Development of a Reliability-Based Design Procedure for Rigid and Flexible Airfield Pavements

by David W. Pittman

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Development of a Reliability-Based Design Procedure for Rigid and Flexible Airfield Pavements

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Preface

The work reported herein was prepared by Dr. David W. Pittman, consulting engineer, Opelika, AL. This work was funded by the U.S. Army Engineer Waterways Experiment Station (WES), Vicksburg, MS, under Purchase Order No. DACA39-95-M-1184, 9 January 1995 to 30 September 1995.

WES personnel in the Airfields and Pavements Division (APD), Geotechnical Laboratory (GL), directly concerned with this work were Mr. Richard Bradley, Civil Engineer, Dr. Reed Freeman, Research Civil Engineer, and Mr. Timothy Vollor, Acting Chief, APD. This work was accomplished under the general supervision of Dr. William F. Marcuson III, Director, GL.

At the time of publication of this report, Dr. Robert W. Whalin was Director of WES and COL Bruce K. Howard, EN, was Commander.
Part I: Introduction

Statement of Problem

The current U.S. Army Corps of Engineers thickness design procedures for rigid and flexible pavements are deterministic in nature; i.e. they use one value (typically the mean value) for each of the design parameters and essentially ignore the inherent variability of the design parameters during the design process. Variability in the design parameters, such as the California Bearing Ratio (CBR) of the subgrade in flexible pavement design, for example, may be recognized during selection of the one design value, but the effects of a subgrade having a relatively consistent CBR versus one that doesn’t cannot be assessed.

The use of probabilistic techniques to characterize the variability of the pavement design parameters, such as with the standard deviation or coefficient of variation (CV*) of the parameter, and translated into an estimated reliability for a particular design, would enable an engineer to more effectively design a pavement for a particular application. These tools would also allow the engineer to evaluate the effects of different degrees of variability in a pavement system, perhaps those determined from quality control and assurance testing during pavement construction, on the reliability of the design, from which appropriate adjustments to compensation for the builder could be more readily quantified.

The Corps of Engineers primarily uses two design procedures for both rigid and flexible pavement design for airfields. The two rigid pavement design procedures are based upon the same performance data taken from the trafficking of pavement test sections constructed for that purpose. One of the procedures uses the Westergaard edge stress equation to calculate the maximum stress due to an applied load (Departments of the Air Force and Army 1988), and the other procedure uses an elastic layer analysis technique to calculate stresses in the slab due to an applied load. The two flexible design procedures were also developed from performance data taken from test section trafficking. The CBR design procedure (Departments of the Navy, the Army, and the Air Force 1978) makes use of the CBR value of the subgrade, subbase, and base course for determining the thickness of pavement necessary to carry a certain number of applied

* CV = standard deviation/mean value
loads before failure (shear failure or rutting of the pavement). The Corps layered elastic design procedure for flexible pavements uses the modulus of elasticity and Poisson’s ratio of each of the pavement layers to estimate the stresses and strains at particular locations in the pavement system that are indicative of the performance of the pavement under repeated load.

**Objectives of the Research**

The objective of this research is to develop a conceptual approach to applying probabilistic techniques to the Corps of Engineers rigid (Westergaard) and flexible (CBR) airfield pavement thickness design procedures. These concepts will then be used to modify the Corps airfield pavement thickness design procedures to consider the effects of variability in the design parameters on the reliability of the design.

**Scope**

The objectives will be accomplished by the following tasks:

1. Conduct a review of existing design procedures that consider variability in the design parameters as part of the design;

2. Investigate statistical methods for characterizing the variability of design parameters and performance equations;

3. Determine reasonable estimates of the variability of the rigid and flexible pavement design parameters;

4. Develop reliability-based design procedures for rigid (Westergaard) and flexible (CBR) airfield pavement thickness design; and

5. Suggest levels of reliability for military airfield pavement thickness design.

**Background**

**Concept of Reliability**

Stated simply, reliability is the probability that something will not fail, or expressed mathematically, reliability is one minus the probability of failure. The AASHTO design procedure (AASHTO 1986) describes the reliability concept for pavements as follows:

The reliability of a pavement design-performance process is the probability that a pavement section designed using the process will perform
satisfactorily over the traffic and environmental conditions for the design period.

Applying these concepts to the Corps of Engineers rigid and flexible pavement design models, the reliability of a pavement may be described as the probability that the pavement will not reach the failure condition before all of the design traffic is applied to the pavement.

One approach for quantifying the reliability is to relate the capacity \( C \) of a pavement system (in terms of the number of applications applied to the pavement before it fails) to the demand \( D \) of traffic to be applied to the pavement (in terms of the number of load applications actually applied to the pavement). Both the capacity and the demand can be represented by a log-normal distribution with a mean value and a variance. The log-normal distribution assumes that the logarithm of the number of passes (or coverages\(^*\)) of traffic applied to a pavement is normally distributed; this assumption is commonly used in other reliability-based design procedures (Harr 1987). The difference in the capacity and the demand, or the safety margin \( (SM) \), is also log-normally distributed, with a mean \( E[SM] \) and a variance \( V[SM] \) expressed as follows:

\[
E[SM] = E[C] - E[D] \quad \text{Eqn. 1}
\]
\[
V[SM] = V[C] + V[D] \quad \text{Eqn. 2}
\]

This model for the \( V[SM] \) assumes that the capacity and demand are independent of each other. These concepts are illustrated in Figure 1.

The variance of the demand \( V[D] \) is simple to apply in this model, although perhaps not simple in practice to estimate. This variability represents the degree of confidence that the designer has in his estimate of how much and what kind of traffic will be applied to the pavement over the life of the pavement. This requires complete knowledge of future events, such as changes in airfield mission requirements or the development of new aircraft, that the designer does not have. Therefore, there is a great chance that what the designer originally estimates for the design traffic and what amount of traffic is actually applied to the pavement over the design life (typically 20 years) is different.

The variance of the capacity \( V[C] \) of the pavement is somewhat more difficult to determine. The capacity of a pavement system is estimated in the pavement design model

\(^*\) The coverages concept is explained in later sections.
Figure 1. Reliability concept using safety margin

by assuming values (usually the mean value) for the input parameters and applying them to the performance equation to get the design thickness. The performance equation relates these design inputs to the capacity expected for that combination of design inputs, and is based (in the Corps of Engineers design procedures) upon regression equations developed from test section data. Therefore, the variability of the capacity of the pavement system is based not only on the variability of the design inputs, but the variability of the regression model itself:
\[ V[C] = f\{V[\text{design parameters}]\} + V[\text{performance equation}] \quad \text{Eqn. 3} \]

This concept is illustrated for the Corps of Engineers rigid pavement design model in Figure 2. The performance model relates the design factor (DF), a ratio of the 90-day flexural strength to the maximum stress (\(\sigma_{WE}\)) in the slab due to load, to the pavement capacity, or the logarithm of the number of applications of load until failure (\(LC_c\)). The design factor contains all of the design parameters, such as the flexural strength, modulus of subgrade reaction, modulus of elasticity and Poisson’s ratio of the concrete, load transfer, and slab thickness, and thus the variability of these inputs results in variability of the design factor. The variance of the performance model equation is a function of the scatter of the data used in developing the equation, and the position along the regression equation; the further from the center of the data, the less reliable the equation is for predicting the relationship between the variables in the equation.

Once the \(E[SM]\) and \(V[SM]\) are known, the reliability of the pavement can be determined by the equation

\[ \text{Reliability} = \text{Prob}[ SM > 0 ] \quad \text{Eqn. 4} \]

which is represented by the shaded area in Figure 1 (the area greater than zero).

**Figure 2.** Conceptual representation of deterministic and stochastic approach to the Corps rigid pavement performance model
This concept is intuitively correct. The larger the expected capacity C of the pavement system is relative to the expected demand D on the pavement, the less likely the pavement will fail before all the demand traffic is applied to the pavement, and therefore the pavement is more reliable. This corresponds to a larger safety margin SM (or E[SM]), which shifts the SM frequency distribution farther away from zero (on the positive side) and increases the area under the curve that is greater than zero, which is the reliability. Note that if the capacity of the pavement system is equal to the demand, the safety margin is zero, and the reliability of the pavement system is 50 percent. This is the typical result of a deterministic design, where the mean value of the design parameters is used, and the expected capacity of the pavement system is automatically assumed to be equal to the expected demand on the pavement.

Another way to increase the reliability of the pavement is to reduce the variability or variance of the design inputs (for instance, the flexural strength of concrete during construction), which in turn decreases the variance of the capacity V[C], which in turn reduces the variance of the safety margin V[SM]. As the V[SM] is reduced, the SM frequency distribution “contracts,” resulting in even more of the area of the curve being greater than zero, which represents an increased reliability. Note that the opposite effect would occur if the demand E[D] is greater than the capacity E[C], and the safety margin E[SM] is less than zero. The reliability in this case is less than 50 percent, and a contraction of the SM frequency distribution resulting from decreased variability in the pavement design parameters actually results in the pavement system being less reliable. These concepts are illustrated in Figure 3.

Before these concepts are applied to the Corps of Engineers rigid and flexible airfield pavement design procedures, the design procedures will be reviewed in sufficient detail to explain how the reliability concept can be applied to them.

**Review of the Corps of Engineers Rigid Pavement Design Procedure using the Westergaard Edge Stress Equation**

As stated previously, this Corps rigid pavement design procedure uses the Westergaard edge stress ($\sigma_{WE}$) equation as a basis for determining the maximum stress in a concrete slab due to a load applied by a single wheel. The equation takes the form:
Case 1: E[C], E[D], E[SM] > 0, Reliability > 50 percent
Reduced variance results in greater reliability

Case 2: E[C] < E[D], E[SM] < 0, Reliability < 50 percent
Reduced variance results in lower reliability

Figure 3. Effect of variability of design parameters on reliability
\[
\sigma_{WE} = \frac{3(1+\mu)P}{\pi(3+\mu)h^2} \left[ \log \frac{Eh^3}{100k\left(\frac{a'+b'}{2}\right)^4} + 1.84 - \frac{4}{3} \mu \right. \\
+ \left. (1+\mu) \frac{a'-b'}{a'+b'} + \frac{2(1-\mu)}{(a'+b')^2} \frac{a'b'}{l} + 1.18(1+2\mu) \frac{b'}{l} \right]
\]

Eqn. 5

where: \( P \) = load (lbs)
\( h \) = slab thickness (in.)
\( l \) = radius of relative stiffness (in.)

\[
= \frac{4}{12(1-\mu^2)} \frac{Eh^3}{k}
\]

Eqn. 6

\( E \) = modulus of elasticity of concrete (psi)
\( \mu \) = Poisson's ratio
\( k \) = modulus of subgrade reaction (pci)
\( a' \) = semi-major axis of footprint of loaded area (in.)
\( b' \) = semi-minor axis of loaded area of footprint (in.)

Pickett and Ray (1951) expanded the scope of the Westergaard equations by developing influence charts which allowed the computation of edge load stresses due to a multiple wheel load, such as an aircraft gear. Kreger (1967) developed a computerized solution of the Pickett and Ray charts, called H51. The Corps of Engineers (Rollings 1989) used the H51 program to develop a regression equation to calculate the Westergaard edge stress under multiple wheel gear loads which took the form:

\[
\sigma_{WE} = \frac{P}{h^2} (a_0 + a_1 l + a_2 (\ln(l))^2)
\]

Eqn. 7

where: \( \sigma_{WE} \) = Westergaard edge stress (psi)
\( P \) = gear load (lbs)
\( h \) = thickness of concrete slab (in.)
\( l \) = radius of relative stiffness (in.)
\( a_0, a_1, a_2 \) = regression constants for a particular aircraft.

This regression equation yielded results that correlated very well with results from the H51 program, and its use greatly simplified the calculation of stresses in the Corps of
Engineer rigid pavement design computer programs, such as the Rigid Airfield Pavement Design (RAD) program (Barker 1987).

Concept of Load Transfer

The concept of load transfer at concrete pavement joints is very important and basic to the Corps of Engineers design procedure for rigid pavements. Load transfer refers to the amount of load (in percent of applied load) that is carried by an unloaded concrete slab due to a load applied to an adjacent slab (Figure 4). Stresses due to the applied load are transferred to the unloaded slab through shear at the vertical interface of the joint between the slabs. The procedures assume that 25 percent of the load applied to the edge of a concrete pavement slab (the most critical loading position) is transferred through the joint to the adjacent unloaded slab (Rollings 1985). This, in effect, reduces the edge stress in the loaded slab by 25 percent from a maximum free edge condition.

![FREE EDGE CONDITION](image1)

![LOAD TRANSFER FROM ADJACENT SLAB](image2)

\[
\text{Load Transfer} = \frac{\Delta u}{\Delta L} \times 100
\]

\[
\text{Load Transfer (\%)} = (1 - \frac{\sigma L}{G E}) \times 100
\]

Figure 4. Concept of load transfer.
thereby allowing for a reduced slab thickness. The 25 percent load transfer assumption in the Corps and FAA design procedures is a simplifying assumption. Load transfer is a complex mechanism that can vary with concrete pavement thickness, joint spacing, temperature, moisture content, aggregate type and size, age, construction quality, magnitude and repetition of load, and type of joint. If the 25 percent load transfer assumption is not met in actuality, the life of the pavement may be significantly reduced from the expected design life (Figure 5).

![Graph showing the effect of reduced load transfer on pavement life](Figure 5. The effect of reduced load transfer on pavement life)

**The Corps Method for Counting Traffic on Airfield Pavements**

The Corps rigid pavement thickness design procedure does not directly use the number of applications of traffic loads, expressed in terms of passes or operations of an aircraft, when determining the thickness necessary to accommodate the traffic. Instead,
the procedure converts the actual number of applications of traffic to coverages of traffic. The coverages concept accounts for the distribution of the traffic (wheel) loads across the width of the traffic lane. According to a Corps study by Brown and Thompson (1973), "a coverage occurs when each point in the pavement within the limits of the traffic lane has been subjected to a maximum stress, assuming that the stress is equal under the full tire print."

The number of passes of a particular aircraft or vehicle required to obtain one coverage of a particular width of traffic lane, called the pass per coverage ratio (p/c), is a function of the tire contact width, number of tires, and tire configuration in a multiple-wheel aircraft gear. The p/c ratios have been defined for a number of aircraft for two "wander widths," or traffic lane widths within which 75 percent of the passes of a gear are expected to occur. For channelized traffic, which is assumed to occur along taxiways where the aircraft can be steered along the centerline of the traffic lane, the wander width is defined as 70 in.; in non-channelized traffic areas, such as runways and aprons, the wander width is assumed to be 140 in. The p/c ratio is divided into the number of passes or operations of an aircraft to get the number of coverages of the aircraft for a particular traffic area.

The Corps procedure divides an airfield into four traffic areas -- Type A, B, C, and D traffic areas -- for purposes of designing pavements for those areas (Figure 6). The

![Figure 6. Corps of Engineers airfield traffic areas](image-url)
traffic areas are distinguished by the percent of the static weight of aircraft used in calculating the stresses due to load, the wander width used in determining the \( p/c \) ratio, and the percent of the total design passes used in calculating thicknesses for that traffic area. These combinations for each traffic area are given in Table 1.

The Corps of Engineers design procedure provides for a mixed traffic analysis, in which a combination of aircraft and pass levels are combined into a single dominant aircraft with an equivalent total number of coverages. The design procedure also has provisions for airfield types and classes which group specific aircraft types and number of passes based upon common experience. Mixed traffic analysis is used to obtain the dominant aircraft and total equivalent coverages for each airfield class and type.

The Corps of Engineers Rigid Pavement Performance Model

The current Corps of Engineers rigid pavement performance model relates the ratio of the concrete flexural strength and the stress due to an applied load to the expected number of coverages of traffic (\( L_{C} \)) that is achieved at the time the failure condition is reached. This relationship was derived from the results of a series of tests conducted in 1943 through 1973 (Parker et. al 1979) in which controlled accelerated simulated aircraft traffic loads were applied to a series of full-scale pavement test sections. The stresses at the edge of the slabs due to load were calculated using plate theory (i.e. the Westergaard

<table>
<thead>
<tr>
<th>Traffic Area</th>
<th>Airfield Location</th>
<th>Wander Width</th>
<th>Percent of Static Weight of Aircraft</th>
<th>Percent of Total Passes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type A</td>
<td>Taxiways, Runway Ends</td>
<td>70 in. (Channelized)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Type B</td>
<td>Aprons</td>
<td>140 in. (Non-channelized)</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Type C</td>
<td>Runway Interior</td>
<td>140 in. (Non-channelized)</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td>Type D</td>
<td>Runway Edges</td>
<td>140 in. (Non-channelized)</td>
<td>75</td>
<td>1</td>
</tr>
</tbody>
</table>
The performance model makes use of a concrete strength/applied load stress ratio called the design factor \((DF)\), expressed as:

\[
DF = \frac{R}{(1 - LT)\sigma_{WE}}
\]

where:
- \(DF\) = design factor
- \(R\) = concrete flexural strength (psi)
- \(LT\) = load transfer (percent)
- \(\sigma_{WE}\) = Westergaard edge stress (psi)

In the performance model, the load transfer \(LT\) is assumed to be a constant 25 percent, so that the Westergaard edge stress is multiplied by a factor of 0.75. The design factor was calculated for each of the test section trials, and plotted as the dependent variable against the number of coverages of traffic applied until the failure condition was reached (Figure 7). A least-squares linear regression was then used to obtain the equation (Rollings 1989):

\[
DF = 0.5 + 0.25 \times LC_c
\]

where:
- \(DF\) = design factor
- \(LC_c\) = Log(coverages of traffic applied until failure, or pavement capacity)

The equation was later modified to account for continued satisfactory performance of concrete slabs on higher-strength foundations, even after the original failure criterion was reached. This essentially changed the failure criteria for slabs on foundations with a modulus of subgrade reaction greater than 200pci. The modified equation was expressed as:

\[
DF = 0.7 - 0.001 \times k + 0.25 \times LC_c
\]

where:
- \(DF\) = design factor
- \(k\) = modulus of subgrade reaction (between 200 and 500 pci).
Figure 7. The Corps of Engineers rigid pavement performance equation

The Corps of Engineers Procedure for Determination of Concrete Pavement Thickness

In the Corps of Engineers procedure for determination of concrete pavement thickness, the design thickness is the minimum thickness which satisfies the following relationship:

\[ DF_{DESIGN} \geq DF_{ALLOWABLE} \]  

Eqn. 11

where: \( DF_{DESIGN} \) = design \( DF \) from Eqn. 10  
\( DF_{ALLOWABLE} \) = allowable \( DF \) from Eqn. 8.

In calculating the \( DF_{ALLOWABLE} \), the desired number of coverages until failure \( LC_D \) is determined from dividing the desired number of passes over the design life of the pavement (typically 20 years for airfield pavements) by the \( p/c \) ratio for the aircraft. The
$L_{C_D}$ is assumed to be equal to the $L_{C_C}$ in Eqn. 10; i.e. the capacity of the pavement is designed to be equal to the demand of traffic to be applied to it.

The modulus of subgrade reaction on top of the base course is used in calculating the $DF_{ALLOWABLE}$. If the modulus of subgrade reaction on top of the base $k_{TB}$ is not known, it may be obtained from the modulus of subgrade reaction $k$ for the compacted subgrade and the base course thickness ($BT$, in inches) by using Figure 8 (Departments of the Army and Air Force 1992), or the following equation:

$$k_{TB} = 10^{[2.69897+(\log(k)-2.69897)-10^{-a_{subbase}}]}$$

Eqn. 12

For modulus of subgrade reactions less than 200 pci, Eqn. 9 is used for calculating the $DF_{ALLOWABLE}$. Modulus of subgrade reactions greater than 500 pci are assumed to be 500 pci for purposes of calculating the $DF_{ALLOWABLE}$ and the $DF_{DESIGN}$.

The concrete flexural strength from the third-point loading test (ASTM 1992) is used in the $DF_{DESIGN}$ calculation; the 90-day strength is usually considered for airfield pavements. The flexural strength in the field is controlled such that 80 percent of the quality control flexural strength test results exceed the design flexural strength.

![Figure 8. Chart for obtaining k on top of base](image)

15
In the $DF_{DESIGN}$ calculation, the load transfer is assumed to be 25 percent for airfield pavement design. The Westergaard edge stress used in the calculation is computed for the design or dominant aircraft using the regression equation expressed in Eqn. 7. The concrete modulus of elasticity and Poisson's ratio are typically assumed in RAD to be 4,000,000 psi and 0.15, respectively, when calculating the radius of relative stiffness $l$. The only remaining variable unaccounted for in the calculation of $DF_{DESIGN}$ is the concrete thickness. In a trial and error process, the RAD program searched for the exact concrete pavement thickness which satisfy the requirements of Eqn. 11. This thickness is then rounded up to the nearest one-half inch to obtain the design thickness.

The Corps rigid pavement design procedure also provides the recommended joint spacing, dowel diameter, and dowel spacing for a particular design. It contains provisions for designing pavements over subgrades subject to weakening under freezing and thawing conditions, for reinforced concrete pavement, and for overlays of concrete or asphalt pavement over an existing concrete pavement. While these are very important aspects of the pavement design procedure, along with the slab thickness, these procedures will not be covered here because their effect was not included in the reliability portion of this study.

**Review of the Corps of Engineers Flexible Pavement Design Procedure using the California Bearing Ratio (CBR)**

**The CBR Equation**

The Corps of Engineers CBR design procedure is much more empirically based than the rigid pavement design procedure, in that it depends little upon calculations of stress, strain, or deflection in determining the pavement thickness. As mentioned previously, the CBR design procedure uses the CBR of the subgrade, subbase, and base course layers under the asphalt concrete to characterize the strength of the layers. The CBR design equation relates the total thickness $t$ of material required to protect a subgrade or subbase material with a given CBR to allow it to withstand 5,000 coverages.

---

* A coverage for flexible pavements occurs when all points on the pavement surface within the traffic lane have been subjected to one application of maximum stress. The number of passes required to accumulate one coverage is called the pass to coverage ratio ($p/c$). As with rigid pavements, the $p/c$ varies with channelized and non-channelized traffic.
of traffic before failure. The CBR design equation was developed from data taken during the trafficking of a series of flexible pavement test sections during the 1940's through the 1970's. The current (cubic) form of the equation is given below (Ahlvin 1991):

$$t = \alpha \sqrt{A} \left[ -0.0481 - 1.1562 \left( \log \left( \frac{CBR}{P_e} \right) \right) - 0.6414 \left( \log \left( \frac{CBR}{P_e} \right) \right)^2 - 0.4730 \left( \log \left( \frac{CBR}{P_e} \right) \right)^3 \right]$$

Eqn. 13

where:

- $t$ = thickness required to cover material of given CBR (using material of a greater CBR), in.
- $\alpha$ = factor relating thickness $t$ to number of coverages of traffic.
- $A$ = contact area of one tire, in.$^2$
- $CBR$ = relative strength of supporting material, percent
- $P_e$ = equivalent single-wheel tire pressure, psi
- $ESWL/A$ = equivalent single wheel load, lbs.

The equation is shown graphically in Figure 9 superimposed on the test section data. The equation line separates the data points of the test sections which had failed at 5,000 or less coverages from the data points of the test sections that had not failed by the end of the test.

![Figure 9. The CBR design equation and test section data](image-url)
Adjusting the Thickness for Various Traffic Levels (the $\alpha$ Curves)

The $\alpha$ term in the equation was developed as a result of multiple-wheel heavy gear load (MWHGL) tests, and is used to adjust the thickness necessary the subgrade for coverage levels ($LC_C$) other than 5,000. The $\alpha$ was found to be a function of the number of wheels in the critical gear, as shown in Figure 10. The data used for developing the curves in Figure 10 were used to obtain sixth-order regression equations of the form

$$\alpha = d_0 + d_1(LC_C) + d_2(LC_C)^2 + d_3(LC_C)^3 + d_4(LC_C)^4 + d_5(LC_C)^5 + d_6(LC_C)^6 \quad \text{Eqn. 14}$$

to predict $\alpha$ for a given $LC_C$, where the $d_i$ are regression constants for a particular number of wheels. The results of the regression analysis for several wheel configurations are given in Table 2. Although the cubic CBR equation (Eqn. 13) was developed as a simplification of two earlier forms of the equation representing different ranges of CBR/$p_a$, it appears to also represent the best-fit equation passing through the test section failure data for 5,000 coverages of traffic (Potter 1985) when the appropriate $\alpha$ is applied to the data, as shown in Figure 11.

![Figure 10](attachment:image.png)

**Figure 10.** The alpha curves for flexible pavement design
Table 2. Regression Constants for Regression Curve Predicting $\alpha$ from $L_{C_{C}}$

<table>
<thead>
<tr>
<th>Aircraft</th>
<th>$d_0$</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$d_3$</th>
<th>$d_4$</th>
<th>$d_5$</th>
<th>$d_6$</th>
<th>$r^2 (%)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F-15</td>
<td>0.1901</td>
<td>0.2689</td>
<td>-0.08039</td>
<td>0.04451</td>
<td>-0.01242</td>
<td>0.001658</td>
<td>-0.000086</td>
<td>99.993</td>
</tr>
<tr>
<td>C-130</td>
<td>0.1808</td>
<td>0.2749</td>
<td>-0.08663</td>
<td>0.04891</td>
<td>-0.01544</td>
<td>0.002286</td>
<td>-0.0001265</td>
<td>99.977</td>
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<tr>
<td>C-141</td>
<td>0.1763</td>
<td>0.2302</td>
<td>0.000364</td>
<td>-0.01280</td>
<td>0.003165</td>
<td>-0.000302</td>
<td>0.0000099</td>
<td>99.973</td>
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<tr>
<td>C-5A</td>
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<td>0.2641</td>
<td>-0.0530</td>
<td>0.01810</td>
<td>-0.006568</td>
<td>0.001088</td>
<td>-0.000063</td>
<td>99.978</td>
</tr>
</tbody>
</table>

Figure 11. The CBR equation, and test section data for failures at 5,000 coverages
The Equivalent Single Wheel Load (ESWL) Concept

The effective tire pressure \( p_e \) provides a means of translating the effect of multiple wheel gear loads into an equivalent single wheel load (ESWL). The ESWL is that load applied to a single wheel of contact area \( A \) that would cause the same deflection as the aircraft gear load (applied to several wheels with contact area \( A \)) at any particular depth below the surface of the pavement. Therefore, the ESWL is a function of the gear configuration as well as the depth below the surface, and is sometimes expressed as a percentage of the gear load. The ESWL has been calculated for several aircraft using a homogeneous, isotropic, linear elastic, half-space model representing the pavement and subgrade; the ESWL curves for the C-130, C-141, and C-5A are presented in Figure 12. Since the critical gear load for the F-15 is a single wheel, the ESWL for any depth is 100 percent of the actual gear load.

The ESWL versus depth data was used to develop a fifth-order multiple linear regression equation to predict the ESWL for any depth (\( t \)) for each of the three aircraft represented in Figure 12. The equation takes the form:

\[
ESWL(\%) = c_0 + c_1(t) + c_2(t)^2 + c_3(t)^3 + c_4(t)^4 + c_5(t)^5 \quad \text{Eqn. 15}
\]

![Figure 12. ESWL curves for C-130, C-141, and C-5A](image-url)
where $c_i$ are the regression constants and the $t$ is the depth in question (in inches). Table 3 contains the $c_i$ and regression constants for several aircraft. The ESWL(%) is expressed as a fraction of the total gear load, so that the ESWL may be determined by multiplying the ESWL(%) by the gear load. The $p_e$ is then calculated by dividing the ESWL by the contact area $A$.

Calculation of Design Thicknesses ($t$) of Flexible Pavement using the CBR Equation

The thickness calculated from Eqn. 13 represents the total thickness $t$ of material (with a higher CBR) required to cover the material represented by the CBR listed in the equation. Since the $p_e$ in the equation is a function of $t$ (because the ESWL is a function of $t$), the equation must be solved by an iterative process to determine $t$. This process is continued to determine the thickness to cover every layer of material above and including the subgrade (except the asphalt concrete), including the subbase and base course layers. The thickness of the subbase and base course layers are obtained by subtraction of the sequential thicknesses to cover, i.e. if 39 inches of material are required to cover a subgrade CBR of 3, and 20 inches are required to cover a subbase material (resting on top

<table>
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<tr>
<th>Aircraft</th>
<th>$c_0$</th>
<th>$c_1$</th>
<th>$c_2$</th>
<th>$c_3$</th>
<th>$c_4$</th>
<th>$c_5$</th>
<th>$r^2$ (%)</th>
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<td>C-130</td>
<td>0.5457</td>
<td>0.001476</td>
<td>4.5781E-05</td>
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<tr>
<td>C-141</td>
<td>0.3232</td>
<td>-0.00203</td>
<td>7.0766E-04</td>
<td>-1.5691E-05</td>
<td>1.4014E-07</td>
<td>-4.6026E-10</td>
<td>99.999</td>
</tr>
<tr>
<td>C-5A</td>
<td>0.0700</td>
<td>0.00053</td>
<td>1.1664E-04</td>
<td>-2.2753E-06</td>
<td>2.1148E-08</td>
<td>-7.9487E-11</td>
<td>99.998</td>
</tr>
</tbody>
</table>
of the subgrade) with a CBR of 15, then the subbase thickness is $39 - 20 = 19$ inches. Minimum thicknesses of the base course and asphalt concrete layer are required depending upon the traffic area, the critical aircraft gear configuration, and the base course CBR.

Similar to the rigid pavement design procedure, the CBR design procedure considers various traffic areas in the airfield (Type A, B, C, and D) with various assumptions of gear load and wander width (and therefore $p/c$ ratios) associated with them. The $p/c$ ratios for flexible pavements are often different than those for rigid pavement, due to the difference in how coverages are counted. The CBR design procedure also contains provisions for designing in frost areas, where the subgrade soil may become weakened during freezing and thawing action; however, these procedures are not covered here because this aspect is not addressed in the reliability-based design procedures developed in later sections of this report. A flexible airfield pavement design computer program entitled FAD has been developed which uses the CBR design procedures outlined above.

**Application of Reliability Concepts to the Corps Rigid and Flexible Design Procedures**

**Methods of Determining the Variance of the Capacity ($V[LC_c]$) of the Pavement**

The variance of the capacity of the pavement system $V[C]$ may be expressed as the variance of the logarithm of coverages of traffic until failure, or $V[LC_c]$. As shown in Eqn. 3, the determination of the variability of the capacity $V[C]$ of the pavement system (or of $V[LC_c]$) depends upon the determination of the variability of the design parameters as expressed in the performance equation. An estimation of $V[LC_c]$ or of the variance of any function of two or more variables can be determined by several techniques, including the simulation (Monte Carlo) method, the point estimation (Rosenblueth) method, and the first-order second-moment (FOSM) method (Harr 1987). A brief discussion of each method follows.

**Monte Carlo Simulation**

In the Monte Carlo technique, the value of the function is calculated many times from random combinations of the independent variables take from a specified frequency distribution (known mean and variance) for each of the independent variables. These calculated dependent variables are then used to determine the mean and variance of the function. The advantage of this technique is that the complete probability distribution of
the dependent random variable is obtained. The disadvantages of this method are that the distribution of the dependent variable is dependent upon an assumed (and perhaps incorrect) distribution of the independent variables (Harr 1987). Also, many repetitions of calculations are needed to accurately estimate the mean and variance of the function. The number of trials necessary to estimate the frequency of the function of \( m \) independent variables so that the simulation is differs no more than \( \varepsilon \) from the estimated value, with a \((1-\xi)\) degree of confidence, can be determined from the equation:

\[
N = \left( \frac{k_{\xi/2}^2}{4\varepsilon^2} \right)^m
\]

Eqn. 16

where \( k_{\xi/2} \) is the value of the standard normal variate with \( \xi/2 \) area in the tails (Harr 1987). For five independent variables, an error of 1 percent, and a degree of confidence of 99 percent, the number of required trials of the Monte Carlo simulation would be approximately \( 1.28 \times 10^{21} \), an extraordinary number of calculations even for today’s computers.

**Point-Estimati**

Method (PEM)

In the Rosenblueth or PEM technique, the mean and variance of the function \( F(x, y) \) are estimated from combinations of the means of the independent variables \( x \) and \( y \) plus or minus the standard deviation of the variable \( (x \pm \sigma_x, y \pm \sigma_y) \) (Harr 1987). This powerful approximation technique can also consider covariance between the variables. Using the notation \( F_{++} \) to indicate the function \( F(x, y) \) evaluated at \( x + \sigma_x \) and \( y + \sigma_y \), \( F_{--} \) to indicate \( F(x, y) \) evaluated at \( x - \sigma_x \) and \( y - \sigma_y \), etc., the estimate of \( F \) (or \( E[F] \)) can be evaluated from the equation:

\[
E[F] = \sum_{i=1}^{2^n} \frac{1 \pm \rho_{\pm\pm}}{2^n} [F_{\pm\pm}]
\]

Eqn. 17

where \( \rho \) is the covariance between \( x \) and \( y \), \( n \) is the number of independent variables, and \( F_{\pm\pm} \) is evaluated for every combination of the \( 2^n \) permutations of the means and standard deviations. The sign of \( \rho \) is determined by the product of the particular set of positive and/or negative signs (±) for that particular permutation; for instance, \( \rho \) would be positive for the permutations \( F_{++} \) and \( F_{--} \), and \( \rho \) would be negative for the permutations \( F_{+-} \) and \( F_{-+} \). If the covariance is zero, then the first portion of Eqn. 17 then becomes \( 1/2^n \); if the covariance is 1, then the combinations containing \( F_{+-} \) and \( F_{-+} \) are zero, since \((1-1)\) is zero.
The $E[F^2]$ is found by evaluating Eqn. 17 with the square of $F_{st}$ instead of simply $F_{st}$. The variance of the function $V[F]$ can then be calculated from the equation:

$$V[F] = E[F^2] - (E[F])^2$$  \(\text{Eqn. 18}\)

This very powerful technique can be used even if the distributions of the independent variables $x$ and $y$ are unknown, and even the $F(x, y)$ is not continuous or is generated from some open-form or iterative analysis technique, such as the finite element method. All that is required is that the function can be evaluated at all the combinations of $(x \pm \sigma_x, y \pm \sigma_y)$, that $\sigma_x$ and $\sigma_y$ are known (\(\rho\) does not have to be known, but is usually assumed to be zero if unknown). In order to obtain $E[F]$ and $V[F]$, the function is evaluated for all $2^n$ combinations of $n$ independent variables, which is typically far fewer evaluations than required by the Monte Carlo simulation.

**First-Order Second Moment (FOSM) Method**

The FOSM provides one of the simplest techniques to evaluate the mean and variance of a function, provided it is continuous over the range of independent variables being considered. Witczak et. al. (1983) used this method in an analysis of the reliability of the Corps of Engineers rigid pavement design procedure. The method makes use of the first-order Taylor series expansion of the function $F(x, y)$ about the expected values of the variables ($\bar{x}$ and $\bar{y}$) to obtain the expected value $E[F(x, y)]$ and variance $V[F(x, y)]$ of the function (Harr 1987):

$$E[F(x, y)] = F(\bar{x}, \bar{y})$$  \(\text{Eqn. 19}\)

$$V[F(x, y)] = \left(\frac{\partial F}{\partial x}\right)^2 V[x] + \left(\frac{\partial F}{\partial y}\right)^2 V[y] + 2\left(\frac{\partial F}{\partial x}\right)\left(\frac{\partial F}{\partial y}\right)\text{cov}[x, y]$$  \(\text{Eqn. 20}\)

The last term in Eqn. 20 allows consideration of the covariance of $x$ and $y$ \(\text{cov}[x, y]\). This term is typically assumed to be zero if the variables are assumed to be independent of each other.

The evaluation of the partial derivatives is the most difficult aspect of the FOSM, especially if the function is complicated. However, once the derivatives are determined, this method offers the simplest and quickest method of determining the reliability of a pavement system, since the estimate and variance of the pavement capacity is calculated with simple closed-form equations.
Review of Reliability-Based Pavement Design Methods

The U.S. Air Force Study

The U.S. Air Force recently undertook a study in which the Monte-Carlo simulation technique was used along with an elastic-layer analysis technique for computing the reliability of flexible pavement systems (Sues et. al, 1993). The procedure features nested loops of simulation in which the inner loop uses stratified sampling to consider different loading conditions during the pavement life, while the outer loop is a direct simulation where each trial represents the entire pavement lifetime. The variable components of the outer loop include environmental and load variation by generating random values for the fraction of passes applied to the pavement for each aircraft, and the pass-fractions for each aircraft load state, discrete lateral distribution of the traffic, and pavement material characteristics for a particular environmental condition. Damage is accumulated by weighting the damage for each pass-fraction and using the Miner’s hypothesis to determine failure. A final distribution of pavement response and pavement reliability is obtained at the end of the outer loop calculations.

The AASHTO Design Procedure

The AASHTO design procedure incorporated reliability concepts in 1986 (AASHTO 1986). In this procedure, reliability is defined as the probability that the pavement will last as long (in terms of passes of equivalent single axle loads, or ESAL’s) as it was designed to. The AASHTO procedure uses a version of the “safety margin technique outlined in the previous section as a basis for defining the reliability of a pavement system. The capacity of the pavement system is determined from the AASHTO design procedure and the demand is assumed to vary between one-half to over two times the predicted demand. The capacity and demand are assumed to be log-normal distributions of the traffic. The failure condition is defined as a terminal serviceability quantified by the pavement serviceability index (PSI). The PSI is an objective measure of the ability of the pavement to carry the design traffic, and depends primarily upon the roughness of the pavement and cracking of the pavement. On a scale of 0 to 5, the terminal serviceability is typically selected as 1.5 to 2.5. The selection of an optimum level of reliability for a particular design is based upon the reliability level which yields the lowest combination of present values of the initial costs (construction costs) and the future costs (maintenance, user delays, rehabilitation, etc.) (Figure 13).
Figure 13. AASHTO procedure for selecting optimum reliability

U.S. Army Studies

Several studies have been completed over the past 20 years (Kennedy et. al. 1975, Hudson et. al. 1975, Brown 1975, Witzczak et. al. 1983, Potter 1985) that have reviewed pavement material variability or presented pavement reliability concepts for the U.S. Army or Federal Aviation Administration (FAA) design procedures. A brief review of each study follows.

Kennedy et. al. (1975) presented the types and distributions of variability of both rigid and flexible pavement materials, including portland cement concrete, asphalt concrete, black-base and asphalt-treated materials, cement-treated materials, lime-treated materials, and untreated subgrade soils. The variability data was derived primarily from
quality control data or samples of in-situ highway and airport pavements. The data included density, indirect tensile strength, compressive strength, flexural strength, modulus of elasticity, slump, air content, and slab thickness of portland cement concrete; and density, percent compaction, indirect modulus of elasticity and Poisson's ratio of asphalt concrete pavement. The report also discussed sources of pavement design variability, including spatial variability of pavement material properties along the pavement, the variation between design and actual material parameters, and variation due to lack-of-fit of the design models.

Hudson (1975) reviewed pavement reliability concepts and how they are applied to highway pavements, based upon the results of the AASHTO Road Tests and the Flexible Pavement Design System (FPS) computer program. Hudson reviewed sources of variability in pavement design, how this variability can be translated to a reliability of a pavement system, and how a level of reliability for a pavement system can accurately be selected.

Brown (1975) outlined a plan for applying statistical quality control procedures for airport pavement construction. In his plan, the variability of pavement material characteristics that would be used in a quality control procedure were presented based upon past construction data at 30 airfields. This data included air content, slump, and flexural strength of portland cement concrete, and gradations, asphalt content, and density of asphalt concrete mixtures.

Witczak et. al. (1983) developed reliability-based design procedures based upon the Corps of Engineers rigid pavement design procedure using both the Westergaard edge stress equation and elastic-layer theory for calculating stresses in slabs. Witczak summarized the variability of several pavement design parameters as researched from other sources in the literature, including those for the slab thickness, modulus of elasticity, Poisson's ratio, and flexural strength of concrete pavements; and the thickness, dynamic modulus, flexural stiffness, and Poisson's ratio of asphalt concrete layers. Witczak used the FOSM technique for characterizing the variance of the design factor equation due to variance in the design parameters, and described the reliability of the pavement system as the probability that the $DF_{DESIGN}$ is greater than the $DF_{ALLOWABLE}$. He also conducted a Monte-Carlo simulation with 300 runs to compare to the FOSM results; the results for the mean and variance of the $DF_{ALLOWABLE}$ using each technique were both within 3 percent of each other.
Potter (1985) examined the Corps of Engineers CBR design procedure to determine the fit of the cubic CBR equation to the test section data used in deriving the curve. Potter determined that the cubic equation apparently was not developed to be a least-squares regression through the failure data (at 5,000 coverages of traffic), but a simplified combination of two previous curves which were developed for different ranges of CBR/\( p_e \) (less than and greater than CBR/\( p_e \) of 0.22). When Potter compared the cubic equation to the test section data (all representing failures at 5,000 coverages) that had been used in developing the \( \alpha \) curves, he found that the cubic equation appeared to fit through the mid-range of the data. From this observation, he concluded that the reliability of the CBR flexible pavement design model is about 50 percent.

The approach similar to the AASHTO (1986) design procedure and Witczak's approach (1983) was used in the development of the reliability-based design procedures presented in this report.

**Applicability of Statistical Methods to the Corps of Engineers Design Procedures**

**Investigation of Statistical Parameter Assumptions**

**Type of Distribution**

Neither the Monte Carlo, FOSM, nor the PEM techniques for determining the expected value (mean) and variance of a function of several independent variables specify what type of distribution the independent variables must have. The only information that is required of the methods is that the mean value and variance of the distribution is known, regardless of the distribution. The selection of a frequency distribution for the input variables, then, should be based upon observation of actual data to determine which distribution (normal, Weibull, uniform, beta, etc.) best characterizes the data set. Most pavement reliability techniques investigated for this report assume that the variation of the pavement design parameters, such as concrete flexural strength or CBR of subgrade, may be described adequately by the normal distribution (with the exception of design traffic passes or coverages, which assume a log-normal distribution). The normal distribution fits in well with the Monte Carlo, FOSM, or PEM techniques, because it also requires that only the mean and variance of the data be known to describe the entire frequency distribution.
Harr (1987) points out that, although the normal distribution may adequately characterize the distribution of most pavement design parameters, its use can lead to (somewhat unlikely) instances of creating an impossible situation, such as a negative flexible pavement thickness. For instance, if the mean of the thickness is very small (say one inch) and the coefficient of variation CV is large (say 50 percent), and if the thickness is assumed to be normally distributed, the pavement has a 2.3 percent chance of having a negative thickness, which is an obviously impossible situation. He recommends the beta distribution be used for defining pavement material variability, which requires that, in addition to the mean and variance, the maximum and minimum values of the parameter be known. This information describes the complete distribution of the function, and eliminates the sometimes untenable situation of predicting a negative thickness.

**Independence of Variables**

The Monte Carlo, FOSM, and PEM techniques consider the possibility that the variables used to determine the value of a function may be related to each other; i.e. the correlation coefficient (or covariance) between any of the variables may be some value other than one. The correlation coefficient is sometimes (usually) assumed to be zero, which typically serves to reduce the total variance of the function (if the correlation is positive), which in turn may result in higher estimations of reliability than actually exist. The opposite effect occurs if the correlation coefficient is negative. Although these effects are recognized in the reliability analysis techniques reviewed earlier, they are usually ignored.
Part II: Identification of Design Parameters

Rigid Pavement Design Procedure

Identification of Pavement Design Parameters

The Corps of Engineers rigid pavement airfield design procedure using the Westergaard edge stress equation uses the 90-day flexural strength $R$, the modulus of elasticity $E$, and the Poisson's ratio $\mu$ of the concrete; the modulus of subgrade reaction $k$ of the foundation material; the load transfer $LT$; and the gear load $P$ in determining the design factor (DF\text{ALLOWABLE}) and ultimately the capacity of the pavement $LC$. The Corps CBR flexible pavement design procedure uses the CBR of the subgrade, the subbase, and the base; the effective tire contact pressure $p_e$ and contact area $A$; and the gear load $P$ in determining the required thickness of flexible pavement to cover the subgrade. The design passes used in determining the demand log coverages until failure $LC_D$ is the same for both the rigid and flexible design procedures; the difference in determining the $LC_D$ for the rigid and flexible pavement design methods is the different pass to coverage ratios $p/c$ for rigid and flexible pavements for a particular aircraft. The variability of the pavement design parameters was found in several sources of literature, and are presented in the following paragraphs.

Variability of Design Parameters — Rigid Pavements

Since the variance of a parameter or variable $x$ is simply the square of the standard deviation of the parameter, the variance may be estimated from the mean $\bar{x}$ and coefficient of variation (CV) for the parameter, since the CV is the standard deviation divided by the mean of the parameter (usually expressed as a percentage); i.e.

$$V[x] = (CV \times \bar{x})^2$$

Eqn. 21

The coefficient of variation is a common way of expressing the variability of design parameters in engineering, such as the design parameters $R$, $LT$, $h$, and $k$. Table 4 contains common CV's for $R$, $h$, $LT$, and $k$ for concrete pavements as found in the literature.

Concrete Flexural Strength

For the examples cited in the literature, the CV for 28-day flexural strength ($R$) ranges from about 4 to 15 percent. The CV for the 90-day flexural strength is probably also in this range, perhaps lower. A $CV_R$ of concrete pavement mixtures of less than 10 percent is considered excellent quality control (Witczak), 10 to 20 percent good control,
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<th>Source</th>
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<td></td>
<td>537</td>
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<td></td>
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<td></td>
<td>8.2 (8.0)</td>
<td>2.9</td>
<td>34</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.2 (8.0)</td>
<td>3.4</td>
<td>31</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>7.6 (8.0)</td>
<td>0.6</td>
<td>10</td>
<td></td>
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</tr>
<tr>
<td></td>
<td>7.6 (8.0)</td>
<td>1.4</td>
<td>9</td>
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</tr>
<tr>
<td>Thickness of PCC: Airfield Pavements (in.)</td>
<td>10.9(10.0)</td>
<td>8.3</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.8 (14.0)</td>
<td>3.3</td>
<td>10</td>
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</tr>
</tbody>
</table>
Table 4. Variability of Rigid Pavement Design Parameters (cont.)

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Mean</th>
<th>Coefficient of Variation (CV) (Percent)</th>
<th>Number of Tests</th>
<th>Source</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>Modulus of Subgrade Reaction (k): (pci)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>62.2</td>
<td>13.2</td>
<td>18</td>
<td>National Research Council (1962)</td>
<td></td>
</tr>
<tr>
<td>Subbase</td>
<td>71.6</td>
<td>13.9</td>
<td>15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>100</td>
<td>16.0</td>
<td>NA</td>
<td>Treybig (1970)</td>
<td></td>
</tr>
<tr>
<td>Load Transfer (LT): (percent)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dowelled Construction</td>
<td>30.6</td>
<td>38</td>
<td></td>
<td>Rollings (1987)</td>
<td></td>
</tr>
<tr>
<td>Dowelled Expansion</td>
<td>30.5</td>
<td>24.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Contraction</td>
<td>37.2</td>
<td>19.2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Keyed Joint</td>
<td>25.4</td>
<td>41.4</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Randolph AFB: Cold</td>
<td></td>
<td></td>
<td></td>
<td>Hammons (1995)</td>
<td>“Cold” refers to winter conditions; “warm” refers to summer conditions</td>
</tr>
<tr>
<td>Transverse Contraction</td>
<td>17.6 to 20.4</td>
<td>30 to 32.9</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Cont.</td>
<td>17.4</td>
<td>37.8</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long. Contraction Tied</td>
<td>25.5 to 27</td>
<td>6.9 to 16.1</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Keyed</td>
<td>13.9 to 15.6</td>
<td>33.3 to 51.9</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long. Dowelled</td>
<td>19.3</td>
<td>9.3</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sheppard AFB: Warm</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transverse Contraction</td>
<td>10.6 to 25.2</td>
<td>8.7 to 29.9</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. Cont. Dowelled</td>
<td>26.6</td>
<td>11.8</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trans. Cont. Tied</td>
<td>24.7</td>
<td>34.9</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Cont.</td>
<td>13.6</td>
<td>11.9</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Keyed</td>
<td>13.7 to 21.8</td>
<td>21.5 to 47.4</td>
<td>24</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Long. Cont. Dowelled</td>
<td>30.7</td>
<td>3.5</td>
<td>12</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Longitudinal Dowelled</td>
<td>28.5</td>
<td>16.2</td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
and greater than 20 percent poor quality control. From this data, the quality control for the concrete flexural strength appears to be very good.

It should be pointed out that the Corps of Engineers specifications for concrete pavement construction (Department of the Army 1987) state that only 2 out of 10 individual flexural strength test results may be lower than the design strength without penalties. If it is assumed that the concrete pavement strength is normally distributed, this would mean that the mean strength of the concrete in place would need to be at least \(0.84 \times CV_R \times R\) greater than the design flexural strength \(R\) so that no penalties would be incurred. For a design strength of 650 psi and a \(CV_R\) of 15 percent, the mean in-place flexural strength would need to be at least 732 psi.

**Concrete pavement thickness**

The CV for concrete pavement thickness \((h)\) is not as commonly reported in the literature; values of about 3 to 8 percent were found for conventional concrete highway and airfield pavements. The concrete pavement thickness also tended to be slightly larger than the specified design thickness, perhaps due to severe penalties imposed in specifications for inadequate pavement thickness. The mean actual thickness being larger than the design thickness would result in a greater reliability for a given design, even though quantifying this effect for a given pavement structure would be difficult to do during the design process.

**Modulus of subgrade reaction**

The CV of the modulus of subgrade reaction \((k)\) determined from plate load tests varied from 13 to 16 percent in the literature. The backcalculated \(k\) value from elastic-layer analysis of deflection basin data using non-destructive testing equipment has suggested that the in-place \(CV_k\) may actually be much higher; results from tests under roller-compacted concrete (RCC) pavements have suggested that the \(CV_k\) may range from 9 to 40 percent (Pittman 1994).

**Load transfer**

Rollings (1987) suggests that the \(CV_{LT}\) of several concrete pavement joint types may range from 24 to 41 percent. Hammons et. al. (1995) suggests that the \(CV_{LT}\) may range from 3 to 52 percent, depending on the joint type and pavement temperature. It should be noted also that the 25 percent load transfer assumption in the rigid pavement design procedure was not reflected in the mean load transfer results obtained by non-
destructive techniques for several joint types (Hammons 1995), especially in cold weather conditions. If the mean load transfer of 25 percent is not obtained as assumed in the design procedure, the reliability of the pavement is significantly reduced. However, as with the concrete pavement thickness, this knowledge is not readily available during the design process; an evaluation of the load transfer after the pavement is constructed would allow a more accurate analysis of the reliability of the pavement system.

**Variability of Design Parameters — Flexible Pavements**

The CV of several flexible pavement design parameters, including the CBR of the subgrade, subbase, and base; the asphalt concrete pavement layer thicknesses; and the tire pressure (of trucks) was found in the literature. Table 5 contains the results of the literature survey; a brief discussion of these results follows.

**CBR**

The CV of the in-place CBR was available from several literature sources, and tended to vary depending upon the layer in question: the subgrade, subbase, or base course. The $CV_{CBR}$ of the subgrade varied from 18 to 35 percent; for the subbase, 9 to 37 percent; and the base, 20 to 38 percent. Part of the variability may be due to the difficulty of obtaining the CBR of granular materials, since the strength of granular materials depends upon the confining stress, and the determination of the in-situ CBR requires at least a temporary release of the confining stress while a test pit is excavated. The high variability of the strength of the in-situ materials, particularly the subgrade, is not entirely surprising, however, considering the variation in types of subgrade materials found under an entire pavement system, and the variability of the degrees of compaction received from point to point. The $CV_{CBR}$ is of the same order of magnitude as that found for the $CV_k$, which lends consistency to this logic.

**Asphalt pavement layer thickness**

The CV of asphalt pavement layer thicknesses tended to be somewhat larger than those obtained for concrete pavement thickness, ranging from 4 to 12 percent (asphalt concrete plus asphalt stabilized base). The CV of the base course thickness was reported as 12 percent, and for the subbase, 15 percent. This reflects the amount of effort taken to control these thickness during the construction process; the subbase layer must also smooth out any deviations in the subgrade surface, the base smoothes out deviations in the
<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Mean</th>
<th>Coefficient of Variation (CV) (Percent)</th>
<th>Number of Tests</th>
<th>Source</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>CBR</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>NA</td>
<td>35</td>
<td>8 lots</td>
<td>Paterson (1979)</td>
<td>&quot;NA&quot; indicates data not available</td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>20</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>22</td>
<td>12 lots</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>NA</td>
<td>8.6</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>Subgrade</td>
<td>1.14</td>
<td>27.5</td>
<td>18</td>
<td>National Research Council (1962)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.445</td>
<td>27.5</td>
<td>11</td>
<td></td>
<td></td>
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<td>Subgrade</td>
<td>7.1</td>
<td>22.3</td>
<td>33</td>
<td>Yoder (1975)</td>
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<tr>
<td></td>
<td>4.2</td>
<td>21.4</td>
<td>33</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>18.2</td>
<td>26.2</td>
<td>7</td>
<td></td>
<td></td>
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<td></td>
<td>7.8</td>
<td>17.9</td>
<td>7</td>
<td></td>
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</tr>
<tr>
<td>Subbase</td>
<td>26.3</td>
<td>31.9</td>
<td>33</td>
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</tr>
<tr>
<td>Base</td>
<td>20.3</td>
<td>36.9</td>
<td>33</td>
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<td></td>
</tr>
<tr>
<td>Base</td>
<td>94.3</td>
<td>37.6</td>
<td>72</td>
<td></td>
<td></td>
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<tr>
<td>Subbase</td>
<td>NA</td>
<td>20</td>
<td>NA</td>
<td>Noureldon (1994)</td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>NA</td>
<td>20</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Subgrade</td>
<td>NA</td>
<td>20</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt Pavement Thickness (in.)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt Concrete</td>
<td>NA</td>
<td>7 (3 to 12)</td>
<td>NA</td>
<td>Noureldon (1994)</td>
<td></td>
</tr>
<tr>
<td>Asphalt Base</td>
<td>NA</td>
<td>10 (5 to 15)</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granular Base</td>
<td>NA</td>
<td>12 (10 to 15)</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Granular Subbase</td>
<td>NA</td>
<td>15 (10 to 20)</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Asphalt Concrete and Stabilized Base</td>
<td>21.66</td>
<td>3.5</td>
<td>NA</td>
<td>Attoh-Okine</td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.37</td>
<td>11.8</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>14.32</td>
<td>1.7</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10.91</td>
<td>9.5</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12.46</td>
<td>7.1</td>
<td>NA</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tire Pressure (Trucks): (psi)</td>
<td></td>
<td></td>
<td></td>
<td>Middleton (1986)</td>
<td></td>
</tr>
<tr>
<td>Radial Tires</td>
<td>94.6 to 99.8</td>
<td>12 to 17</td>
<td>17 to 730</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Bias Tires</td>
<td>78 to 95</td>
<td>12 to 20</td>
<td>11 to 265</td>
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<td></td>
</tr>
</tbody>
</table>
subbase surface, and the asphalt concrete layer smooths out deviations in the base course surface.

An interesting observation was made by Darter et al. (1973) that spoke to the effect of contracting practices on the thickness of the asphalt concrete pavement layer. The contractor tends to keep the asphalt concrete layer to a minimum if the materials are paid for by the square yard, minimizing the cost to the contractor and increasing profits. If the materials are paid for by the ton, the asphalt pavement layer tends to be thicker, increasing the payment per square yard and again increasing profits for the contractor. A thinner asphalt pavement would reduce the reliability of the pavement, all other factors being equal, while a thicker pavement would have the opposite effect.

**Tire pressure**

No information on the CV of tire pressure for aircraft was found in the literature. However, a study to determine the effects of truck tire pressure on highway pavement performance revealed that the $CV_p$ ranged from 12 to 17 percent for radial tires and 12 to 20 percent for bias-ply tires. Assuming that aircraft receive a higher degree of maintenance than the typical tractor-trailer truck, and that the tolerance specifications for aircraft tire pressure during maintenance is at most 10 percent of the average tire pressure, then this range of $CV_p$ is probably higher than that for aircraft.

**Variability of Aircraft Load Magnitude and Repetitions of Load**

An estimate of the variability of the magnitude of aircraft loads was not found in the literature. The rigid pavement and CBR design procedures assume that the aircraft is at maximum take-off weight for each repetition of aircraft load. This is a very conservative assumption, considering that the aircraft loses some fuel weight during flight and therefore lands at a lower weight than takeoff, and that the aircraft, particularly for training exercises, is probably not loaded to capacity weight for each operation of the aircraft. Therefore, estimates of the variability of aircraft weight were made based upon the range between the maximum take-off weight and the basic mission take-off weight, as reported by Holliway (1988). These weights for four aircraft are given in Table 6. If it is assumed that the basic mission take-off weight is the average weight of the aircraft during operation of the aircraft, and that the weight of the aircraft over many operations is normally distributed, then the range between the average and maximum take-off weights represents three standard deviations of take-off weight. Therefore an estimate of the CV of the takeoff weight can be found by dividing the difference in the basic mission and
maximum take-off weights by three, and dividing that dividend by the basic mission (average) weight. This approximation lead to CV_F’s ranging from 3 to 10 percent, depending upon the type of aircraft. These estimates are likely on the low side of the true CV_F for an aircraft, considering the number of applications of traffic during which the fuel tanks are almost empty (landing aircraft) and therefore the weight is even less.

An estimate for the CV of the design passes or coverages (LC_D) was also difficult to obtain for aircraft. The AASHTO design procedure assumes that the variance of the difference between the actual and predicted (or design) logarithm of passes of ESAL traffic is about 0.194 for flexible pavements and 0.114 for rigid pavements. This translates to a CV of predicted design passes (CV_{DP}) of about 275 percent for a flexible pavement, or a CV_{DP} of about 220 percent for a rigid pavement. These estimates of the variability are likely higher than that experienced with military airfields, since the traffic on airfields is controlled to a certain extent by air traffic controllers and mission requirements. However, the difference in the predicted and actual LC_D is likely to be high, due to factors unknown to the designer predicting the LC_D, such as changing mission requirements or the development of new aircraft.
Suggested Levels of Variability of Pavement Design Parameters

Table 7 lists a range of coefficients of variation (CV’s) that appeared to be most representative of those found in the literature. These values were used in a sensitivity analysis of the reliability-based design models, which are presented in later sections of this report.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Coefficient of Variation (CV), percent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rigid Pavements</strong></td>
<td></td>
</tr>
<tr>
<td>1. Load Magnitude, ( P ) (lbs)</td>
<td>5</td>
</tr>
<tr>
<td>2. Design Passes, ( DP )</td>
<td>10</td>
</tr>
<tr>
<td>3. Thickness of PCC, ( h ) (in.)</td>
<td>2</td>
</tr>
<tr>
<td>4. 28-day Flexural Strength, ( R ) (psi)</td>
<td>10</td>
</tr>
<tr>
<td>5. Modulus of Subgrade Reaction, ( k ) (pci)</td>
<td>10</td>
</tr>
<tr>
<td>6. Load Transfer, ( LT ) (percent)</td>
<td>10</td>
</tr>
<tr>
<td><strong>Flexible Pavements</strong></td>
<td></td>
</tr>
<tr>
<td>1. Load Magnitude, ( P ) (lbs)</td>
<td>5</td>
</tr>
<tr>
<td>2. Design Passes, ( DP )</td>
<td>10</td>
</tr>
<tr>
<td>3. Thickness of Flexible Pavement (AC+Base+Subbase), ( t ) (in.)</td>
<td>5</td>
</tr>
<tr>
<td>4. CBR (percent)</td>
<td>15</td>
</tr>
<tr>
<td>5. Tire Pressure, ( p ) (psi)</td>
<td>10</td>
</tr>
</tbody>
</table>
Part III: Development of Reliability-Based Design Procedures

Reliability-Based Design Procedure for Rigid Airfield Pavements

Because the performance equation for rigid pavements is expressed as a function that is continuous over the range of independent variables normally considered in design, the first-order second moment (FOSM) procedure was selected as the means of determining the variance of the log coverages until failure (or the capacity \( LC_C \)) of the pavement system (or \( V[LC_C] \)) in terms of the design input variables. The first step in this process is to express the design factor versus \( LC_C \) equation (Eqn. 10) in terms of \( LC_C \), as follows:

\[
LC_C = 4DF + 0.004k - 2.8
\]  
Eqn. 22

where the terms are described in Eqn. 10. From the FOSM procedure, the expected value \( E[LC_C] \) and the variance \( V[LC_C] \) of the \( LC_C \) can be determined by applying Eqns. 19 and 20 to Eqn. 22 as follows:

\[
E[LC_C] = -2.8 + \left[ \frac{4 \bar{R}}{(1-\bar{L} \bar{T})} \left( \frac{P}{\bar{h}^2} \left[ a_0 + a_1 \ln(l) + a_2 (\ln(l))^2 \right] \right) \right] + 0.004k \]  
Eqn. 23

\[
V[LC_C] = \left( \frac{\partial LC_C}{\partial R} \right)^2 V[R] + \left( \frac{\partial LC_C}{\partial LT} \right)^2 V[LT] + \left( \frac{\partial LC_C}{\partial h} \right)^2 V[h] + \left( \frac{\partial LC_C}{\partial k} \right)^2 V[k] + \left( \frac{\partial LC_C}{\partial P} \right)^2 V[P] \]  
Eqn. 24

For the purpose of developing the reliability model for rigid pavements, the variances of the concrete flexural strength \( (R) \), the load transfer \( (LT) \), the pavement thickness \( (h) \), the modulus of subgrade reaction \( (k) \), and the load \( (P) \) were considered to be the most critical in evaluating the variance of \( LC_C \), since their influence in the calculation of the design stress and the design factor is most pronounced. The influence of the variances of the concrete modulus of elasticity \( (E_c) \) and the Poisson's ratio \( (\mu) \) were considered negligible in the calculation of \( V[LC_C] \), since their influence is minor in the calculation of the design stress -- their values are often approximated in the calculation of the design stress. For this reason, the variances of \( E_c \) and \( \mu \) are ignored in the calculation of \( V[LC_C] \). Also, if the variables \( R, LT, h, P, \) and \( k \) are considered independent of each other, then the covariance of these variables is zero, and the last term of Eqn. 20 is zero.
The assumption that these variables are independent was considered reasonable for purposes of this analysis.

The partial derivatives in Eqn. 24 are expressed as follows:

\[
\frac{\partial LC_C}{\partial R} = \frac{4h^2}{P(1-LT)(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)} \quad \text{Eqn. 25}
\]

\[
\frac{\partial LC_C}{\partial LT} = \frac{4Rh^2}{P(1-LT)^2(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)} \quad \text{Eqn. 26}
\]

\[
\frac{\partial LC_C}{\partial h} = \frac{4Rh}{P(1-LT)(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)} \left[ 2 - \frac{(0.75a_1 + 1.5a_2 \ln(l))}{(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)} \right] \quad \text{Eqn. 27}
\]

\[
\frac{\partial LC_C}{\partial k} = \frac{Rh^2(a_1 + 2a_2 \ln(l))}{Pk(1-LT)(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)^2} + 0.004 \quad \text{Eqn. 28}
\]

\[
\frac{\partial LC_C}{\partial P} = \frac{4h^2}{P(1-LT)(a_0 + a_1 \ln(l) + a_2(\ln(l))^2)} \quad \text{Eqn. 29}
\]

To estimate the variance of the performance equation (DF vs LC_C), the data used in developing the original equation was used in performing a linear least-squares regression between the actual applied LC_C and the predicted LC_C from the performance equation (Figure 14). From this regression analysis, the estimated variance of the regression \(V[\text{Reg}]\), which is analogous to the variance of a variable, was determined for the regression (Draper and Smith 1981) as follows:

\[
V[\text{Reg}] = \frac{SS_e}{(n-2)} \left( 1 + \frac{(LC_C - E[LC_C])^2}{\sum (LC_{Ci} - E[LC_C])^2} \right)
\]

where the \(SS_e\) is the sum of the squares of the errors for the regression (1.44), \(n\) is the number of data sets used in the regression (50), and \(LC_{Ci}\) is each value of \(LC_C\) used in the regression. The variance of the regression follows a hyperbolic function on either side of the expected value, with the minimum variance occurring at the mean of the LC_C values used in the regression. The variance increases as the estimate is obtained for values farther away from the mean LC_C; the standard deviation of the regression (\(\sqrt{V[\text{Reg}]}\)) relative to the expected value is shown in Figure 14 as the dashed lines above and below the regression line.
The variance of the log coverages for the demand $V[L_{CD}]$ can be determined simply:

$$V[L_{CD}] = (CV_{D} \times L_{CD})^2$$  \hspace{1cm} \text{Eqn. 31}$$

where $CV_{D}$ is the coefficient of variation for the demand (estimated log coverages of applied traffic) and the $L_{CD}$ is the expected log coverages of traffic (demand) to be applied to the pavement $E[L_{CD}]$.

Once the $E[L_{C}]$, $V[L_{C}]$, $E[L_{D}]$, $V[L_{D}]$, and $V[Reg]$ are calculated, the safety margin $SM$ can be calculated as:

$$E[SM] = E[L_{C}] - E[L_{D}]$$  \hspace{1cm} \text{Eqn. 32}$$

and the variance of the safety margin as

$$V[SM] = V[L_{C}] + V[L_{D}] + V[Reg]$$  \hspace{1cm} \text{Eqn. 33}$$
These values are represented in Figure 1 (V[Reg] is included within V[LCc]). The value of the standard normal variate \( Z \) can then be calculated as:

\[
Z = \frac{E[SM]}{\sqrt{V[SM]}} \tag{Eqn. 34}
\]

Note that the square root of the \( V[SM] \) in the denominator of the equation is the standard deviation of \( SM \).

\( F(Z) \) represents the cumulative distribution function of the standard normal variate, which is simply the area under the normal distribution curve in Figure 1 greater than \( Z \) standard deviations from \( E[SM] \). The reliability of the pavement system is then simply \( F(Z) \); that is:

\[
\text{Reliability} = \Pr[SM \geq 0] = F(Z) \tag{Eqn. 35}
\]

Note that if the LCc and LCce are equal, the SM is zero and the reliability of that design is 50 percent. This is the assumption made in the deterministic design procedure. Values of \( F(Z) \) can be obtained from many probability and statistics texts; the \( Z \) values for several levels of reliability (\( F(Z) \)) are given in Table 8.

<table>
<thead>
<tr>
<th>Reliability</th>
<th>( Z )</th>
</tr>
</thead>
<tbody>
<tr>
<td>50%</td>
<td>0</td>
</tr>
<tr>
<td>60%</td>
<td>0.26</td>
</tr>
<tr>
<td>70%</td>
<td>0.53</td>
</tr>
<tr>
<td>80%</td>
<td>0.84</td>
</tr>
<tr>
<td>90%</td>
<td>1.28</td>
</tr>
<tr>
<td>95%</td>
<td>1.645</td>
</tr>
</tbody>
</table>

**Table 8. Values of the Standard Normal Variate \( Z \) for Various Levels of Reliability**

*The Reliability-Based Rigid Airfield Design Program (RRAD)*

These concepts were applied in the development of a computer software spreadsheet program called RRAD.XLS. The spreadsheet was written using the
Microsoft Excel® spreadsheet and Visual Basic programs, and may be executed on any personal computer containing this software along with the Solver analytical package. The program calculates for a single aircraft the thickness of a concrete slab necessary for a Type A traffic area (deterministic design), assuming non-frost conditions, and develops reliability versus thickness and thickness versus LC_C plots as output. RRAD allows the selection of one of four different aircraft, the F-15E, C-130, C-141, and C-5A, on the input screen. The modulus of elasticity and Poisson's ratio of the concrete are assumed constant at 4,000,000 psi and 0.15, respectively. The load transfer is assumed to be 25 percent, and the edge stress equation constants (a_o, a_i, and a_j) and the p/c ratio for a Type A traffic area are automatically selected for the design aircraft.

A flowchart illustrating the algorithms used by the RRAD program is shown in Figure 15. The program requires the deterministic inputs of type of aircraft, load magnitude P, the design passes DP, the 90-day flexural strength R, the modulus of subgrade reaction k, and the base thickness BT. Variability inputs are the CV’s of the load (CV_P), the design passes (CV_DP), the flexural strength (CV_R), the thickness (CV_h), and the load transfer (CV_LT). The program suggests the design load for the selected aircraft, and automatically calculates and displays the effective k on top of the base used in calculating the design thickness from Eqn. 12. The program also automatically calculates the minimum mean concrete field strength necessary to meet the construction specification requirements from the following equation:

\[
Field \text{ } R = R \times (1 + CV_R \times 0.84)
\]

Eqn. 36

where 0.84 (or -0.84) is the value of the standard normal variate Z at which 20 percent of a normally distributed population would be less. An example of the RRAD input screen with selected design and variability parameters is shown in Figure 16.

RRAD uses the same default equations and assumptions for calculating the thickness of the rigid pavement as is presented in the computer source code for RAD version 1.0, the Corps of Engineers rigid pavement design program. The primary difference in the RAD and RRAD programs is minor differences in the stress equation coefficients which can lead to slight differences in the design thicknesses determined by the methods.

* Microsoft and Visual Basic are registered trademarks of the Microsoft Corporation. Microsoft Excel Solver code is copyrighted by Frontline Systems, Inc. and Optimal Methods, Inc.
Figure 15. Flowchart for RRAD
Reliability-Based Rigid Airfield Pavement Design (RRAD)

This program calculates the thickness of an airfield concrete pavement slab for a range of reliability levels, based upon the Corps of Engineers criteria for rigid airfield pavements. The thicknesses determined are for a Type A traffic area only, non-frost conditions.

Step 1: Insert values for design inputs (50% reliability):

| Type of aircraft (use all capital letters) | C-130 | Enter "C-130", "C-141", "F-15", or "C-5A" |
| Design load (lbs): | 175,000 | Default value (lbs) = 175,000 |
| Design passes: | 10,000 | Use range of 10 to 10,000,000 passes |
| Modulus of Subgrade Reaction k (pci): | 100 | Use range of 50 (clay) to 500 (sand) |
| Design 90-day flexural strength ft (psi) | 650 | Typical range: 500 to 800 Field ft avg: 732 |
| Thickness of non-frost base (in.) | 4 | Min. 4 inches k on top of base 123 |

Step 2: Enter expected coefficient of variation (CV) in percent for the following variables:

| Design load: | 10 | Suggested range: 5 to 15 (%) |
| Passes: | 50 | Suggested range: 10 to 90 (%) |
| Total thickness: | 5 | Suggested range: 2 to 8 (%) |
| Flexural strength: | 15 | Suggested range: 10 to 20 (%) |
| Modulus of Subgrade Reaction (pci): | 25 | Suggested range: 10 to 40 (%) |
| Load Transfer: | 25 | Suggested range: 10 to 40 (%) |

Step 3: Press the tab at the bottom of the page entitled "Calculations" to begin calculations.

Figure 16. Input screen for RRAD with example problem

The program uses the deterministic design inputs in an iterative nonlinear optimization routine in Solver to find the thickness \( h \) necessary to satisfy the conditions of Eqn. 11. The thickness is rounded up to the nearest one-half inch to obtain the design thickness. The reliability analysis is then conducted to find a set of thicknesses and associated reliability's for the given deterministic and variability inputs. The program uses the iterative technique to calculate a range of thickness for a range of \( L_C \) of about 2 to 10, this time with the same deterministic design inputs except that the field \( R \) is used instead of the design \( R \) in calculating the thicknesses. The field \( R \) is used because it is closer to the actual mean flexural strength in the field, and the mean field flexural strength was used in the development of the performance equation represented in Eqn. 8.

The reliability analysis begins by calculating the variance of each of the design parameters \( V[\text{parameter}] \) by using the CV's input at the beginning of the program:

\[
V[\text{parameter}] = (CV_{\text{parameter}} \times \text{parameter})^2
\]

Eqn. 37
The variance of the $LC_C$, $LC_D$, and the regression of the predicted versus actual $LC_C$ is then calculated from Eqns. 24, 30, and 31 for each set of assumed $LC_C$ and accompanying $h$ values. The variance and expected value of the safety margin $SM$ are then calculated for each set by Eqns. 32 and 33. The value of the standard normal variate $Z$ is calculated for each $LC_C/h$ set using Eqn. 34, and the cumulative normal distribution function $F(Z)$, which represents the reliability of that combination of $LC_C$ and $h$, is determined by a spreadsheet function. The program then plots the design thickness versus the reliability of the design thickness (represented as 50 percent reliability), and the range of thicknesses with their accompanying reliability’s on the same plot. An example of this plot for the design input parameters given in Figure 16 is shown in Figure 17.

Since the reliability curve uses the mean field flexural strength instead of the design flexural strength when calculating the thickness $h$ for a particular $LC_C$, the reliability curve will not pass through the 50 percent reliability line at the design thickness, but will instead be at a higher reliability at the design thickness (Figure 17). This is intuitively correct; if the actual mean flexural strength in the field is greater than the input design strength (which it is if the contractor meets requirements of the existing construction guide

![Image of reliability curve](image.png)

**Figure 17.** RRAD output screen: thickness vs. reliability plot
specifications), then it is obvious that the pavement would last longer (or have a greater \( LC_c \)) than if the mean field strength were lower, or equal to the input design strength. If the CV of the flexural strength is input as zero, the reliability curve will pass through the plotted design thickness-50 percent reliability point.

A plot of the thicknesses \( h \) calculated for each \( LC_c \) is also generated. These thicknesses correspond to a reliability of 50 percent, since the mean values of the design parameters (and the estimated mean field \( R \)) are used in calculating \( h \). An example of this plot generated for the design input values shown in Figure 16 is shown in Figure 18.

Sensitivity Analysis of RRAD

To determine the effects of the variance of the design factor parameters on the reliability of concrete pavements designed with the Corps of Engineers procedure, a sensitivity analysis was performed using a range of CV values for the parameters. In this analysis, a matrix of low, medium, and high CV values for \( R, LT, h, k, DP \) and \( P \) were

![Thickness of Concrete Slab at 50% Reliability vs Passes until Failure](image)

**Figure 18.** RRAD output screen: thickness vs. passes until failure

47
used to determine the effects of the resulting individual variances on the reliability of various thicknesses. In the analysis, a design thickness concrete of 11.5 in. was determined for 10,000 passes of a C-130 aircraft, assuming a load $P$ of 175,000 pounds, an $R$ of 650 psi, a $LT$ of 25 percent, a $k$ of 100 pci, a modulus of elasticity of concrete of 4,000,000 psi and a Poisson's ratio of 0.15. As previously stated, this design thickness is plotted at a reliability of 50 percent. The CV values used in the analysis are given in Table 7.

Figures 19 through 26 present the results of the sensitivity analysis in graphical form. The first figure represents the individual effect of the variance of the performance equation only ($V[\text{Reg}]$), with no variance of the input parameters. Figures 20 through 26 show the individual effects of the variances of $R$, $LT$, $h$, $k$, $P$, and design passes $DP$, respectively, on the reliability of a range of concrete thicknesses. In these figures, the CV of the parameters not shown on the graph is zero; however, the effect of the variance of the regression ($V[\text{Reg}]$) is included. The characteristic S-shape curves are similar to the cumulative distribution function of the normal variate $Z$. The curves are steeper for the smaller values of the CV for each parameter, suggesting that a small change in concrete

![Graph showing reliability of design (%) versus thickness of concrete slab (in.)](image)

**Figure 19.** RRAD sensitivity analysis: effect of $V[\text{Reg}]$
Figure 20. RRAD sensitivity analysis: effect of CV of flexural strength

Figure 21. RRAD sensitivity analysis: effect of CV of load transfer
Figure 22. RRAD sensitivity analysis: effect of CV of thickness

Figure 23. RRAD sensitivity analysis: effect of CV of k
Figure 24. RRAD sensitivity analysis: effect of CV of load

Figure 25. RRAD sensitivity analysis: effect of CV of passes
Figure 26. RRAD sensitivity analysis: effect of CV of all parameters

thickness would have a relatively large effect on the reliability of the pavement. As the CV increases, the variance increases, and the slope of the reliability curve decreases, indicating a decreased sensitivity of reliability to changes in thickness.

Note that Figure 20 representing the effect of changing $C V_R$ has reliability curves that pass above the design thickness-50 percent reliability data point. These curves can be used to obtain the estimated reliability of the design thickness by passing a vertical line through the design thickness point; the intersections of this line with the reliability curves represent the estimated reliability for the corresponding levels of variability (low, medium, or high CV). From the graph, it is apparent that the design thickness has a reliability of about 70 percent for all levels of $C V_R$. The intersections of the reliability curves with the 50 percent reliability line occur at the thicknesses for an estimated 50 percent reliability. These thicknesses, ranging from 10.6 in. for the high $C V_R$ to 11.2 in. for the low $C V_R$, will always be less than the deterministic design thickness if the $C V_R$ is non-zero.

The relative sensitivity of the reliability of the pavement systems to the variance of the individual design parameters can be estimated by observation of the slope of the
reliability curves. If the slope of the curve is changed (flattened) a relatively large amount when the variance of a particular design parameter is considered, then the sensitivity of the reliability to that parameter is relatively large. If the reliability is not sensitive to the variability in the design parameter, then the slope of the curve would remain unchanged and nearer to vertical; if there is no variability in any of the design parameters, or no error in the regression equation, and no difference in LCc or LCd, the reliability curve would be vertical, or equivalent to a deterministic design. For the range of CV's used in this analysis, the parameters whose variance appear to have the largest effect on the reliability, in order of largest to smallest effect, appear to be R, P, LT, h, k, V[Reg], and design passes DP. Again, these relative effects are based in part on the range of CV's assigned to the variables; if the CVDP was assumed to be higher than the assumed 10 to 90 percent (say the 200 percent range suggested by AASHTO), its relative effect on the design reliability might be much greater.

Figure 26 illustrates the combined effects of the variances of the parameters and the regression on the reliability of the example pavement system. The variances of the parameters were calculated from the low, medium, and high CV values in Table 8 and combined in those groupings to calculate the reliability for a range of pavement thicknesses. It is at first apparent that the slopes of the reliability curves are relatively flatter than the counterpart curves from the consideration of the individual variances; this indicates that the variance of SM is generally greater when the variance of more than one of the individual design factor parameters is considered. For this particular pavement system, the reliability of a 13-in.-thick concrete pavement ranges from about 79 to 92 percent as the variances of the parameters range from the high to the low values. Interpreted in a different way, the analysis shows that, to obtain a reliability of 90 percent, the required thickness ranges from about 12.8 in. to about 14.3 in. as the variances of the parameters range from the low to the high values.

Regardless of the method of interpreting the results of the sensitivity analysis, it is apparent that the reliability of a given pavement system is sufficiently sensitive to the variability of R, LT, h, k, P, and design passes to merit their consideration in pavement design and construction. This type of analysis is particularly useful in quantifying the effects of quality control on the performance of the pavement.

**Reliability-Based Design Procedure for Flexible Pavements**

The FOSM method was also used for the CBR design procedure, again because the CBR equation and the alpha curves are continuous functions over the range of input
variables normally considered for design. In the first step of the development of the reliability procedure for CBR design, the performance equations were transformed to express the capacity of the pavement (LC_c) in terms of the design parameters. This must be done in a two-step process, because the CBR equation was developed for a fixed number of coverages (5,000), and the alpha curves were developed to make any adjustments in the thickness necessary for any other coverage level, depending upon the number of wheels in the critical gear. Therefore, the CBR equation was expressed in terms of alpha (α), as follows:

\[
\alpha = \frac{t}{\sqrt{A} \left[ -0.0481 - 1.1562 \times \log \left( \frac{CBR}{P_e} \right) - 0.6414 \times \log \left( \frac{CBR^2}{P_e} \right) - 0.4730 \times \log \left( \frac{CBR^3}{P_e} \right) \right]}
\]

Eqn. 38

Next, the α curves were expressed in terms of LC_c. To accomplish this, fourth-order multiple linear regression equations were developed of the form:

\[
LC_c = b_0 + b_1(\alpha) + b_2(\alpha^2) + b_3(\alpha^3) + b_4(\alpha^4)
\]

Eqn. 39

for each of the α curves. The values of the regression coefficients \(b_n\) along with the correlation coefficient \(r^2\) for each of the α curves is given in Table 9.

<table>
<thead>
<tr>
<th>α curve</th>
<th>(b_0)</th>
<th>(b_1)</th>
<th>(b_2)</th>
<th>(b_3)</th>
<th>(b_4)</th>
<th>(r^2) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1-wheel</td>
<td>-0.65293</td>
<td>2.597798</td>
<td>5.109429</td>
<td>-4.83375</td>
<td>1.181418</td>
<td>99.981</td>
</tr>
<tr>
<td>2-wheel</td>
<td>-0.81468</td>
<td>4.74599</td>
<td>-1.19977</td>
<td>2.487597</td>
<td>-0.84312</td>
<td>99.976</td>
</tr>
<tr>
<td>4-wheel</td>
<td>-1.08453</td>
<td>7.96767</td>
<td>-3.43099</td>
<td>5.076565</td>
<td>-1.91868</td>
<td>99.984</td>
</tr>
<tr>
<td>6-wheel</td>
<td>-1.14345</td>
<td>8.80797</td>
<td>-2.72023</td>
<td>3.83910</td>
<td>-1.27587</td>
<td>99.979</td>
</tr>
<tr>
<td>8-wheel</td>
<td>-1.53273</td>
<td>13.43212</td>
<td>-4.11326</td>
<td>5.733025</td>
<td>-2.19275</td>
<td>99.941</td>
</tr>
<tr>
<td>12-wheel</td>
<td>-0.35165</td>
<td>-0.34886</td>
<td>1.829152</td>
<td>-3.62041</td>
<td>2.89996</td>
<td>99.969</td>
</tr>
<tr>
<td>16-wheel</td>
<td>0.788701</td>
<td>-14.8595</td>
<td>5.329309</td>
<td>-9.72765</td>
<td>6.538819</td>
<td>99.946</td>
</tr>
</tbody>
</table>
Using the FOSM procedure, the expected value of \( \alpha (E[\alpha]) \) and the variance \( V[\alpha] \) can be determined from the CBR equation (Eqn. 38) as follows:

\[
E[\alpha] = \frac{t}{\sqrt{A} \times \left[-0.0481 - 1.1562 \log \left(\frac{CBR}{p_e}\right) - 0.6414 \log \left(\frac{CBR^2}{p_e}\right) - 0.4730 \log \left(\frac{CBR^3}{p_e}\right)\right]}
\]

Eqn. 40

\[
V[\alpha] = \left(\frac{\partial \alpha}{\partial t}\right)^2 V[t] + \left(\frac{\partial \alpha}{\partial CBR}\right)^2 V[CBR] + \left(\frac{\partial \alpha}{\partial p_e}\right)^2 V[p_e]
\]

Eqn. 41

For the purpose of developing the reliability model for the CBR design procedure, the variances of the thickness of flexible pavement above the subgrade \( t \), the CBR, the tire pressure \( p_e \), the load magnitude \( P \), and the design passes \( DP \) were considered to be the most critical in evaluating the variance of \( \alpha \). Consideration of the influence of the variance of the tire contact area \( A \) was considered unnecessary since its value is considered fixed when performing the CBR design procedure, allowing the tire pressure to vary directly with the load \( P \). The covariance of the design parameters was considered zero; even though the load \( P \) and the tire pressure \( p_e \) are positively correlated, the correlation may vary for different values of \( A \). The simplicity of ignoring the covariance in the equation was considered more important than the influence of the covariance on the reliability results.

The partial derivatives of Eqn. 41 are expressed as follows:

\[
\frac{\partial \alpha}{\partial t} = \frac{1}{\sqrt{A} \times \left[-0.0481 - 1.1562 \log \left(\frac{CBR}{p_e}\right) - 0.6414 \log \left(\frac{CBR^2}{p_e}\right) - 0.4730 \log \left(\frac{CBR^3}{p_e}\right)\right]}
\]

Eqn. 42

\[
\frac{\partial \alpha}{\partial CBR} = \frac{t \times \left[0.50213 + 0.24195 \ln \left(\frac{CBR}{p_e}\right) + 0.11623 \ln \left(\frac{CBR^2}{p_e}\right)\right]}{CBR \times \sqrt{A} \times \left[-0.0481 - 1.1562 \log \left(\frac{CBR}{p_e}\right) - 0.6414 \log \left(\frac{CBR^2}{p_e}\right) - 0.4730 \log \left(\frac{CBR^3}{p_e}\right)\right]^2}
\]

Eqn. 43
\[
\frac{\partial \alpha}{\partial p_e} = \frac{-t \times \left[ 0.50213 + 0.24195 \times \ln \left( \frac{CBR}{p_e} \right) + 0.11623 \times \ln \left( \frac{CBR}{p_e} \right)^2 \right]}{p_e \times \sqrt{A} \times \left[ -0.0481 - 1.1562 \times \log \left( \frac{CBR}{p_e} \right) - 0.6414 \times \log \left( \frac{CBR}{p_e} \right)^2 - 0.4730 \times \log \left( \frac{CBR}{p_e} \right)^3 \right]^2}
\]

Eqn. 44

To include the influence of the variance of the load \( P \) in the total variance of \( \alpha \), the variance of \( p_e \) with respect to \( P \) was determined, using the relationship \( p_e = P/A \) and the FOSM technique:

\[
V[p_e] = \left[ \frac{\partial p_e}{\partial A} \right]^2 V[A] + \left[ \frac{\partial p_e}{\partial P} \right]^2 V[P]
\]

Eqn. 45

Assuming that the \( V[A] \) is zero, as stated earlier, the \( V[p_e] \) then becomes

\[
V[p_e] = \frac{1}{A^2} \times (CV(P))^2
\]

Eqn. 46

and the entire expression for \( V[p_e] \) then becomes

\[
V[p_e] = \frac{1}{A^2} \times (CV(P))^2 + (CV(p_e))^2
\]

Eqn. 47

Once the \( V[\alpha] \) is determined, the \( V[LCC] \) can be determined from the \( \alpha \) regression equations represented in Eqn. 39 and the FOSM technique. The \( V[LCC] \) then becomes:

\[
V[LCC] = \left[ \frac{\partial LCC}{\partial \alpha} \right]^2 V[\alpha]
\]

\[
= \left[ b_1 + 2b_2\alpha + 3b_3\alpha^2 + 4b_4\alpha^3 \right]^2 V[\alpha]
\]

Eqn. 48

where the coefficients \( b_i \) are obtained from Table 9.

To estimate the variance of the cubic CBR equation, the data used to validate the cubic equation as represented in Figure 11 was used to determine a least-squares linear regression between the actual and predicted \( \alpha \) values (Figure 27). From this regression analysis, the estimated variance of the regression \( V[Reg] \) was obtained in the same manner as represented in Eqn. 30, where the \( SS_e \) for the regression was 0.3126 and the number of data points \( n \) was 28. Just as was shown with the rigid pavement performance equation analysis, the \( V[Reg] \) is a hyperbolic function of the distance away from the mean \( \alpha \), and is represented by the dashed lines in Figure 27.
The variance of the demand $V[LC_D]$ is calculated using Eqn. 31. The $E[SM]$, $V[SM]$, and the standard normal variate $Z$ are calculated in the same procedure used for the rigid pavement analysis, and the reliability calculated using Eqn. 35.

**The Reliability-Based Flexible Airfield Pavement Design Program (RFAD)**

The concepts expressed in the preceding paragraphs were applied in developing a computer spreadsheet design program based on the Corps CBR design procedure called RFAD. Like the RRAD program, RFAD requires the use of Microsoft Excel and the Solver analytical package. The program calculates the total thickness $t$ of flexible pavement required above a subgrade with a specified CBR for a Type A traffic area, assuming non-frost conditions. The thicknesses of the individual subbase, base, and asphalt concrete layers are not computed in RFAD. The program also develops reliability versus thickness and thickness versus $LC_C$ plots as outputs, similar to RRAD.

A flowchart illustrating the process incorporated by RFAD is shown in Figure 28. The program requires the deterministic inputs of type of aircraft, gross weight of aircraft $P$, design passes $DP$, and subgrade CBR. Variability inputs include the CV’s of the load

---

**Figure 27. Actual vs. predicted alpha**

- The variance of the demand $V[LC_D]$ is calculated using Eqn. 31.
- The $E[SM]$, $V[SM]$, and the standard normal variate $Z$ are calculated in the same procedure used for the rigid pavement analysis, and the reliability calculated using Eqn. 35.

**The Reliability-Based Flexible Airfield Pavement Design Program (RFAD)**

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A flowchart illustrating the process incorporated by RFAD is shown in Figure 28. The program requires the deterministic inputs of type of aircraft, gross weight of aircraft $P$, design passes $DP$, and subgrade CBR. Variability inputs include the CV’s of the load.
Figure 28. Flowchart for RFAD
(CV_p), the design passes (CV_{DP}), the CBR (CV_{CBR}), the total thickness (CV_t), and the tire pressure (CV_p). The appropriate ESWL curve constants, \( \alpha \) curve constants, tire contact area, and the \( p/c \) ratio for a Type A traffic are assumed constant and are automatically retrieved when the design aircraft is entered. RFAD allows the selection of one of four different aircraft, the F-15E, C-130, C-141, and C-5A, on the input screen. An example of an RFAD input screen with a trial set of design and variability input parameters is shown in Figure 29.

RFAD uses the same default equations and assumptions for calculating the thickness of the flexible pavement as is presented in the computer source code for FAD version 1.0, the Corps of Engineers flexible pavement design program. The primary difference in the FAD and RFAD programs is the method used to obtain the ESWL (%) with depth; FAD uses a cubic equation to obtain the ESWL with depth, and RFAD uses a fifth-order equation (Eqn. 15). This difference may lead to slight differences in the ESWL determination, which in turn can lead to slight differences in the design thicknesses determined by the methods.

<table>
<thead>
<tr>
<th>Reliability-Based Flexible Airfield Pavement Design (RFAD)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>This program calculates the total flexible pavement thickness above a subgrade with a specified CBR for a range of reliability levels. The deterministic thickness design (50% reliability) is also determined for comparison. The design procedure is based upon the Corps of Engineers CBR criteria for airfield pavements. The thicknesses determined are for a Type A traffic area only, non-frost conditions.</strong></td>
</tr>
</tbody>
</table>

**Step 1: Insert values for design inputs (50% reliability):**

<table>
<thead>
<tr>
<th>Type of aircraft:</th>
<th>C-130</th>
<th>Enter &quot;C-130&quot;, &quot;C-141&quot;, &quot;F-15&quot;, or &quot;C-5A&quot; (use capital letters!)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design load (lbs):</td>
<td>175,000</td>
<td>Default value (lbs) = 175,000</td>
</tr>
<tr>
<td>Design passes:</td>
<td>10,000</td>
<td>Use range of 10 to 10,000,000 passes</td>
</tr>
<tr>
<td>Subgrade CBR:</td>
<td>3</td>
<td>Use range of 1 (highly plastic clay) to 20 (sand)</td>
</tr>
</tbody>
</table>

**Step 2: Enter expected coefficient of variation (CV) in percent for the following variables:**

<table>
<thead>
<tr>
<th>Design load:</th>
<th>10</th>
<th>Suggested range: 5 to 15 (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Passes:</td>
<td>60</td>
<td>Suggested range: 10 to 110 (%)</td>
</tr>
<tr>
<td>Total thickness:</td>
<td>10</td>
<td>Suggested range: 5 to 15 (%)</td>
</tr>
<tr>
<td>Subgrade CBR:</td>
<td>25</td>
<td>Suggested range: 15 to 35 (%)</td>
</tr>
<tr>
<td>Tire pressure:</td>
<td>15</td>
<td>Suggested range: 10 to 20 (%)</td>
</tr>
</tbody>
</table>

**Step 3:** Press the tab at the bottom of the page entitled "Calculations" to begin calculations.

**Figure 29. Input screen for RFAD with design example**
RFAD uses an iterative nonlinear optimization process in Solver to calculate the thickness of flexible pavement required to solve Eqn. 38, given the design input variables. As explained previously, the iterative process is necessary because the ESWL used in calculating the $p_x$ is a function of $t$. The $\alpha$ required for Eqn. 38 is obtained for the appropriate number of wheels using Eqn. 14. The $t$ which is obtained from the iterative process is then rounded up to the nearest half-inch to obtain the design thickness.

In a manner similar to RRAD, the RFAD reliability analysis is then conducted to find a set of thicknesses and associated reliability's for the given deterministic and variability inputs. The program uses the iterative technique to solve for $t$ for an assumed set of $LC_c$ values ranging from 2 to 10. For each set of $LC_c$ and $t$, the reliability analysis then calculates the variances of the design parameters using Eqn. 37. The variance of the $LC_c$, $LC_d$, and the regression of the actual versus predicted $\alpha$ are determined from Eqns. 48, 31, and 30, respectively. The $E[LC_c]$, $E[LC_d]$, and $E[SM]$ are calculated from Eqns. 39, 37, and 1, respectively, the $Z$ is calculated from Eqn. 34, and $F(Z)$ (the reliability of the system) is determined from a built-in Excel function. The program then plots the design thickness versus the reliability of the design thickness (50 percent), and the range of thicknesses (for the assumed $LC_c$) and their respective reliability's on the same graph. An example of this plot for the trial design parameters shown in Figure 29 is shown in Figure 30.

A plot of the thicknesses $t$ calculated for each $LC_c$ is also generated. These thicknesses correspond to a reliability of 50 percent, since the mean values of the design parameters are used in calculating $t$. An example of this plot generated for the design input values shown in Figure 29 is shown in Figure 31.

**Sensitivity Analysis of RFAD**

To determine the effects of the variance of the design parameters on the reliability of flexible pavements designed with the Corps of Engineers CBR procedure, a sensitivity analysis was performed using a range of CV values for the parameters. In this analysis, a matrix of low, medium, and high CV values for $CBR$, $t$, $p$, $DP$, and $P$ were used to determine the effects of the resulting individual variances on the reliability of various thicknesses. In the analysis, a total flexible pavement thickness of 38.5 in. was determined for 10,000 passes of a C-130 aircraft, assuming a load $P$ of 175,000 pounds, and a subgrade CBR of 3. A contact area of 400 square inches and a $p/c$ ratio of 2.09 are automatically retrieved for the design example. As previously stated, this design thickness
Figure 30. RFAD output: thickness vs. reliability

Figure 31. RFAD output: thickness vs. passes of traffic until failure
is plotted at a reliability of 50 percent. The CV values used in the analysis are given in Table 7.

Figures 32 through 38 present the results of the sensitivity analysis in graphical form. The first figure represents the individual effect of the variance of the performance equation only, with no variance of the input parameters. Figures 33 through 37 show the individual effects of the variances of $P$, $DP$, $t$, $CBR$, and $p$, respectively, on the reliability for a range of flexible pavement thicknesses. In these figures, the CV of the parameters not shown on the graph is zero; however, the effect of the variance of the regression is included. As with the RRAD analysis, the characteristic S-shape curves are similar to the cumulative distribution function of the normal variate. The curves are steeper for the smaller values of the CV for each parameter, suggesting that a small change in flexible pavement thickness would have a relatively large effect on the reliability of the pavement. As the CV increases, the variance increases, and the slope of the reliability curve decreases, indicating a decreased sensitivity of reliability to changes in thickness.

![Graph showing the effect of CV of V[Reg] only](image)

**Figure 32.** RFAD sensitivity analysis: effect of CV of V[Reg] only
Figure 33. RFAD sensitivity analysis: effect of CV of load

Figure 34. RFAD sensitivity analysis: effect of CV of Passes
Figure 35. RFAD sensitivity analysis: effect of CV ofThickness

Figure 36. RFAD sensitivity analysis: effect of CV of tire pressure
Figure 37. RFAD sensitivity analysis: effect of CV of CBR

Figure 38. RFAD sensitivity analysis: effect of CV of all variables
Using a ranking procedure similar to the one used in the RRAD sensitivity analysis, the design parameters which appear to have the most effect on the reliability of the pavement system, in order of greatest to least effect, are the CBR, \( t \), V[Reg], \( p_{s} \), \( DP \), and \( P \). These relative effects are based in part on the range of CV's chosen for each parameter; for instance, if a much larger range for \( CV_{p} \) was used than the assumed 5 to 15 percent range, the load \( P \) would have a greater effect than indicated.

Figure 38 illustrates the combined effects of the variances of the parameters and the regression on the reliability of the example pavement system. The variances of the parameters were calculated from the low, medium, and high CV values in Table 7 and combined in those groupings to calculate the reliability for a range of pavement thicknesses. It is apparent that the slopes of the reliability curves are relatively flatter than the counterpart curves from the consideration of the individual variances; this indicates that the variance of \( SM \) is generally greater when the variance of more than one of the individual design factor parameters is considered. For this particular pavement system, the reliability of a 50-in.-thick flexible pavement ranges from about 85 to 97 percent as the variances of the parameters range from the high to the low values. Interpreted in a different way, the analysis shows that, to obtain a reliability of 90 percent, the required thickness ranges from about 46 to about 52 in. as the variances of the parameters range from the low to the high values.

**Comparison of Monte Carlo, PEM, and FOSM Results**

The Monte Carlo simulation technique and the PEM technique were used in a flexible pavement design example to compare the results of the variability analysis of these techniques to the FOSM method. The design example considered 100,000 passes of a C-130 with a gross weight of 175,000 pounds, and the subgrade CBR was 5. The coefficient of variation assumed for the example were: \( CV_{P} \), 20 percent; \( CV_{DP} \), 20 percent; \( CV_{t} \), 15 percent; \( CV_{CBR} \), 25 percent; and \( CV_{p_{s}} \), 15 percent. The Monte Carlo simulation was conducted for 100 trials, with the design parameters distributions obtained by a spreadsheet function designed to use the input variable as the mean and the standard deviations calculated from the CV's. The PEM was conducted for five dependent variables using the same mean and variances of the design parameters as were assumed for the Monte Carlo and FOSM methods. The results of the analysis are shown in Figure 39. It is apparent from the graph that the Monte Carlo, PEM, and FOSM methods yielded virtually the same distributions of thickness, with only a slight horizontal shift in the curves representing slight differences in the estimates of the mean thickness values.
Figure 39. Comparison of Monte Carlo, PEM, and FOSM results
Part IV: Determination of Appropriate Levels of Reliability for Airfields

Assessment of Risk of Failure

The capability of designing a pavement for various degrees of reliability is useless to the designer unless he understands what a given degree of reliability really means, and what degree of reliability is then necessary for a particular application. The following discussion addresses the first issue, and suggests methods of obtaining appropriate values of reliability for the second issue.

The reliability of a design thickness as specified in this report simply means that the pavement has about X chance (for a reliability of X) of achieving the design level of traffic before the pavement reaches the “failure” condition. The failure condition as defined for the rigid pavement design procedure simply means that half of the concrete slabs have sustained at least one structural crack. For the CBR design procedure, the failure condition is defined as that point during the life of the pavement at which the pavement has experienced a certain degree of surface deformation (rutting) due to shear failure and/or densification of the pavement layers or subgrade. These are considered structural failures of the pavement, and should be distinguished from a functional failure. A functional failure occurs when the pavement can no longer provide safe, comfortable, and adequate service to those using the pavement. It is typically governed primarily by the smoothness (or roughness) of the pavement as felt by the user of the pavement while riding along the pavement. The two methods of defining failure do not necessarily coincide at a common point during the life of the pavement (in terms of passes or coverages of traffic). A concrete pavement may be structurally failed according to the Corps rigid pavement criteria, i.e. one half of the slabs may have one or more structural cracks, but still provide good service to the users of the pavement. Conversely, the pavement may be functionally classified as failed due to excessive roughness of the surface or faulted joints, but may not have reached the point of structural failure. Therefore, the reliability derived from the procedure presented in this report does not necessarily translate to similar degree of functionality or serviceability of the pavement for a given level of traffic, and should not be construed to be a reflection of the probability of sustained serviceability over the life of the pavement.

The selection of an appropriate degree of a reliability for a pavement system should be based upon the assessment of risk associated with failure of the pavement. A pavement design with a fifty-percent reliability implies that the designer is comfortable
with the knowledge that there is a 50 percent chance that the pavement will last for the number of load applications it has been designed for before failing structurally, and a 50 percent chance that it will not. This degree of reliability has been used for many years by engineers using the Corps CBR design procedure, and according to the analysis in this report, a somewhat higher degree of reliability has inadvertently been enjoyed by those designing rigid pavements using the Corps procedure. The example in this report indicates that a reliability of about 70 percent might actually be achieved with the deterministic rigid pavement thickness design. However, the risk of failure in these cases was more that likely perceived to be rebuilding or rehabilitating the pavement sooner than expected, and did not mean that an important military mission would be curtailed due to an early failure. Therefore, risk of failure (the complement of reliability) for a military pavement should not be determined solely on the basis of the economics of increased maintenance, rehabilitation, or reconstruction of the pavement, but also on the probability that these events might occur during a critical mission or national security event. This problem was realized during the deployment of heavy cargo aircraft from Campbell Army Airfield to Saudi Arabia in support of Operation Desert Storm in the summer and fall of 1990 and early 1991 (Dyer 1993). The heavy aircraft began failing the 1940’s vintage airfield pavement at such an alarming rate in February 1991 that complete reconstruction of critical sections of the airfield had to be completed in emergency situations to sustain the viability of the critical mission from Ft. Campbell.

Knowledge of Alternative Strategies

Another factor that should be considered in the selection of a reliability level for a military airfield is the accessibility and viability of using alternate airfield sites or even alternate pavement areas on the same airfield. Multiple runways at a single facility that are capable of supporting the aircraft necessary for a critical mission may not need to be designed at the same degree of reliability as a single runway in a remote airfield. Likewise, a location with several airfields in reasonably close proximity, such as San Antonio, may be able to divert critical mission aircraft to a nearby airfield in the event of a pavement failure, and the pavements therefore may be designed at a lesser degree of reliability than a remote airfield.

Existing Recommendations for Levels of Reliability

The Texas flexible pavement design program FPS has recommended various degrees of reliability of a pavement depending upon three criteria (Hudson 1975); the volume of traffic expected over the lifetime of the pavement, the functional classification
of the pavement (arterial or collector), and the availability of alternative routes. These recommendations were developed from the results of a survey of pavement designers who indicated their degree of confidence in several pavement designs at various levels of reliability. The recommendations are shown in Table 10. Of note is the relatively high degree of reliability assigned to even the lowest risk pavement: 95 percent for several combinations of relatively low traffic levels and a reasonable ability to divert traffic. It should also be noted that the failure mode for the FPS design system is based upon functional failure, and not simply structural failure.

<table>
<thead>
<tr>
<th>Traffic Handling Situation</th>
<th>Functional Classification</th>
<th>Traffic Level</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>&lt; 500,000 passes</td>
</tr>
<tr>
<td>Satisfactory</td>
<td>Collector</td>
<td>95</td>
</tr>
<tr>
<td></td>
<td>Arterial</td>
<td>95</td>
</tr>
<tr>
<td>Some</td>
<td>Collector</td>
<td>95</td>
</tr>
<tr>
<td>Problem</td>
<td>Arterial</td>
<td>95 or 99</td>
</tr>
<tr>
<td>Considerable</td>
<td>Collector</td>
<td>95 or 99</td>
</tr>
<tr>
<td>Problems</td>
<td>Arterial</td>
<td>99</td>
</tr>
</tbody>
</table>

**Proposed Recommendations for Level of Reliability for Military Airfields**

From observation of this table developed for highway pavements, and recognition of the relatively good performance of airfields designed using existing criteria, a framework for assessing the necessary reliability for military airfield pavements was devised. The key components of this simple framework include the criticality of the airfield mission to current or future national security or wartime operations; the type of aircraft assigned to the airfield; the traffic area classification; and the proximity of redundant airfield systems. A brief discussion of each of these classifications follows.

The criticality of the mission of the airfield to national security or wartime operations would be relatively high for a strategic bomber base or an airfield supporting forward operations, such as the Campbell Army Airfield. The criticality of the mission to national security might be lower for an airfield with the primary mission of storing mothballed aircraft. The type of aircraft using the pavement might also influence the reliability level chosen, due to the capability of the aircraft for operating on an airfield that has "failed" structurally. Certain aircraft, such as fighter aircraft or some passenger aircraft,
particularly those with jet engines, require a relatively smooth and defect-free surface to maintain operational effectiveness. However, some cargo aircraft, such as the C-130, can operate on relatively rough airfield surfaces, even an unsurfaced airfield, and therefore the consequences of “failure” of the pavement on the operational effectiveness of the aircraft are less drastic.

The traffic area classification of an airfield pavement is certainly relevant to its relative importance to an airfield mission. The reliability of a runway or taxiway should reflect their importance in getting aircraft on the ground (or back in the air) and out of the way of other incoming aircraft needing to land or take off. An apron for parking and servicing aircraft is also important, but the immediate risk of someone dying from a plane crash does not exist for aprons as it does for runways or even taxiways. An apron is typically large enough such that aircraft can maneuver around defective areas in the pavement, while a defective area in a the middle of a runway is more difficult to avoid.

Finally, the proximity of redundant airfield pavement systems with a similar capability should be considered in selecting the design reliability; whether the redundancy is on the same airfield or at a nearby airfield with a similar mission capability. According to the laws of probability of redundant (or parallel) systems (Harr 1987), an airfield pavement system with two runways having a reliability of 50 percent each has the same probability of failure as an airfield pavement system with one runway having a reliability of 75 percent.

Considering the relatively high levels of suggested reliability for highway pavements along with the performance of airfield pavements designed with existing criteria (i.e. 50 to 70 percent reliability), a table of possible levels of reliability for designing military airfield pavements was developed (Table 11), using the suggested framework for classification of relative needs for reliability. This table was developed purely upon the judgment of the author in light of the previously stated considerations, and is not based upon any analysis of data of quantitative assessment of risk. While the reliability levels suggested in the table should not be taken at face value (nothing replaces the judgment of the design engineer), they may provide a framework for a designer to assess the relative degree of risk (and therefore the degree of reliability) that should be used for a particular design application.
<table>
<thead>
<tr>
<th>Proximity of Redundant Systems</th>
<th>Traffic Area</th>
<th>Classification of Aircraft that is Primary and Critical to Airfield Mission</th>
<th>Assessment of Current or Future Criticality of Airfield Mission for National Security or Wartime Operations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Remote airfield site and/or no redundant systems</td>
<td>Runway or Taxiway</td>
<td>Fighter or Passenger</td>
<td>98 % plus</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cargo</td>
<td>98 % plus</td>
</tr>
<tr>
<td></td>
<td>Apron</td>
<td>Fighter or Passenger</td>
<td>95 % plus</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cargo</td>
<td>95 % plus</td>
</tr>
<tr>
<td>Airfield in close proximity or existing availability of redundant systems</td>
<td>Runway or Taxiway</td>
<td>Fighter or Passenger</td>
<td>90 to 95 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cargo</td>
<td>90 to 95 %</td>
</tr>
<tr>
<td></td>
<td>Apron</td>
<td>Fighter or Passenger</td>
<td>85 to 90 %</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Cargo</td>
<td>85 to 90 %</td>
</tr>
</tbody>
</table>
Part V: Conclusions and Recommendations

Conclusions

This study accomplished the stated objectives by providing the means of assessing the reliability of an airfield pavement design using the Corps of Engineers rigid (Westergaard) or flexible (CBR) design procedures. The procedures consider the variability of the design parameters, and the variability in the performance models, in assessing the reliability of a design. The reliability of the deterministic thickness design derived from the CBR design procedure was found to be around 50 percent, while the deterministic thickness design value obtained from the rigid pavement design procedure (Westergaard) was found to be in excess of 50 percent, and may actually be closer to 70 percent. These procedures were used in developing simple design spreadsheet programs for rigid (RRAD) and flexible (RFAD) pavements. The RRAD and RFAD models indicated a relative sensitivity of the reliability of the pavement to the variance of the design parameters as follows (in order of greatest to least influence): $R$, $P$, $LT$, $h$, $k$, $V[Reg]$, and $DP$ for rigid pavements, and CBR, $t$, $V[Reg]$, $p_v$, $DP$, and $P$ for flexible pavements. It was noted that these relative effects might change for different assumed values of CV for the parameters. A framework of assessing the reliability level needed for a particular military pavement application was also presented, along with possible ranges for those reliability levels.

Strengths and Limitations of the Reliability Models

The strengths and limitations of the reliability procedures developed in this report should be recognized. The advantages of a reliability-based design procedure over a deterministic design system are numerous. First and foremost is the capability to design at other than relatively fixed levels of reliability such as the 50 to 60 percent typically achieved with the deterministic approach. The ability to characterize the effect of variability in the design inputs parameters on the reliability of a pavement system has important ramifications in the area of quality control criteria development or the assessment of penalties or rewards based upon various degrees of variability in the design parameters as measured in quality assurance operations. The reliability-based approach also allows the assessment of the relative sensitivity of the expected pavement performance due to realistic variations in the design parameters.
However, the limitations of the reliability-based design procedure are also numerous. The accuracy of a predicted reliability relative to the actual performance of the pavement is no better than the ability of the design model to predict the performance of the pavement given the design parameters. While some of this difference in actual and predicted performance was quantified in the analysis of the variance of the rigid pavement and CBR performance equations, the performance equations themselves are limited by the range and type of data used to model the performance. For instance, the largest number of coverages until failure used in the development of the rigid pavement performance model was about 10,000, but the same model is routinely used to predict the performance of pavements with design coverages several orders of magnitude larger than this. Another potential problem with the performance models is that the data used to obtain them came from test sections that were constructed under controlled situations and experienced a limited degree of variation in climatic and environmental conditions during the period of accelerated trafficking. While the variations in the climate during the testing most certainly contributed to the scatter in the performance data, and therefore are indirectly accounted for in the development of the performance equations, extreme climatic conditions or variations in climate at the actual construction site might cause the performance of the pavement to be different than expected. The performance models are also limited in that the parameters and analytical models used in characterizing the behavior of the pavement materials are limited in their ability to predict the actual states of stress, strain, deflection, or fatigue in the pavement.

A significant problem in the accuracy of the reliability procedure, however, may be realized in the ability (or inability) of the designer to know the actual mean and variance values of the in-place materials during the design process. Differences in the assumed and actual in-place means and variances in the design parameters can have a significant effect on the ability of the reliability-based design procedure to accurately assess the reliability of the pavement system. One example of this effect was illustrated by using the estimated in-place flexural strength of the concrete pavement instead of assuming that the design value was the mean value. As was seen in Table 4, the mean values of the load transfer, which is always assumed to be 25 percent in rigid pavement design for airfields, can be as low as 14 percent in certain conditions of climate and joint type. The mean thickness of concrete and asphalt concrete pavements as constructed is often slightly larger than the design thickness (the assumed mean thickness) due to severe penalties for inadequate thickness. This increased thickness would result in a somewhat greater reliability of the pavement system than expected. The relative effects of probable differences in the assumed and actual means of the pavement design parameters on the reliability of the
pavement system is shown in Table 12. As can be seen from the table, the effects on the reliability are mixed, and therefore their combined effects on the reliability may tend to cancel each other out, resulting in a more accurate assessment of reliability than may be assumed if only one parameter is investigated.

<table>
<thead>
<tr>
<th>Design Parameter</th>
<th>Probable Relative Position of Actual Mean Value to Assumed Mean Value</th>
<th>Effect of Relative Difference on the Reliability of the Pavement System</th>
</tr>
</thead>
<tbody>
<tr>
<td>Load Magnitude</td>
<td>Less</td>
<td>Higher</td>
</tr>
<tr>
<td>Number of Design Passes</td>
<td>More</td>
<td>Lower</td>
</tr>
<tr>
<td><strong>Rigid Pavements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of PCC Slab</td>
<td>More</td>
<td>Higher</td>
</tr>
<tr>
<td>Flexural Strength of Concrete</td>
<td></td>
<td>Higher</td>
</tr>
<tr>
<td>Modulus of Subgrade Reaction</td>
<td></td>
<td>Higher</td>
</tr>
<tr>
<td>Load Transfer</td>
<td>Less</td>
<td>Lower</td>
</tr>
<tr>
<td><strong>Flexible Pavements</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thickness of Asphalt Concrete (AC)</td>
<td>Less if the AC is paid for by the square yard</td>
<td>Lower</td>
</tr>
<tr>
<td>CBR of Subgrade</td>
<td>More</td>
<td>Higher</td>
</tr>
<tr>
<td>Tire Pressure</td>
<td>Less</td>
<td>Higher</td>
</tr>
</tbody>
</table>
Another aspect of the variability associated with the design model is the accuracy and precision of some of the assumed relationships built into the model, such as the function for estimating the $k$ on top of the base ($k_{TB}$ in Eqn. 12), the ESWL curves, the $p/c$ ratios, and the regression curves developed specifically for RRAD and RFAD (such as Eqn. 14 or 15). While the regression equations developed for this report had very high correlation coefficients ($r^2$ typically above 99.9 percent), there is some error associated with the predictions, and these errors are not considered in this analysis. However, these errors are probably small compared to the accuracy and precision of the $k_{TB}$ approximation. These errors should be investigated further to determine their effect on the reliability analysis.

One additional limitation of the design models is the presumption that the materials used for the construction of the pavement are sufficiently durable to last the full design period, whether the pavement is loaded or not. This may not always be the case. If a concrete pavement is not properly air-entrained, for instance, the pavement may deteriorate prematurely in areas where freezing and thawing and saturation of the pavement is frequent and common. Asphalt concrete pavements that have not been compacted properly, contain too much asphalt cement, or contain relatively weak aggregate are more likely to fail sooner than expected if these properties were assumed to be satisfactory in the design process.

**Recommendations**

**Applications for the Reliability Models**

The proposed reliability-based design procedure should be used as a tool by the airfield pavement designer who is interested in characterizing the effect of pavement parameter variability on the reliability of the pavement design. The user of the method should be aware of the limitations of the model, however, and should use the output of the proposed design procedure as input to a judgment-based decision on the necessary design thickness.

The FOSM procedure was used in the development of this model, because of the relative simplicity of applying the method to the closed-form performance equation. However, the advantages of the PEM are also outstanding, and this method along with the Monte Carlo simulation technique are the only practical methods for assessing the variability of a complex computational analysis procedure such as the elastic layer or finite element methods. Of these two procedures, the PEM is the most computationally efficient
in terms of assessing the variance of the design system with a reasonable degree of accuracy. It is therefore recommended that the PEM be used for further development of reliability-based design procedures that use these computational methods in estimating the pavement behavior and performance.

**Future research needs**

It is also recommended that further research be conducted in the following areas to improve the accuracy and usefulness of the reliability-based design procedure:

a) The relationship between the actual and assumed mean values of the design parameters should be investigated in detail so that these effects can be compensated for in the reliability analysis.

b) A survey of airfield pavement designers along with an assessment of the requirements of various military airfield operational situations should be undertaken to develop realistic recommendations or criteria for levels of reliability needed for military airfield pavement design. A framework for such an investigation has been proposed.

c) A more accurate reliability-based design procedure would include more directly the effects of spatial variability in the pavement system, such as variability of the concrete strength with depth in the slab, or variability in the lateral wander of various aircraft in different situations. This would require the use of more sophisticated analytical techniques than the Westergaard equation, for example, which does not consider the effects of friction at the bottom of the pavement slab, curling or warping of the slabs due to temperature or moisture gradients in the slab, or non-linear behavior of the support of the foundation material under the load. The adoption of a functionally-based failure criteria that would consider various modes of failure would also lead to more realistic assessments of reliability. A more sophisticated analytical model that would consider directly such effects, such as a finite-element or particulate mechanics approach, and would also allow various distributions of pavement material and geometric properties to be reflected directly in the pavement system in all directions, would contribute greatly to a more accurate reliability-based approach. Such a model should also consider the effects of dynamic loading of aircraft on the pavement, as well as any lateral distribution of traffic that may be encountered. The performance of the materials in terms of durability in different environmental conditions should also be considered in the model, since failures can result from a lack of durability of the pavement material in particular environments. The rapid progress of computational capabilities in both the research and pavement design arena are making the realization and use of such an approach more likely than ever before.
References


Rollings, Raymond S. "Developments in the Corps of Engineers Rigid Airfield Design Procedures," Fourth International Conference on Concrete Pavement Design and Rehabilitation," Purdue University, West Lafayette, Indiana, 405-418, April 1989.


### REPORT DOCUMENTATION PAGE

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13. **ABSTRACT (Maximum 200 words)**

   This report documents the application of reliability-based techniques to the U.S. Army Corps of Engineers rigid (Westergaard) and flexible (CBR) airfield pavement design procedures. Pavement reliability is defined as the probability that the expected capacity of the pavement system exceeds the expected demand of traffic to be applied to the pavement, in terms of coverages until failure. A literature review briefly outlines existing procedures and previous studies related to reliability-based pavement design procedures. The variability of pavement materials and design inputs for the Corps rigid and flexible pavement design procedures is presented. The first-order second moment (FOSM), Monte-Carlo simulation, and point estimation method (PEM) for characterizing the variability of the pavement capacity are reviewed. The FOSM and Monte-Carlo techniques are applied to the rigid and flexible design procedures, and the results are compared for a particular design example. A spreadsheet program is presented which uses the FOSM technique to estimate the pavement capacity variability, and a sensitivity analysis of the variability of the rigid and flexible design inputs on the reliability of the pavement is presented. A suggested framework for determining levels of design reliability for airfield pavements is presented, and advantages and limitations of the reliability method are discussed.

14. **SUBJECT TERMS**

   - CBR
   - Flexible pavements
   - Pavement design
   - Reliability
   - Rigid pavements

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