**New Computational Analytical Methods for Scattering and Propagation in Random Media**

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**Scattering in inhomogeneous environments with irregular boundaries has ramifications for a number of practical problems of interest to the U.S. ARMY and other government agencies. Subdividing the problem into smaller and thereby more easily managed units has been successfully exploited. Under this grant new methods were developed that are particularly well suited to highly directed propagation. Propagation at low grazing angles relative to the boundary affords further simplification when backscatter can be neglected all together. New computation methods have been developed that can be used in place of the parabolic-wave-equation methods which remain restricted to low grazing angles.**

**Enclosure 1**
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NEW COMPUTATIONAL/ANALYTICAL METHODS FOR SCATTERING & PROPAGATION IN RANDOM MEDIA—— FINAL REPORT

by
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I Statement of the Problem

Propagating wave fields are scattered by inhomogeneities, obstacles, and boundaries. Scattering by weak inhomogeneities, smooth boundaries, discrete scatterers, and rough surfaces are generally characterized in isolation. Many practical problems, however, involve mutual interactions among these different types of scatterers. Under this research grant we have developed new techniques that effectively combine the cannonical methods to build more realistic models.

Any propagation medium can be formally partitioned into discrete elements whose local response to impinging waves can be calculated in isolation. The response to an arbitrary excitation can then be calculated as a mutual interaction problem. Wayne Chew [1] and his coworkers at the University of Illinois have demonstrated substantial computational improvements by applying this principle to cylindrical- and spherical-wave interactions in a variety of applications. In their approach waves impinge upon and are scattered from regions that are bounded by spheres or cylinders. In our approach, we use slab-like partitions oriented normal to the principal propagation direction.

The motivation for pursuing the slab approach came from the multiple-phase-screen model as it is used for propagation in unbounded randomly irregular media. The mathematical basis for the model is the split-step algorithm in which a propagation step is followed by a phase perturbation. Each phase screen represents a slab whose idealized scattering characteristics are particularly simple; however, the phase screen itself is an approximation restricted to restricted to narrow scattering angles. If the phase-screen is replaced by the generalize scattering function for the slab a formally exact system of equations results. Each wave component entering the slab generates a family of scattered waves in both the forward and backscatter directions. These scattered waves impinge upon the adjacent slabs as incident fields. The complete interaction is characterized by a pair of mutual interaction equations.

The remainder of this final report summarizes the theoretical development and applications of the mutual interaction method (MIM), which are described in detail in the publications cited.
II Summary of Principal Results

II.1 Project Overview

Continuing work that was initiated under a previous ARO grant (DAALO3-87-C-002), we developed a rigorously correct phase-screen-like formalism for both discrete and continuous random media. For example, the mutual interaction form of the propagation equations for continuous random media can be written as follows:

\[
\frac{\partial \psi^+(K; x)}{\partial x} = ikg\psi^+(K; x) + \frac{i k}{2g(K)} \int \int \delta \epsilon(K - K'; x) \psi(K'; x) \frac{dK'}{(2\pi)^2} \tag{1}
\]

\[
-\frac{\partial \psi^-(K; x)}{\partial x} = ikg\psi^-(K; x) + \frac{i k}{2g(K)} \int \int \epsilon(K - K'; x) \psi(K'; x) \frac{dK'}{(2\pi)^2} \tag{2}
\]

where \(\psi(K; x)\) and \(\tilde{\delta \epsilon}(K; x)\) are transverse spatial Fourier transforms of the field and the relative permittivity variation, respectively. The integral term involves the total wave field

\[
\psi(K; x) = \psi^+(K; x) + \psi^-(K; x). \tag{3}
\]

Insofar as we know, these equations had not been developed previously, although spatial-domain forms involving formal operators are known and have been used in similar applications.

In Rino [2], we established conditions under which a hierarchy of moment equations can be derived from (1) and (2). We showed that two approximations are involved. The first approximation requires small local perturbations, which is sufficient condition for evaluating the functional derivatives that appear in the Novikov-Furutsu (NF) theorem. This assumption permits the development of a closed hierarchy of differential equations for the signal moments. The second approximation requires that over short distances the Fourier components propagate like plane waves in a homogeneous medium. Under these approximations integral equations for the second order-signal moments can be generated that depend only on the spectral density function of the permittivity fluctuations. Upon simplifying the resulting transport equations by using the narrow-angle scatter approximation we reconfirmed an earlier finding, namely that backscatter enhancements are negligibly small in continuous random media under the usual conditions of the Markov approximation. More recently other researchers have been using the method to extend results that have long been known from standard parabolic wave equation methods.

We have also applied the MIM method to practical problems involving an object scattering near a highly rough surface. Under a separate contract we applied MIM to investigate the hypothesis that enhanced acoustic surface reverberation in the ocean is
caused by subsurface bubble clouds [3]. These computations provide a good example of the comparative ease with which the results from numerical simulations and known scattering functions can be combined via MIM. A surface-scatter simulation was developed for dynamically evolving nonlinear ocean surface realizations as one input to the MIM computation [4]. The bubble cloud was modeled as a cylindrical void with known scattering characteristics. The general problem of a particle scattering near a rough surface was reviewed in [5].

We also treated the scattering of an object occulted by a phase screen by using MIM. The usual treatments of this problem had created some confusion about energy conservation particularly with regard to well-known cross section enhancements. These results were published in Rino [6].

More general applications of MIM demand a careful assessment of equivalent or generalized scattering functions. This topic was treaded in Ngo and Rino [7]. With the general properties of scattering functions for MIM established, we investigated a class of two-scatterer problems. The first involved two discrete scatterers. The problem is simplified considerably when the wave-space integration required to evaluate the scattering interaction can be replaced by a simple product. In Ngo and Rino [8], we describe a class of such scattering objects for which the scattering operator can be evaluated algebraically. In Ngo and Rino [9], we considered the interaction of a discrete scatterer near a smooth reflecting boundary. We were able to show the relation of MIM to image theory for both perfectly conducting and dielectric interfaces. Because monopole and dipole scatterers are the simplest representatives of the class of product MIM scatterers, we were also able to generalize the standard solutions to include the reflections from the radiating monopole or dipole element. These effects change the impedance of radiating elements that are raised above the surface.

We have completed the work supported under this research grant with an investigation of methods that can be used when backscatter is neglected in the calculation of the forward wave field. The results are described in detail in Rino and Ngo [10] and summarized below.
II.2 Forward Propagation

One of the most challenging problems in wave propagation is to evaluate the combined effects of irregular surfaces and inhomogeneous atmospheres. Equations (1) and (2), which can be written in the equivalent spatial domain form

$$\pm \frac{\partial \psi^\pm(\rho; x)}{\partial x} = \Theta \psi^\pm(\rho; x) + 2k^2 \iint \frac{i}{4} H_0^{(1)}(k|\Delta \rho|) \delta \eta(\rho', x) \psi(\rho', x) d\rho',$$

where

$$\Theta = \sqrt{1 - \frac{1}{k^2} \nabla^2}.$$  \hspace{1cm} (5)

We refer to (1) and (2) or (5) as generalized transmission-line equations (TLEs) because they characterize the interaction in bidirectional modal form.

Although the TLE equations are exact, they are most useful when the initial wavefield is directed. In that case, one can usually neglect backscatter or backward propagation in the calculation of the forward interaction. Formally, $\psi(\rho, x)$ in the forward integral term is replaced by $\psi^+(\rho, x)$, whereby the forward equation is decoupled from the backward equation and can be integrated directly. Backscatter can then be estimated by substituting the solution of the forward equation for $\psi$ in the backward equation. The backscatter can then be added to the forward scatter and the process repeated. Each iteration can be evaluated by using a generalized split-step approach. That is, the propagation term is evaluated first after which the media interaction term is evaluated and added to the solution.

If an irregular boundary is incorporated, however, the problem becomes much more complicated. Nonetheless, we have found that the same technique of neglecting backscatter can also be used to solve the surface boundary integral equation. Furthermore, the causal form of the resulting allows a marching solution. The field in any transverse plane can be used to calculate the interaction from that point on. Finally, we combined the marching solution. The following figures, which are taken from Rino and Ngo [10], show the effect of smooth and periodic surfaces bounding a region with a strongly ducting atmosphere. The calculation was performed over an 8 km surface with a 1 GHz source 20 m above the surface.

The new method has considerable promise for solving problems that involve both propagation in weakly inhomogeneous media and irregular boundaries. For example, the only additional computation time required to include atmospheric turbulence is that required to generate the turbulent structure.
Figure 1. Effect of strong duct over a smooth surface.
Figure 2. Effect of strong duct over a sinusoidal surface.
References


