_4$ should scale as its cube root and $B_4$ should be yield independent. In Table 3 we list his laws for alluvium and granite, but these laws are only
loosely comparable to the others given for Pahute Mesa and Amchitka. He also gave laws for alluvium and salt. His form for the laws has become known as cube root scaling.

Mueller and Murphy (1971) concluded that simple cube root scaling did not satisfy their observations. They introduced depth into the scaling laws which they were able to do because they considered a more detailed physical model. We give the laws in terms of the most fundamental free parameters following Murphy (1977). We also give a convenient depth scaling rule which is appropriate for Pahute Mesa. The explicit depth dependencies appear in the expressions for elastic radius, cavity radius and peak shock pressure. They have the effect of the system corner frequencies having a yield dependence which is measurably different from the simple inverse cube root of yield. The parameters given in Table 3 are related to $B_5$ and $B_6$ in Table 2.

As it was initially introduced, the von Seggern-Blandford representation was presented as an equivalent to the Mueller-Murphy source and scaled to fit teleseismic observations from Amchitka. An important equivalence was inferred between $k_0$ and $k_2$, and between $B_2$ and $2p = (2p_{0s}/p_{0c})$ in an attempt to unify the two formulations. We will suggest some very different relationships in the following.

No scaling law relationships were proposed in the initial Helmberger and Hadley (1981) study. They simply measured near field source strength in terms of their representation, and determined $t^*(t^*_a = 1.3s)$ for the Pahute Mesa test site. The important scaling law behaviors were developed in the investigations of Burdick et al. (1984) and Lay et al. (1984) for Amchitka and Barker (1991) for Pahute Mesa. The form for the yield dependence of $\Psi_\infty$ and $k_3$ were chosen to be the same as in the Mueller-Murphy representation. $B_3$ was chosen to be constant by Burdick et al. (1984), as in the Haskell representation because its yield dependence is predicted to be weak by the Mueller-Murphy laws and was not resolvable from any of the data available. Lay et al. (1984) determined a $B_3$ dependence opposite in sense to the yield dependence in the Mueller-Murphy laws. In Table 3, we present the former because of the internal consistency with the other laws. We do not intend to imply that there are
TABLE 3: SCALING LAWS

Haskell Source
Tuff - Rainier Mesa

\[ \Psi_\infty = 1024 Y \text{meters}^3 \]
\[ k_4 = 40.2 / y^{1/3} \text{hz} \]
\[ B_4 = \text{const} = 0.05 \]

Granite - HARDHAT

\[ \Psi_\infty = 500 Y \text{meters}^3 \]
\[ k_4 = 54.0 / y^{1/3} \text{hz} \]
\[ B_4 = \text{const} = 0.24 \]

Mueller-Murphy Source
Pahute Mesa - Wet Tuff Murphy (1977)

\[ r_{el} = 1490 Y^{1/3} / h^{0.42} \text{meters} \]
\[ r_c = 31.4 Y^{0.29} / h^{0.11} \text{meters} \]
\[ k_5 = v_p / r_{el} \text{hz} \]
\[ k_0 = 1.5 k_5 \text{hz} \]
\[ P_{0c} = 0.8 \mu (r_c / r_{el})^2 \]
\[ P_{0s} = 1.5 \rho gh - P_{0c} \]
\[ v_p = 3.5 \text{km/s} \]
\[ v_s - v_p / \sqrt{3} \]
\[ \rho = 2.0 \text{gm/cm}^3 \]
\[ h = 122 Y^{1/3} \]

Amchitka

\[ r_{el} = 1490 Y^{1/3} / h^{0.42} \text{meters} \]
\[ r_c = 24.7 Y^{0.29} / h^{0.11} \text{meters} \]
\[ k_5 = v_p / r_{el} \text{hz} \]
\[ k_0 = 1.5 k_5 \text{hz} \]
\[ P_{0c} = 0.8 \mu (r_c / r_{el})^2 \]
\[ P_{0s} = 1.5 \rho gh - P_{0c} \]
\[ v_p = 3.5 \text{km/s} \]
\[ v_s - v_p / \sqrt{3} \]
\[ \rho = 2.0 \text{gm/cm}^3 \]
Von Seggern - Blandford Source
Pahute Mesa (wet tuff) (see Der et al., 1985 for cube root scaling)

\[ \Psi_\infty = 6200Y^{0.87}/h^{1/3} \]
\[ k_2 = \gamma \nu_p/\tau_{el} \]
\[ B_2 = (P_{0s} - P_{0c})(2P_{0c}) \quad \text{see Mueller-Murphy above} \]

Amchitka

\[ \Psi_\infty = 4000Y^{0.87}/h^{1/3} \]
\[ k_2 = \gamma \nu_p/\tau_{el} \]
\[ B_2 = (P_{0s} - P_{0c})(2P_{0c}) \]

Helmerger - Hadley Source
Pahute Mesa (Barker et al., 1991)

\[ \Psi_\infty = 1600Y/h^{0.27} \text{meters}^3 \]
\[ k_3 = 3.75h^{1/2.4}/Y^{1/3}h_2 \]
\[ B_3 = \text{const} = 1.0 \]

Amchitka (Burdick et al., 1984) (see Lay et al. (1984) for alternate \( B_3 \) scaling).

\[ \Psi_\infty = 950Y/h^{0.27} \text{meters}^3 \]
\[ k_3 = 4.7h^{1/2.4}/Y^{1/3}h_2 \]
\[ B_3 = \text{const} = 1.0 \]
any reasons to disregard the observations of Lay et al. (1984).

We thus have four source representations unified by a single expression (Table 1) with four sets of scaling laws. These laws have 10 independent parameters which we can reduce through the techniques discussed in the following. The necessary input parameters for predicting a source time function from Tables 1 and 3 are just event yield and depth. To estimate a source time function starting with only an \( m_b \) value, we recommend the use of an \( m_b \) yield curve such as

\[
m_b = 3.695 + 0.8019 \log(Y)
\]

(3)

which is appropriate for Pahute Mesa (Burger et al., 1987). We recommend the ISC values for \( m_b \) since these were used in many of the initial studies. Depth can be estimated from a scaling law such as the one given for Pahute Mesa in Table 3. The source time history is then specified by the formalism of choice.

### Measured Correlations Between the Source Representations

The goal of this section will be to reduce the number of free parameters in the general source representation through some simple practical considerations. In doing so, we acknowledge the special position of the Mueller-Murphy and the Helmberger-Hadley representations. The former is important because of its more complex physical model and inclusion of empirical observations of cavity size scaling. The latter model is general enough to allow modeling of near field velocity records. Both are being used in a variety of contemporary investigations such as intercorrelation and network-averaged spectra studies.

The approach pursued is to assume that the Mueller-Murphy representation is in fact exact and to measure in a quantitative sense how well the other source functions could mimic it. We determine simple ratios between the Mueller-Murphy parameters and those of the other source representations. This accomplishes our goal of drastically reducing the number of free parameters in the general representation, and focuses the direction of future work on
where it properly belongs; that is, which are appropriate yield scaling relations. Mueller-Murphy sources were accordingly generated for a suite of yield values between 10 and 1000 kt and best-fitting time histories determined using a simple parameter search approach. The true source is assumed to be a Mueller-Murphy in saturated tuff rhyolite. The results are summarized in Figures 1 and 2. The initial \( k \) and \( B \) values for Mueller-Murphy are shown in light line and the measured values for the other three representations are shown in dark.

The basic tool we utilize for measuring the variations between pulses is a simple least squares measurement of the difference between the far field displacement signals of the various source types. More precisely, we measure the maximum of the correlation function between predicted pulses which is the same as the minimum of the standard least squares difference between spectra in the frequency domain. The optimization of the correlation and frequency domain least squares norms were the techniques used in the validation studies of the Helmberger-Hadley source and the Mueller-Murphy source by Lay (1985) and Murphy et al. (1989) respectively. We choose the far field displacement pulse because it is predicted to be continuous by each of the four source representations. We again emphasize the special relevance of the Mueller-Murphy and the Helmberger-Hadley sources. The former has two basic corner frequencies which are independent and cannot be linked. The first of these is related to the arbitrary choice of a pressure time history at the elastic radius and the second to the value of the elastic radius itself. The Helmberger-Hadley source is continuous to velocity and was recently used in a number of near field velocity record studies. It is also intermediate in character between the other two source representation functions.

Figure 1 compares the measured \( k \) values for the three polynomial-based source representations to the analytic \( k_0 \) and \( k_5 \) values of the Mueller-Murphy source. As we noted earlier, von Seggern and Blandford (1972) postulated that their \( k \) (that is \( k_2 \)) should be the same as the Mueller-Murphy \( k_5 \). They discussed the possibility that the Mueller-Murphy \( k \) could be some fraction of theirs, but concluded that the ratio was essentially one. As shown in the figure, though the shape of the two curves is essentially the same, for the Pahute Mesa
Figure 1: The $k$ values measured for the von Seegern-Blandford ($k_2$), Helmerberger-Hadley ($k_3$), and Haskell ($k_4$), sources shown with dark lines, are compared to the two $k$'s for Mueller-Murphy shown in light.
The $B$ values measured for the von Seggern-Blandford ($B_2$), Helmberger-Hadley ($B_3$), and Haskell ($B_4$), sources shown with dark lines, are compared to the two $B$'s for Mueller-Murphy shown in light. It has been claimed in the literature that the von Seggern-Blandford $B$ would be approximately $2p$. 
tuff, the ratio is about 1.66. The correlation coefficient never falls below 0.9921 which is far higher than the correlation norms found in the intercorrelation studies. As might have been expected, it falls somewhere between the two Mueller-Murphy characteristic frequencies. The ratio of the Helmberger-Hadley \( k \) to \( k_5 \) is about 0.298 with correlation coefficients all higher than 0.9969. The Haskell \( k \) ratio is 2.92 with coefficients greater than 0.9959.

The patterns of \( B \) scaling are much more complex and interesting than those of the characteristic frequencies. We begin in Figure 2 with the three source-shaping parameters of the Mueller-Murphy source. These are \( B_0 \) which modifies the exponential term, \( B_5 \) which modifies the cosine term and \( B_6 \) which modifies the sine term. Though \( B_6 \) is generally the largest, and even though it is just \((1 - B_6)\), we select it as our basis parameter as we selected \( k_5 \) in the previous section. The term \( 2p \), which von Seigern and Blandford suggest is the same as \( B_2 \) is also shown in light line. Clearly, there is a major offset between the two curves. In fact, we find that \( B_2 \) is \( X \) times larger than \( 2p \) if we neglect the negative values of the latter. An important point regarding \( B_3 \) and \( B_4 \) is that they are essentially independent of yield, just as they were proposed to be by Haskell and Helmberger-Hadley. Nonetheless, we can estimate their average ratio with respect to \( B_6 \). The results are summarized in Table 4.

The minimum correlation coefficients are the same as discussed in the previous section since the optimal \( k \) and \( B \) values were searched for in tandem.

**Discussion**

The fundamental point of the preceding is that the unified form of the four source representations is much too general, and that it needs to be simplified. In particular, there is no reason to select one of the four representations over the other. The only two real restrictions are the capacity of the Helmberger-Hadley source to predict continuous near field records and of the Mueller-Murphy to incorporate cavity scaling and other in-situ information. For the purposes of inverting large amounts of teleseismic short period waveform data into a
TABLE 4: UNIFIED SOURCE PARAMETERS

Mueller-Murphy Formulation: $k_0$, $k_5$ and $B_6$ Independent

\[ B_0 = 1 - B_6 \]

Equivalent Parameters for von Seggern-Blandford

\[ k_2 = 1.66k_5 \]
\[ B_2 = 0.797B_6 \]

Equivalent Parameters for Helmberger-Hadley

\[ k_3 = 2.36k_5 \]
\[ B_3 = 0.298B_6 \]

Equivalent Parameters for Haskell

\[ k_4 = 2.92k_5 \]
\[ B_4 = 0.0823B_6 \]
yield estimation scheme, the four representations are interchangeable. The tangible physical differences are embodied in the scaling laws listed in Table 3. Recall that Haskell (1967) used only dimensionality arguments and limited observations from a few different emplacement media. His cube root scaling laws did not account for depth. It certainly seems intuitively obvious that identical explosions under very different levels of confining pressure should have different source time histories. The fundamental considerations regarding the effects depth on yield scaling were postulated by Mueller and Murphy (1975). The von Seggern and Blandford (1972) exercise simply preceded this one in attempting to unify the previous two. The Helmberger-Hadley scaling laws as developed by Burdick et al. (1984) and Barker et al. (1991) were also designed to follow Mueller-Murphy. Table 3 shows that $k_3$ and $k_5$ have a virtually identical depth dependence.

There is still room for additional work in this area, but it should be focused on refining the scaling laws from both an observational and theoretical standpoint. It is well known that the Mueller-Murphy and Helmberger-Hadley laws predict significantly different telesismic amplitudes. This results in different estimates of $t^*$ (Burger et al., 1987). There is certainly a great need to further pursue the $\Psi_\infty$ paradox brought to light by Lay et al. (1984). Also, the recent tests by the French and the continuing experiments by China provide an important new data base.

Conclusions

In Table 1 we show five $k$'s or characteristic frequencies and five shaping parameters or $B$'s required in the unified source model. In Table 4 we show that these can for all practical purposes be reduced to just one of each. The correlation norms between the Mueller-Murphy source and the three alternates are so high that they do not justify the choice of any one over the other. There are very valid physical constraints on the shape of the nuclear source time function, but these originate from the particular experiments designed to measure them and
are embodied in the scaling laws, not the form of the fitting functions. The two most direct observations come from the in-situ experiments of Mueller-Murphy and the near field records which constrain Helmberger-Hadley. The investigations of large teleseismic data bases, such as intercorrelation or network-averaged spectra, measure changes in source time histories but no absolutes and depend on assumptions regarding the frequency dependence of $Q$. 
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