Heat Transfer and Frost-Thaw Penetration in Soil Surrounding an Inclusion of Sand
Numerical Model Results Relevant to Electromagnetic Sensor System Performance

Lindamae Peck and Kevin O'Neill

July 1995
Abstract
One- and two-dimensional numerical simulations of heat flow in silty soil with and without a sand inclusion (15 cm thick, variable width) have determined the magnitude and the lateral extent of the disruption in frost and thaw penetration attributable to the presence of the inclusion. Four different soil temperature histories, derived from field data at a Vermont site, were used as the surface boundary condition for the winter-long simulations. This identified differences in frost depth and soil temperature resulting solely from an overall colder or warmer soil surface condition. For a given surface boundary condition, the moisture content of the soil was varied (10, 17 or 25%, by weight) to contrast the changes in frost penetration caused by the moisture-dependent differences in soil thermal conductivity and latent heat. The drier sand (3% moisture content by weight) with its smaller latent heat freezes more rapidly than does the soil under identical conditions; so initially (early winter) frost penetration is greater (by 5–6 cm) when the sand inclusion is present because the freezing front proceeds rapidly through the sand. Subsequently, the freezing front is deeper (by a maximum of 11 cm) in soil without a sand inclusion. The less conductive sand impedes heat flow toward the soil surface; resulting in higher soil temperatures beneath the inclusion, which in turn retards freezing of the soil. Frost penetration beneath a sand inclusion is deeper the drier the soil is; with no sand inclusion present, frost depth is greater the more moist the silty soil is. Under the conditions of this study, maximum frost penetration is 61 cm (*coldest* surface boundary condition, 25% moisture content soil, no sand inclusion). The change in maximum frost depth because of the presence of a wide sand inclusion is large relative to overall frost penetration. Similarly, the difference in soil temperature at a given depth, although small, can correspond to a large difference in unfrozen moisture content of the silty soil. Under either relatively mild or "normal New England" winter conditions, the presence of a sand inclusion is probably beneficial to the performance of a buried electromagnetic sensor system, which is more reliable in dry or frozen soil because of the soil's lower electrical conductivity. A sand inclusion may not lead to improved sensor performance under more severe winter conditions, which cause much lower soil temperatures and much deeper frost penetration.

Cover: Depth of the freezing front in silty soil of 10, 17 or 25% moisture content, by weight. The *warm* boundary condition (BC-Warm) was used as the soil surface temperature history for the winter-long simulation. The sand inclusion is shown stippled.

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PREFACE

This report was prepared by Dr. Lindamae Peck, Geophysicist, Geophysical Sciences Division, and Dr. Kevin O'Neill, Research Civil Engineer, Civil and Geotechnical Engineering Research Division, Research and Engineering Directorate, U.S. Army Cold Regions Research and Engineering Laboratory, Hanover, New Hampshire.

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INTRODUCTION

This study was motivated by field data taken at a Vermont site over three winters that indicated a slight temperature difference throughout each winter in the sandy loam soil surrounding an inclusion of sand (16.5 cm wide, 15 cm deep, and 15 cm thick), compared to soil at the same depth in a location where there was no sand inclusion. The difference in soil temperature at the two nearby locations (~24-m separation) was not explainable by either a different near-surface soil condition or a different soil condition at depth (52.5 cm), since soil temperatures at these depths at both locations were similar during the winters.

To investigate the differences in wintertime soil temperature histories that are attributable to the presence of a body of sand within the soil, numerical simulations of heat flow in silty soil with and without a sand inclusion have been done. The sand inclusion is 15 cm thick, its top is 15 cm deep, and its width is variable. The moisture content of the soil and the temperature history of the soil surface are varied to explicitly examine their influences on soil temperature profiles. The surface boundary conditions are a sequence of freeze and thaw episodes from October to April (BC-1); a freezing episode lasting from December into April, with a minimum soil surface temperature of −4°C (BC-Warm); a freezing episode from December to April, with a minimum soil surface temperature of −9°C, that is interrupted by a January thaw during which the soil surface warms to 0°C (BC-Cold); and a freezing episode from January into March, with minimum surface temperature of −1°C (BC-Warmest).

The silty soil is assigned a moisture content of 10, 17 or 25% (by weight); the sand, 3%. Because variation in the unfrozen moisture content of the soil affects buried electromagnetic sensor systems, the depth and the timing of freezing and thawing of the silty soil, with and without a sand inclusion, are emphasized in the presentation of results.

BACKGROUND

When a sensor cable is placed at shallow depth in soil subjected to freezing, a precaution often taken is to surround the cable with sand. The low moisture content of the sand reduces the possibility of ice lens formation and associated frost heave adjacent to the cable. This is important when it is necessary that the depth of the cable remain constant along its length. A related consideration is that, should there be any differential motion of the cable, the potential for abrasive or puncture damage to the cable is less when a rock-free material such as sand surrounds the cable.

If the cable is intentionally constructed to leak radio frequency energy as part of an electromagnetic sensor system, then sand is an advantageous burial medium because of the low electrical conductivity associated with its low moisture content. The higher the electrical conductivity of the medium, the greater the loss of the electromagnetic signal with distance from the cable, and the less effective the sensor system is.

Because the unfrozen moisture content of soil
decreases as the soil temperature drops below 0°C, the performance of the electromagnetic sensor system improves when the soil is frozen. An example of seasonal variation in the response of such a sensor system as the frozen–unfrozen status of the surrounding soil changes is given in Appendix A.

Any situation that delays the initial freezeup of the soil above the sensor cables has a negative impact on the reliability of the sensor system. Similarly, any situation that causes the soil to transition through numerous freeze–thaw–freeze episodes during the winter results in inconsistent performance by the sensor system. Therefore, when field measurements at a Vermont site indicated that, under apparently identical physical conditions, both temperature profiles and the number of freeze–thaw transitions were different for a sandy loam soil with a sand inclusion compared with nearby soil without a sand inclusion, a series of numerical modeling experiments of heat flow in soil with and without a sand inclusion was begun. Because sand must always be used to surround sensor cables when the local soil is clay or any other high electrical conductivity soil, the modeling was restricted to the case of a silty soil, a situation for which security designers have a choice of whether to place the sensor cables in sand.

NUMERICAL MODELING OF HEAT FLOW

Computer program

A version of XYFREZ (O’Neill 1987) is used to simulate conduction of heat in idealized situations of soil with and without a sand inclusion. The program XYFREZ uses finite elements in space and finite differences in time to model heat conduction. Latent heat effects due to a phase change in water within the soil or sand are included through a singularity in the heat capacity of each material. Moisture diffusion due to a temperature gradient in the materials is not considered. The numerical formulation has been tested with good agreement against some exact solutions (O’Neill 1983, 1991) and some laboratory measurements of soil freezing.

Both one- and two-dimensional simulations are conducted. For the two-dimensional studies (Fig. 1a, b, c), the left-hand side of the mesh corresponds to a vertical plane through the center of the sand inclusion. The half-width of the sand inclusion is initially set at 8.25 cm, a common configuration with sensor systems, and subsequently increased to 45.5 cm to examine the lateral extent of the disruption of frost and thaw penetration in the soil adjacent to a wider sand inclusion. For the one-dimensional studies, the sand inclusion is treated as a 15-cm-thick layer that is

Figure 1. Cross-section (a) and mesh (b, c) of the soil with sand inclusion model for two-dimensional simulations. Only the near-surface portion of the mesh is shown. The half-width of the sand inclusion, shown stippled, was 8.25 cm (b) or 45.5 cm (c). x is distance from the centerline of the inclusion, and y corresponds to depth. a–Cross-section of the soil/sand/soil layered section representing the physical situation of the one-dimensional simulations.
infinite in lateral extent (Fig. 1d). It is covered by a horizontally infinite soil layer of the same thickness and underlain by another soil layer that is infinite in depth and in lateral extent. A piecewise linear finite element mesh of 75 unequally spaced nodes is used for the one-dimensional simulations, with node 1 at the surface (0 cm) and node 75 at a depth of 20.52 m. Node spacings (and element sizes) range from 2.5 cm (one-dimensional simulations) or 5 cm (two-dimensional simulations) within and above the sand inclusion to some meters in size in distant regions.

A simulation proceeds in hourly time steps. The output is soil or sand temperature at selected nodes. Locations of freezing and thaw fronts are linearly interpolated from the temperature data.

**Properties of the soil and inclusion**

For simulations the silty soil and inclusion are assigned the properties listed in Table 1. The physical and thermal conditions of the inclusion (sand or sandy soil) and surrounding silty soil were selected as being representative of situations relevant to installations of electromagnetic sensor systems, and intentionally did not duplicate conditions at the field site in Vermont. The moisture content of the silty soil was varied, as was the temperature at the soil surface, to investigate the sensitivity of the results to these two factors, other influences on heat flow being unchanged.

Heat capacity and thermal conductivity, both for the frozen and unfrozen state, and latent heat are specific input parameters of the program. The dry densities of the sand and soil are calculated from their mineral content per volume by assuming a mineral density of 2.65 g/cm³. The mineral contents used are 0.6 (sand) and 0.55 (soil) based on assumed porosities of 0.4 and 0.45, respectively. The resultant dry density of the sand, 1.59, is within the range reported by Miller et al. (1992) for uniform sand that is loose, 1.45, to dense, 1.75.

The moisture content (weight percent) of the sand, 3%, is that of a concrete sand buried in sandy loam soil at the CRREL field site, known as SOROIDS, in Vermont. The moisture content of the SOROIDS sand was determined from the reduction in sample weight upon drying. The intermediate soil moisture content, 17%, is that of the medium-grained soil at SOROIDS in November/December, prior to freezing; 10% and 25% moisture contents are representative of dryer

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**Figure 1 (cont'd).**
and wetter conditions, respectively, at the onset of winter. These three moisture contents are suggestive of three different climates in terms of relative rainfall per year. The moisture contents are multiplied by the material’s dry density to convert them to moisture contents by volume.

The latent heat released upon freezing a unit volume of soil or sand depends on volumetric moisture content. For pure water, the latent heat released is the latent heat per unit mass of water (80 cal/g) times the density of ice (0.917 g/cm³), or 73.36 cal per cm³ of ice. For a partially saturated soil or sand, the latent heat released is 73.36 cal/cm³ times the volumetric moisture content. This assumes that all the water within the soil or sand freezes at 0°C, and so ignores freezing point depression. The unfrozen moisture content of silt, however, is ≤ 5% by weight at temperatures less than −1°C (Williams 1967, Anderson and Morgenstern 1973); for a coarser, silty soil, the unfrozen moisture content would be less. The simplification of having all soil moisture freeze at 0°C instead of exhibiting a (below-freezing) temperature dependence does not detract from the usefulness of numerically investigating frost penetration in silty soils of distinctly different moisture content (10, 17, or 25% by weight).

Heat capacities of the soil and sand depend on their porosities and volumetric moisture contents (eq 1).

\[
C = 0.54 \phi_s + 1.0 \phi (\text{unfrozen})
\]

\[
C = 0.54 \phi_s + 0.46 \phi (\text{frozen})
\]

where \(C\) = heat capacity
\(\phi_s\) = solid fraction (1 - porosity)
\(\phi\) = volumetric moisture content.

This is based on a heat capacity of the mineral solids equal to 0.54 cal/cm³, of liquid water equal to 1 cal/cm³, and of ice equal to 0.46 cal/cm³.

The thermal conductivities of the soil for each moisture content are taken from plots of the average frozen and unfrozen thermal conductivity of silt and clay soils as a function of water content and dry density in Andersland and Anderson (1978, Figures 3.8 and 3.9, respectively).

Two materials are used for the inclusion. The first is a quartz sand. The thermal conductivity of unfrozen quartz sand is taken from Farouki (1981, Fig. 53) who reports De Vries’s (1974) data on thermal conductivity of quartz sand as a function of the volume fraction of water. The thermal conductivity of the sand when frozen is assumed to be 16% lower; this is based on the ratio of thermal conductivity of Lowell sand below freezing to that above freezing as a function of moisture content, as reported by Farouki (1981, after Kersten 1963). The second inclusion material is a sandy soil; thermal conductivities for this material frozen and unfrozen are taken from Figures 3.6 and 3.7, respectively, of Andersland and Anderson (1978). The thermal conductivity of the soil increases upon the soil freezing, but that of the sand and sandy soil decreases. Farouki notes that saturated soils and soils with a high degree of saturation have a higher thermal conductivity when frozen because the thermal conductivity of ice is approximately four times that of water. At low degrees of saturation, however, heat conduc-
tion at contact points between soil particles may decrease upon freezing of the soil because water at the contact points is removed to form ice in the soil pores (Farouki 1981, and others). Since the thermal conductivity of air is approximately one-twentieth that of unfrozen water, the loss of water from contact points causes the thermal conductivity of the frozen sand or sandy soil to be less.

Initial temperatures

The one- and two-dimensional meshes were initialized with the temperatures plotted in Figure 2 by depth. As no measured soil temperatures were available for depths greater than 60 cm, a temperature profile for depths of 0.6 to 20 m was calculated as follows. Thermal properties consistent with a moisture content of 17% were assigned to the soil in that region. Daily averages of SOROIDS soil temperatures at 60-cm depth, recorded half-hourly throughout 1990, were used as an upper boundary condition to drive the calculations over a one-year time span. This was repeated for a 20-year time period, assuming a uniform initial temperature approximately equal to the yearly average at 60 cm and assuming that the temperature profile at a depth of ~20 m remains flat. By the end of the simulation a stable yearly pattern of temperature profiles for soil depths of 0.6 to 20 m had become established (Fig. 3), as well as a stable temperature value at depth; any transient startup effects and influences of initial conditions had been filtered out.

The final temperature profile used to initialize the meshes consists of the simulated profile for day 300 (28 October) from Figure 3 for the depth range 0.6 to 20 m spiked to a constant temperature profile from 0.6 m to the surface. The flat, near-surface profile is a computational convenience that disappeared quickly and did not significantly affect the results.

Boundary conditions

Four boundary conditions for the temperature of the soil surface (0 cm) have been derived from SOROIDS soil temperature data. The different surface boundary conditions were created to investigate the dependence of the "sand inclusion

![Figure 2. Initial temperature condition for the numerical simulations, by depth.](image)

![Figure 3. Calculated soil temperature profiles for depths of 0.6 to 20 m on selected days (day 1 is 1 January). The temperature profiles were obtained from numerical simulations of heat flow at depth during a 20-year period based on measured soil temperatures at a depth of 0.6 cm.](image)
effect" (the difference in frost penetration with and without a sand inclusion present) on the relative severity of the winter, as expressed through the temperature history at the soil surface. The first boundary condition (BC–1) is a simplification of the time-series record of near-surface soil temperatures for July 1990 through June 1991. The soil temperature was determined with a copper-constantan thermocouple located within the root zone of the grass cover, a few mm below the soil surface. The temperature was recorded automatically every half hour. These data have been averaged to create a record of daily average temperatures of the near-surface soil, from which the 28 October 1990 through 30 June 1991 portion was extracted. The resultant boundary condition applied to the soil surface is shown in Figure 4. The averaging and the hourly time steps used in establishing BC–1 eliminated some of the occasions when the actual temperature of the soil passed through 0°C. Whereas the soil surface temperature crosses 0°C ~120 times during this period, BC–1 does so only 34 times, which simplified the computations while retaining the overall character of the data. More frequent and more rapid freezing and thawing would have required greater attention to control of numerical resolution in both time and space.

The second boundary condition, BC–Warm, is derived mainly from the near-surface soil temperature data for 22 October 1991 to 6 May 1992. The half-hourly data were averaged for a daily value, and then the daily (averaged) temperature record was sampled every eight days. Temperature differences between samples were distributed evenly over the eight-day period. This produced a soil surface temperature record that had no transitions through 0°C other than the initial freezeup in December and the thaw in April.

Figure 5 compares BC–Warm with near-surface soil temperature data from field measurements during three winters. BC–Warm is a good representation of the variation in temperature through the course of a northern New England winter when frost penetration is moderated by the existence of a snow cover for extended periods.

The third boundary condition, BC–Cold, is derived from soil temperature data acquired near the SOROIDS driveway. Snow is removed from this area throughout the winter when the driveway is plowed. The lack of a persistent snow cover results in deeper frost penetration and lower temperatures at depth than at the snow-covered location where the soil temperature data used to create BC–1 and BC–Warm were obtained. It also results in more erratic changes in the temperature of the exposed soil, and in higher near-surface temperatures on winter days of high incident solar radiation. Similarly, under autumn and spring weather conditions, the lack of a vegetative cover to shelter the soil results in higher soil temperatures at this site.

BC–Cold is based on soil temperatures at 7.5-cm depth recorded approximately every eight days. (No thermocouple was placed at a shallower depth because of the risk of damage when the driveway was plowed.) This BC, applied to the soil surface during simulations, is intended to represent a more severe winter condition of generally lower surface temperatures. Alternatively, a comparison of soil temperatures from simulations with BC–Warm and BC–Cold surface condi-

![Figure 4. Soil surface temperatures corresponding to BC–1. This is the 28 October through 30 June portion of a simplified time series record of measured near-surface soil temperatures at the SOROIDS field site for July 1990 through June 1991.](image-url)
Figure 5. Comparison of BC-Warm and time series records of measured near-surface soil temperatures for 1990–91, 1991–92 and 1992–93 winters. From late March into May, BC-Warm is cooler than the measured soil temperatures shown in the figure because BC-Warm is derived from a daily average of soil temperatures measured every half hour; the nighttime low temperatures “drag down” the daytime high temperatures. The soil temperatures shown for the three winters were single measurements taken during daytime site visits when the near-surface soil was warm because of solar heating.

Figure 6. Soil surface temperatures corresponding to BC-Cold, BC-Warm and BC-Warmest.

Simulations highlights the influence of a snow cover on frost/thaw penetration in the underlying soil. Figure 6 compares BC-Cold and BC-Warm.

The fourth boundary condition, BC-Warmest, represents a shortened winter season during which the surface soil is colder than 0°C for ~50 days. It is created from BC-Warm by eliminating the period corresponding to 1 January through 17 March. This is based on a comparison of air temperature extremes during the winter at SORD-IS, at a coastal site in Washington state, and at a site in southern England. BC-Warmest is the mildest winter condition used for the simulations, in terms of both the duration and the magnitude of the cold weather (Fig. 6).

RESULTS OF NUMERICAL SIMULATIONS

Two-dimensional simulations

A limited number of two-dimensional simulations have been done under the BC-Warm and BC-Cold conditions for a sand inclusion of 16.5- or 91-cm width, and soil of 25% moisture content. The results show that the disruption to frost penetration due to the presence of a sand inclusion varies with the width of the inclusion. This is intuitively reasonable since a two-dimensional simulation accommodates the flow of heat around the sides of the inclusion, which is not realizable with the infinitely wide layers of one-dimensional simulations. The wider the inclusion is, the more this “by-passing” flow of heat will be possible only for soil locations close to the sides of the inclusion until, eventually, heat flowing along a vertical profile that bisects the inclusion effectively encounters an infinitely wide inclusion.

A comparison of frost penetration along a vertical profile centered on the inclusion for two cases, two-dimensional simulations with the 16.5-cm-wide inclusion and one-dimensional simulations with no inclusion present, shows
that the difference in frost depth at any time is zero within the computational accuracy of the one- and two-dimensional simulations. The 16.5-cm-wide intrusion is narrow enough that it causes minimal disruption of the upward flow of heat.

When the sand inclusion is 5.5 times wider, 91 cm, the history of frost penetration under BC-Warm and BC-Cold conditions is quite similar to that when an infinitely wide inclusion is present. Table 2 compares frost depth under the center of the inclusion on selected days. Heat flow in a vertical profile through the center of this wider inclusion is significantly disrupted because the inclusion is too wide for the heat to effectively escape the influence of the dry, low thermal conductivity sand by flowing around the sides of the inclusion.

Temperature profiles on days 120 and 140 for BC-Cold and BC-Warm, respectively, are shown in Figure 7. On each plot, one profile extends vertically through the center of the inclusion, one is through the edge of the inclusion, and one is effectively in undisturbed soil (4.4 m from the center of the inclusion). Temperature profiles through a portion of the sand inclusion show a break in slope at depths corresponding to the upper and lower boundaries of the inclusion, and at the frozen/unfrozen condition. These kinks are realistic, and are due to the discontinuities in thermal properties. In all cases the soil above (below) the inclusion is colder (warmer) than it would be were no inclusion present (see discussion of one-dimensional results, below). On day 120, under BC-Cold conditions, the soil is -0.35°C warmer immediately beneath the center of the narrow inclusion but -0.9°C warmer immediately beneath the center of the wide inclusion. On day 140, under BC-Warm conditions, the temperature differences are -0.15°C and 0.3°C, respectively.

At a depth of 30 cm, the soil at the edge of the wide sand inclusion is 0.11°C warmer under BC-Warm conditions and 0.35°C warmer under BC-Cold conditions, compared to 30-cm-deep soil remote from the inclusion. The temperature difference caused by the disruption of heat flow when the inclusion is present diminishes rapidly with lateral distance from the inclusion. It is negligible at a distance of 45 or 55 cm under BC-Warm or BC-Cold conditions, respectively.

The two-dimensional simulations show that a sand inclusion of the size (~16.5 cm wide) associated with trenching machines has little effect on heat flow and frost penetration due to its small width. As the width of the inclusion increases, the frost depth decreases.

### Table 2. Frost depths in silty soil of 25% moisture content on selected days beneath inclusions of various widths.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>Simulation type</th>
<th>Inclusion width</th>
<th>Frost depth (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC-Warm</td>
<td>1-D</td>
<td>0 cm</td>
<td>40.0</td>
</tr>
<tr>
<td></td>
<td>2-D</td>
<td>91 cm</td>
<td>32.2</td>
</tr>
<tr>
<td></td>
<td>1-D</td>
<td>infinite</td>
<td>31.9</td>
</tr>
<tr>
<td>BC-Cold</td>
<td>1-D</td>
<td>0 cm</td>
<td>53.0</td>
</tr>
<tr>
<td></td>
<td>2-D</td>
<td>91 cm</td>
<td>46.3</td>
</tr>
<tr>
<td></td>
<td>1-D</td>
<td>infinite</td>
<td>44.8</td>
</tr>
</tbody>
</table>

* Along the vertical centerline of the inclusion.

![Figure 7](image-url)  
**Figure 7. Temperature profiles on selected days at various distances from the vertical midplane of a sand inclusion in silty soil (25% moisture content), based on two-dimensional numerical simulations under BC-Cold (a,b) or BC-Warm (c,d) conditions. The sand inclusion is 16.5 (a,c) or 91 (b,d) cm wide.**
the disruption in heat flow becomes more significant until eventually, at the vertical center plane of the inclusion, frost penetration is indistinguishable from that of an infinitely wide inclusion even though the inclusion itself is of finite width. Accordingly, the computationally simpler one-dimensional simulations have been done to investigate the effects of soil moisture content and soil surface temperature history on frost penetration for a wide inclusion.

**One-dimensional simulations**

The first simulation is that of the freezing and thawing of a 17% moisture content silty soil (no sand inclusion present) under BC–1 conditions.

Figure 7 (cont’d).

Figure 8. Temperature profiles for soil depths of 7.5, 22.5 and 37.5 cm, obtained from one-dimensional numerical simulations of heat flow in a silty soil (17% moisture content) with no inclusion, under BC–1 conditions. The imposed temperature history of the soil surface is also shown.
frozen—thawed status of deeper soil. Surficial freezing or thawing will not have the significant impact on electromagnetic sensor system performance that deeper penetration of the freezing or thawing front will.

The effect of a sand inclusion on soil temperature is easily seen in Figure 9. The difference in soil temperature, with and without an inclusion, is plotted for both types of inclusions (sand and sandy soil) and three depths. Above the inclusion (7.5 cm, Fig. 9a) the soil is generally cooler when a sand inclusion is present, as indicated by a negative temperature difference. The material of the inclusion makes a difference only in early winter (days 53 to 60), when following a period of sustained below-freezing temperatures, the soil surface cycles between being frozen and unfrozen. During this period, soil above an inclusion of sandy soil is sometimes warmer than it would be in the absence of the inclusion, while soil above an inclusion of sand is sometimes colder than it would be without the inclusion. Subsequent temperature differences for both types of inclusion are consistent in sign and generally similar in magnitude (less than 1°C different). That the dependence of the disruption in heat flow on the material of the inclusion lessens during the winter is a consequence of the freezing of the inclusions. Unfrozen, the sand and sandy soil have significantly different thermal conductivities, with that of the sandy soil being 27% higher; frozen, the two materials’ conductivities are similar (Table 1). While unfrozen, sand would less readily (than sandy soil) conduct heat from depth to the overlying soil to counteract cooling events initiated at the soil surface.

Below the inclusion, the soil is generally warmer (positive temperature difference). The temperature difference at 37.5-cm depth (Fig. 9c) is most dynamic in midwinter (days 75 to 100). During this period, for the case of no inclusion, the soil shows large fluctuations in temperature. Soil below an inclusion does not experience temperature changes of similar magnitude. Instead, it is warmer (positive temperature difference) following a cooling episode and colder (negative temperature difference) following a warming episode. This is a consequence of the temperature perturbations not propagating as readily through the low conductivity frozen inclusion (sand or sandy soil) to the soil beneath.

The temperature differences are largest when the temperatures of the soil and the inclusion are compared. At a depth of 22.5 cm, which corre-

Figure 9. Temperature differences (inclusion minus no inclusion) obtained from one-dimensional numerical simulations of heat flow in a silty soil (17% moisture content) with no inclusion, with a sand inclusion present, or with a sandy soil inclusion present, under BC-1 conditions. At the 22.5-cm depth, the temperature comparison is between sand or sandy soil and silty soil.
sponds to the center of the inclusion, the inclusion can be more than 4°C colder (Fig. 9b). Over the course of the winter, the trend and magnitude of the temperature differences are largely independent of whether the inclusion is sand or sandy soil. During an early freezing episode following a surface thaw (for example, days 56 to 64), however, both inclusions respond more rapidly to the cold wave. Other things being equal, the lower thermal conductivity of the inclusion would act to retard the influence of boundary temperature changes. More importantly, however, the lower heat capacities and lower latent heats of the dryer sandy materials promote more rapid freezing than in the soil because a smaller change in thermal energy is required to lower the temperature of or to freeze the sandy materials. For these materials, a smaller loss of heat in response to a cold wave is sufficient to change the temperature of the inclusion material (heat capacity) or to change its phase (latent heat). Also, since the less conductive inclusion diminishes upward heat flow, that upward flow is less able to mitigate the near-surface effects of the cold wave.

When both the soil and the inclusion material are frozen, the inclusion material is warmer (positive temperature difference) during extended periods when the soil surface temperature is less than 0°C, such as days ~72-78 and ~86-92. The higher thermal conductivity of the frozen soil, twice that of either frozen sand or frozen sandy soil, promotes more rapid cooling of the soil.

The complexity of the time series record of soil temperatures resulting from BC-1 surface temperatures, although a realistic representation of the potential variation in frost depth over the course of a northern New England winter, makes it difficult to examine the significance of factors such as the moisture content of the soil or the severity of the weather. Instead, the boundary conditions designated cold, warm and warmest will be used for simulating heat flow in silty soil with a moisture content of 10, 17 or 25%, and in inclusions of sand or sandy soil.

1. BC-Warmest

Under this mildest winter condition, expressed as both a shorter duration of below-freezing soil surface temperatures and as warmer surface temperatures, frost penetration is confined to the upper 15 cm of the soil. Figure 10 shows the depth of the freezing front in soil (no sand inclusion) through day 100. Three moisture contents of the soil are represented. Surface thawing begins shortly after this (day 101), and the thickness of the frozen layer decreases under the joint action of warming from below and from the surface. The soil is completely unfrozen (Table 3) by day 103 (10% and 17% moisture contents) or day 105 (25% moisture content).

Frost penetration is deeper in moister soil, being 13, 14 or 15 cm in 10, 17 or 25% moisture content soil, respectively (Table 3). This is a consequence of the differences in thermal conductivity of the frozen soils (Table 1). The thermal conductivity increases 57% due to a rise in soil moisture content from 10 to 17%, and 36% upon a further rise in soil moisture content to 25%. Although the latent heat of the soil increases with moisture content, so that to freeze a unit volume of soil requires the release of more energy the more moist the soil is, once the soil is frozen, it conducts heat from the freezing front more readily when moist. The removal of the pulse of heat released upon a
Table 3. Maximum frost depth, its earliest occurrence, and time of complete thaw.

<table>
<thead>
<tr>
<th>BC-Warmest</th>
<th>Maximum frost penetration (cm)</th>
<th>Earliest occurrence of max. frost penetration (day)</th>
<th>Complete thaw of system (day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% mc</td>
<td>no inclusion</td>
<td>13.18</td>
<td>84.88</td>
</tr>
<tr>
<td></td>
<td>sand inclusion</td>
<td>13.23</td>
<td>84.92</td>
</tr>
<tr>
<td>17% mc</td>
<td>no inclusion</td>
<td>14.50</td>
<td>85.96</td>
</tr>
<tr>
<td></td>
<td>sand inclusion</td>
<td>14.64</td>
<td>86.12</td>
</tr>
<tr>
<td></td>
<td>sandy soil inclusion</td>
<td>14.58</td>
<td>86.00</td>
</tr>
<tr>
<td>25% mc</td>
<td>no inclusion</td>
<td>14.94</td>
<td>86.75</td>
</tr>
<tr>
<td></td>
<td>inclusion</td>
<td>15.53</td>
<td>86.25</td>
</tr>
</tbody>
</table>

| BC-Warm    | no inclusion                  | 36.06                                         | 141.50                        | 183.20                        |
|            | sand inclusion                | 35.34                                         | 140.90                        | 181.20                        |
| 17% mc     | no inclusion                  | 39.24                                         | 142.20                        | 184.50                        |
|            | sand inclusion                | 33.39/33.85                                   | 115.70/140.80                 | 180.00                        |
|            | sandy soil inclusion          | 33.03/33.20                                   | 115.60/140.60                 | 180.20                        |
| 25% mc     | no inclusion                  | 40.14                                         | 142.80                        | 185.50                        |
|            | inclusion                     | 31.90/31.84                                   | 115.30/140.50                 | 181.00                        |

| BC-Cold    | no inclusion                  | 58.95                                         | 139.10                        | 183.40                        |
|            | sand inclusion                | 57.83                                         | 139.00                        | 181.00                        |
| 17% mc     | no inclusion                  | 61.17                                         | 139.20                        | 184.60                        |
|            | sand inclusion                | 54.43                                         | 138.90                        | 183.20                        |
|            | sandy soil inclusion          | 53.94                                         | 138.80                        | 182.60                        |
| 25% mc     | no inclusion                  | 60.89                                         | 139.30                        | 186.00                        |
|            | sand inclusion                | 49.97                                         | 138.80                        | 184.10                        |

mc: moisture content.

The volume of soil freezing can be considered in terms of the soil's diffusivity, which also increases with moisture content. These two effects, higher conductivity and higher diffusivity of released latent heat in the moist soil when frozen, combine to offset the high latent heat and heat capacity of the moist soil, which are impediments to the advance of the freezing front. The net result is deeper frost penetration the more moist the soil is.

Frost penetration in 10% or 17% moisture content silty soil under BC-Warmest conditions is little different (less than 0.25 cm deeper) when a sand inclusion is present (Fig. 11a, b; Table 3). In these soils the freezing front does not reach the top of the sand inclusion. Frost penetration in the 25% moisture content silty soil does extend to the top of the inclusion (Fig. 11c). A rapid advance of the freezing front, from 15 cm to 15.5 cm, follows due to the lower latent heat and heat capacity of the dryer sand. The sand remains frozen for only a short time, ~4 days, because the boundary condition is already moderating, i.e., the soil surface has begun warming.

The BC-Warmest simulations show that when no inclusion is present, not all the soil above a sensor cable will freeze during the winter. Depending on the soil's moisture content, a 7- to 9-cm-thick layer of unfrozen soil overlying the sensor cable will persist. Although the frost penetration is only slightly greater when a sand inclusion is present, the thickness of the unfrozen soil layer is much less, only ~0 to 2 cm, because the remaining depth is occupied by the relatively dry, less lossy sand. For this type of winter condition, the presence of the sand inclusion should result in better sensor system performance. The advantage of having the sand inclusion present is essentially unrelated to its effect on heat flow.

2. BC-Warm

Simulations with this boundary condition are representative of frost penetration in soil that becomes snow-covered during the winter. The maximum frost depth is slightly dependent on the wetness of the soil; it is 36, 39 and 40 cm for soil moisture contents of 10, 17 and 25%, respectively, and no sand inclusion (Fig. 12, Table 3). By the time (~100 to 106 days, depending on moisture content of the soil) that the freezing front has reached a depth of 30 cm, the soil sur-
face is experiencing an overall warming trend. The brief cooling episodes that follow are superimposed on the longer-term warming trend. The higher surface temperatures effectively reduce the driving force for further reductions in soil temperature at depth. Frost penetration continues at a reduced rate in the soil, until approximately day 142. Thawing from the surface down begins late on day 176. The soil is fully thawed by day 184 (10% moisture content), 185 (17% moisture content) or day 186 (25% moisture content).

Given that when there is no sand inclusion the frost depth reaches at least 30 cm for all three soil moisture contents, which is a highly favorable situation for an electromagnetic sensor system, it is particularly important to know what differences in frost penetration result because a sand inclusion is present. Accordingly, simulations were done to investigate the effect of a sand inclusion on frost and thaw penetration for this boundary condition. The freezing histories are identical, for the sand inclusion and no inclusion cases, until the freezing front encounters the sand inclusion (Fig. 13). Frost penetration proceeds more rapidly through the relatively dry sand than it does through the soil, reaching a depth of 30 cm 12, 9 or 8 days sooner when a sand inclusion is present, for soil moisture contents of 10, 17 or 25%, respectively. Differences in latent heat and heat capacity between the soil and sand determine the relative rates of advance.
of the freezing front from 15- to 30-cm depth. A smaller change in heat content is required to reduce the temperature of and eventually freeze the sand.

Whether frost penetration below the sand inclusion occurs is the net result of the diminished cooling from the surface downward and further warming of the basal soil due to heat flow from depth. As discussed above, when there is no sand inclusion, frost penetration proceeds more slowly below 30 cm because of the overall warming trend at the surface. There is also heat flow from the warm soil at depth, as defined by the initial temperature condition (Fig. 2). After that heat has been conducted through the soil below the sand inclusion, at a rate determined by the thermal properties of unfrozen soil, its upward flow is retarded by the low thermal conductivity of the frozen sand; therefore, more heat is retained in the soil at the base of the sand inclusion and it warms. Further frost penetration is inhibited by the resistance to upward heat flow caused by the presence of the sand inclusion. Frost depth is least in the soil with the highest moisture content (Fig. 14) because a reduction in temperature of that soil requires the largest change in thermal energy, and so is most impeded by warming of the soil as a consequence of the presence of the sand inclusion. The inclusion is like a layer of insulation at this stage, impeding the heat flow towards the ground surface that further freezing requires. A consequence of this is that 17 and 25% moisture content soil at 37.5-cm depth remains above freezing when the sand inclusion is present (Table 4). Maximum frost penetration (Table 3) is ~3 cm deeper in the relatively dry soil (10% moisture content) than it is in the wettest soil (25% moisture content).

For BC-Warm winters, the advantage for sensor system performance of having a sand inclusion present is related to soil temperature, not frost depth, since the soil above a sensor cable (22.5-cm depth) freezes in both cases. When an inclusion is present, the frozen soil is colder (Table 4). Although the temperature difference is small, ~0.5°C, the soil temperature is in the range (0 to ~3.5°C) where a small change in temperature can result in an appreciable change in unfrozen moisture content (Williams 1967, Anderson and Morgenstern 1973).

The disadvantage of having a sand inclusion under BC-Warm conditions is that there can be a large relative decrease in frost depth beneath the inclusion. Frost depth is 6 cm (14%) or 8 cm (20%) less in 17% or 25% moisture content soil,
respectively. Without the sand inclusion, an approximately 17-cm-thick layer of frozen, low electrical conductivity soil would underlie a sensor cable in 17% moisture content soil; the layer would be ~1 cm thicker in 25% moisture content soil. With a sand inclusion present, the sensor cable would be underlain by a 7.5-cm-thick layer of low conductivity sand but by only ~2-3 cm of frozen soil. There is a net reduction in the extent of the region of low electrical conductivity soil and/or sand beneath the sensor cable when a sand inclusion is present.

The material of the inclusion does influence frost and thaw penetration. The maximum depth of the freezing front in 17% moisture content soil is slightly greater, on the order of 1 cm, beneath a sand inclusion compared to that when the inclusion is sandy soil (Fig. 15). The sandy soil thaws more rapidly, which could be detrimental in situations of frequent reversals between warming and cooling trends. The performance of a buried electromagnetic sensor system would be more stable if any fluctuations in surface temperature were effectively smoothed at depth by the slow response of the inclusion material. Together with the previous observa-

Table 4. Least soil temperature (°C) and earliest day of occurrence for soil depths of 7.5, 22.5 and 37.5 cm.

<table>
<thead>
<tr>
<th>Boundary condition</th>
<th>7.5 cm Day</th>
<th>22.5 cm Day</th>
<th>37.5 cm Day</th>
</tr>
</thead>
<tbody>
<tr>
<td>BC-Warmest</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% mc no inclusion</td>
<td>-0.34</td>
<td>79.2</td>
<td>0.44</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-0.34</td>
<td>78.9</td>
<td>0.46</td>
</tr>
<tr>
<td>17% mc no inclusion</td>
<td>-0.37</td>
<td>77.3</td>
<td>0.35</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-0.38</td>
<td>77.5</td>
<td>0.47</td>
</tr>
<tr>
<td>25% mc no inclusion</td>
<td>-0.38</td>
<td>78.4</td>
<td>0.34</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-0.39</td>
<td>78.3</td>
<td>0.53</td>
</tr>
<tr>
<td>BC-Warm</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% mc no inclusion</td>
<td>-2.48</td>
<td>93.2</td>
<td>-0.76</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-2.83</td>
<td>93.1</td>
<td>-0.88</td>
</tr>
<tr>
<td>17% mc no inclusion</td>
<td>-2.52</td>
<td>93.1</td>
<td>-0.87</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-3.10</td>
<td>93.1</td>
<td>-1.23</td>
</tr>
<tr>
<td>25% mc no inclusion</td>
<td>-2.53</td>
<td>93.0</td>
<td>-0.90</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-3.23</td>
<td>93.1</td>
<td>-1.36</td>
</tr>
<tr>
<td>BC-Cold</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10% mc no inclusion</td>
<td>-6.96</td>
<td>112.1</td>
<td>-4.01</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-7.15</td>
<td>112.2</td>
<td>-4.11</td>
</tr>
<tr>
<td>17% mc no inclusion</td>
<td>-7.03</td>
<td>112.1</td>
<td>-4.13</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-7.39</td>
<td>112.1</td>
<td>-3.95</td>
</tr>
<tr>
<td>25% mc no inclusion</td>
<td>-7.02</td>
<td>112.1</td>
<td>-4.06</td>
</tr>
<tr>
<td>sand inclusion</td>
<td>-7.56</td>
<td>112.1</td>
<td>-3.83</td>
</tr>
</tbody>
</table>

mc: moisture content.
tion that, under BC-1 conditions in early winter, soil above an inclusion is colder (warmer) when the inclusion material is sand (sandy soil), the more rapid thawing associated with a sandy soil inclusion suggests that sand, rather than sandy soil, is the preferred material for surrounding the sensor cables.

3. BC-Cold

The final round of one-dimensional simulations was conducted with a boundary condition representative of the most severe winter conditions considered. Equivalently, it corresponds to frost and thaw penetration in a soil subjected to “normal New England” conditions (i.e., BC-Warm) but without a snow cover. As would be expected, the maximum frost penetration is greatest for this case, −60 cm (Fig. 16; Table 3). Deepest frost penetration occurs ~30 days after the period of lowest soil surface temperatures.

Frost penetration also varies with the moisture content of the soil, with least penetration in 10% moisture content soil, as previously noted for soil with no sand inclusion.

There is a transient surface thaw during days 76 to 82. The 17% and 25% moisture content soils thaw completely. The thaw penetrates 4.5 cm into the 10% moisture content soil, leaving a 4-cm-thick frozen layer below the thawed layer. End-of-winter thawing from the surface down begins on day 158. The 25% moisture content soil is completely thawed on day 186, the 17% moisture content soil on day 184, and the 10% moisture content soil on day 183.

For all moisture contents of the soil, the freezing front advances more rapidly through the sand inclusion (Fig. 17). Unlike the BC-Warm case, when frost penetration to the base of the sand inclusion coincided with the onset of a warming trend, in this situation the soil surface
is still experiencing a strong cooling trend when the sand inclusion has frozen. This overrides any inhibition of further frost penetration due to reduced rate of upward heat flow at the base of the inclusion. Instead, the soil below the inclusion continues to cool, although at a slower rate than does soil of the same moisture content when no inclusion is present. For each soil moisture content, the frost penetration is deeper without the sand inclusion; the freezing front extends ~1, 7 or 11 cm deeper in soil of 10, 17 or 25% moisture content, respectively.

Beneath the sand inclusion, where more upward flowing heat is retained, frost penetration is greater the drier the soil is (Fig. 18; Table 3). This is consistent with freezing occurring more readily in drier soil because its latent heat and heat capacity are smaller. The maximum difference in soil temperatures (inclusion versus no inclusion) is greater at 37.5-cm depth under BC-Cold conditions than it is under BC-Warm conditions for all moisture contents of the soil (Table 4).

Whether a sand inclusion is present is least important to sensor system performance under BC-Cold conditions. During the harshest portion of the BC-Cold winter, days 100 to 135, the soil above the sensor cables remains several degrees below freezing in both cases. In response to the subsequent increase in soil surface temperature, to ~0.1°C by day 143, the soil at 7.5-cm depth warms to ~0.1°C regardless of moisture content and, for 17% moisture content soil, regardless of whether the inclusion material is sand or sandy soil.

Although frost depth is less when a sand inclusion is present (Table 3), the relative reduction is slightly smaller, 12–18%, than under BC-Warm conditions. This, plus the overall deeper frost penetration under BC-Cold conditions, makes the effect of a sand inclusion on heat flow less significant to sensor system performance under BC-Cold conditions.

Discussion of inclusion effects on soil temperatures

Temperature profiles based on one-dimensional simulations of heat flow through silty soil with and without a sand inclusion show that the temperature relationship is variable. For all moisture contents, the soil at 37.5-cm depth is colder under the sand inclusion for the portion of the early winter corresponding to rapid frost penetration through the dry sand inclusion. The duration of this period decreases with increasing moisture content of the soil,
from 44 to 17 to 15 days for BC-Warm conditions, and from 18 to 10 to 7 days for BC-Cold conditions (moisture content of 10, 17 or 24%, respectively).

Following this initial period, the soil at 37.5 cm depth is warmer under a sand inclusion. The 10% moisture content soil is warmer by a maximum of 0.04 or 0.16°C under BC-Warm or BC-Cold conditions, respectively. In these cases the greatest depth of the freezing front differs only slightly (~1 cm) whether or not a sand inclusion is present, and the freezing front does not even penetrate to 37.5 cm under BC-Warm conditions. For 17% moisture content soil, the maximum temperature difference is ~0.2°C under BC-Warm conditions, but it is as much as 0.8°C under BC-Cold conditions. There is a larger dependence of frost depth on the presence of a sand inclusion in this soil, with maximum frost penetration being 6–7 cm less when the sand inclusion is present; this is reflected in greater disparity in soil temperature with depth. Finally, for the wettest soil considered (25% moisture content), the maximum temperature difference is ~0.5°C under BC-Warm conditions, and as large as 1.17°C under BC-Cold conditions. The reduction in frost penetration because of the presence of a sand inclusion is greatest, 7–11 cm, or 18–20%, for this soil.

The heat flow simulations indicate that soil at a depth of 7.5 cm is generally colder above a sand inclusion. For all moisture contents and both BC-Warm and BC-Cold conditions, the shallow soil is noticeably colder (0.3 to ~1°C) during the period of rapid frost penetration in the sand inclusion. Following this, for the remaining ~80 days of the BC-Warm winter and the remaining ~50 days of the BC-Cold winter (that period when the temperature of the soil surface is consistently less than 0°C), the temperature difference is smaller, 0.2°C or less.

The small differences in temperature attributable to the presence of the sand inclusion would not be significant if the soil temperatures were greatly less than zero, on the order of ~10°C. If the temperature of a silty soil that cold varied by even a few degrees, the unfrozen moisture content of the soil would not change appreciably (Williams 1967, Anderson and Morgenstern 1973). Under BC-Warm conditions, however, the lowest temperature that the soil at 7.5-cm depth attained was ~2.5°C (Table 4). For silty soil under conditions that normally (i.e., when there is no sand inclusion) would result in a temperature of 0 to ~2°C for much of the winter, the decrease in unfrozen moisture content associated with a drop in temperature of several tenths of a degree may be large (Anderson and Morgenstern 1973). The decrease in unfrozen moisture content in turn causes a decrease in the electrical conductivity of the frozen soil (e.g., Hoekstra 1969) that potentially improves electromagnetic sensor system performance.

Although the soil types are different (silty soil in the simulations and sandy loam at SOROIDS), field measurements agree with the temperature relationships predicted by the simulations. During a BC-Warm type of winter (snow cover present), the SOROIDS soil with no sand inclusion is infrequently colder than ~3°C at a depth of 7.5 cm. Half-hourly records of soil temperature indicate that the soil typically is colder during only a few days of the freezing season. Comparisons of soil temperatures at the sand inclu-
sion location and at an undisturbed (control) location indicate that at 7.5-cm depth, the soil temperature is slightly colder, by 0 to 0.3°C, when it overlies the center of the 16-cm-wide sand inclusion, and that soil at 37.5-cm depth is slightly warmer, 0 to 0.2°C, when it underlies the center of the same sand inclusion. The measurements of soil temperatures at the inclusion location were made weekly. The magnitudes of the differences in soil temperature are small, but the differences can be quite large (15–30%) relative to soil temperatures when no inclusion is present.

**SUMMARY AND CONCLUSIONS**

For a silty soil of variable wetness (10, 17, 25% moisture content by weight) subjected to freezing conditions ranging from mild (BC-Warmest) to severe (BC-Cold), numerical simulations have quantified the disruption in frost and thaw penetration caused by the presence of a sand inclusion in the soil. For the narrowest inclusion considered, 16.5 cm, two-dimensional simulations show that frost penetration beneath a surface point centered on the inclusion is insignificantly different from the case of no inclusion. Heat effectively flows around the sides of an inclusion; penetration of an inclusion this narrow, resulting in small temperature differences relative to the no-inclusion case. When the inclusion is wider, 91 cm, frost penetration below the inclusion center is effectively that associated with an infinitely wide inclusion (one-dimensional simulations). For the cases of the 91-cm-wide and infinite-width inclusions, both the rate and depth of frost penetration are disrupted by the presence of the sand inclusion. The low latent heat content of the dry (3% moisture content) sand causes it to freeze more rapidly than the surrounding soil. Therefore, however, the low conductivity sand acts as insulation, causing the soil above it to be colder and the soil below it to be warmer.

The dependence of frost penetration on soil moisture content demonstrates the potential for inconsistent performance by an electromagnetic sensor system within its detection zone. If the moisture content of the soil surrounding a buried sensor system varies along its cable length, frost penetration will not be uniform within the detection zone. There will be significant location-dependent differences in sensor performance that persist until the overlying soil throughout the entire detection zone is frozen.

The complexity of the soil temperature histories obtained from BC–1 simulations highlights the difficulty of reliably knowing either the frozen-unfrozen status of soil at depth or when relative changes in unfrozen moisture content of frozen soil occur, unless soil temperature is monitored directly by making real-time measurements or indirectly from numerical simulations. Otherwise, the occurrence of changes in electromagnetic sensor system effectiveness that result from short-term variations in temperature-dependent unfrozen soil moisture may be unrecognized.

The differences in soil temperature and in frost depth in response to a change in soil moisture content or a change in boundary condition (Warmest, Warm, Cold) demonstrate the potential for variability in sensor system performance from year to year. All other things being equal, an anomalous moisture content at the beginning of winter, or winter weather of greater or lesser severity, can result in such different soil temperatures and frost depths that the likelihood and duration of improved sensor system performance, related to freezing of the soil, may be affected.

The presence of a sand inclusion potentially improves electromagnetic sensor system performance in certain situations. If frost depth is less than cable depth (BC-Warmest simulations), then the thickness of the layer of unfrozen, lossy soil above the cables is reduced by an amount equal to the half-thickness of the sand inclusion. If frost depth exceeds cable depth, but the frozen soil overlying the cables normally (no inclusion) is not very cold (BC-Warm simulations), then the fact that the soil is colder when a sand inclusion is present may mean that the soil’s unfrozen moisture content is appreciably less. However, if normally the soil overlying the cables is much colder (BC-Cold simulations), the difference in soil temperature that results from the presence of the sand inclusion will not significantly affect sensor system performance. Finally, if the soil surface temperature reaches or crosses 0°C during the winter (other than at initial freeze-up and final thaw), such as late in the BC-Cold winter or throughout the BC–1 winter, the sand inclusion does not prevent 7.5-cm-deep soil from warming to 0°C or higher. Its presence, however, does result in complete thaw of the soil/sand/soil section at the end of winter occurring earlier than does complete thaw of just the soil section (no sand inclusion); this is jointly because the soil underlying the sand inclusion is already warmer.
before thawing from the surface down begins, and because the latent heat of the sand is less than that of the soil it replaced. The sooner the soil is completely unfrozen, the quicker it will dry since perched thawed layers, generally of high moisture content, will be eliminated. The sensor system's performance will be better in dryer soil than in damp soil having a high electrical conductivity.

One unresolved question is exactly how significant to an electromagnetic sensor system is the difference in electrical conductivity associated with the colder soil above a sand inclusion. The lateral extent of the soil temperature differences (two-dimensional simulations) must be considered when determining potential changes in sensor system performance. If the sensor system does function more reliably as a consequence of the changes in wintertime soil temperature history caused by the presence of the sand inclusion, then it is necessary to assess whether the duration of that improved performance during winter justifies the time and money required to incorporate a sand inclusion during the installation of the cable system, and whether a wider or narrower inclusion is desirable.

LITERATURE CITED


A radio-frequency (41 MHz) electromagnetic sensor system consisting of a processor and two buried coaxial cables is in operation at SOROIDS. The sensor system is designed to generate an alarm when a person crosses the buried cables. The cables are placed 1.5 m apart at a depth of 22.5 cm in the local soil, a sandy loam. The electrical conductivity of the soil at 40 MHz ranges from 2 to 41 mhos/m for moisture contents of 3 to 37% by volume (Malone, pers. com.). Because this soil has a low electrical conductivity, only the excavated soil—no sand—was used as backfill around the cables.

Both coaxial cables are manufactured with gaps in their shields. One cable, the transmitter, leaks the pulsed signal sent down it by the processor into the surrounding soil. The other cable, the receiver, conducts a signal induced by changes to the electromagnetic field set up in the soil surrounding the transmitter cable. The processor analyzes the received signal for variations that would be caused by a person crossing the cables, and generates a voltage in response. If the voltage is negative, the sensor system does not generate an alarm and the person crossing the cables proceeds undetected. A voltage of 0.4 V is regarded as the minimum response for reliable detection of a person because that voltage corresponds to a margin of 8 dB above the alarm threshold.

One factor determining the magnitude of the processor voltage is the strength of the electromagnetic field in the zone defined by the cables. If the electromagnetic field generated by the transmitter cable is weak, as it would be in soil of high electrical conductivity, the disturbance to it caused by a person crossing the cables may be insufficient to produce a processor voltage

![Diagram](image-url)

**Figure A-1. Maximum responses (voltages) of buried electromagnetic sensor system to a person crossing the sensor cables at various locations on the dates shown fall within the shaded band.**
that is higher than 0.4 V, or even non-negative. Because the soil's electrical conductivity varies directly with its unfrozen moisture content, freezing and thawing of the soil will affect sensor system performance.

An example of the sensor system response to a walking person over the course of a winter is given in Figure A-1. On each day shown, the person crossed the zone defined by the cables at 15 to 18 locations along the 141-m lengths of the cables (Peck 1992). The resultant processor voltages define the shaded band. In November 1990, the system response to some crossings was unacceptably low, either negative or less than 0.4 V. Soil temperatures during this month were above freezing, as determined with a vertical array of copper-constantan thermocouples in the soil at depths of 0 to 60 cm. By 2 January 1991, when the soil was frozen to a depth of at least 22.5 cm (temperature of the 30-cm-deep soil was 0°C), all crossings produced a voltage greater than 0.4 V. The contrast between non- or marginal detection in November and reliable detection from 2 January through 13 March is consistent with differences in the unfrozen moisture content of the soil. The soil froze in January to between 37.5- and 52.5-cm depth and in late February to between 52.5 and 60 cm; it remained so on 13 March. This means that the soil between the sensor cables had a low electrical conductivity and consequently the strength of the electromagnetic field was high.

On 28 March the shallow soil was moist under the combined effects of the melting of a 3-cm-deep snow cover over the previous two days, rainfall on this day, and the surface thawing of the soil. The near-surface soil warmed to above 20°C at midday, and thawing (soil temperatures of ~1 to 0°C) had penetrated to 22.5 cm depth. Some of the crossings on 28 March were only marginally detected.

By 29 April the soil was completely thawed (temperatures of 12 to 6°C at depths of 7.5 to 60 cm) and there were both marginally detected and undetected crossings. In response to this unacceptable detection capability, the alarm threshold of the system was lowered. The effect of this was to change the equivalency between sensor signal and processor-produced voltage, such that a smaller sensor signal now caused a positive voltage. With this change in system sensitivity, acceptable detection capability was restored.

Had the sensor system been operating in soil having a thermal history similar to Figure 8, then daily repetitions of the crossings would have revealed more variable sensor performance in early winter when the soil above the sensor cables was still experiencing short-term freeze-thaw-freeze episodes.
**TITLE AND SUBTITLE**
Heat Transfer and Frost/Thaw Penetration in Soil Surrounding an Inclusion of Sand: Numerical Model Results Relevant to Electromagnetic Sensor System Performance

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**ABSTRACT (Maximum 200 words)**
One- and two-dimensional numerical simulations of heat flow in silty soil with and without a sand inclusion (15 cm thick, variable width) have determined the magnitude and the lateral extent of the disruption in frost and thaw penetration attributable to the presence of the inclusion. Four different soil temperature histories, derived from field data at a Vermont site, were used as the surface boundary condition for the winter-long simulations. This identified differences in frost depth and soil temperature resulting solely from an overall colder or warmer soil surface condition. For a given surface boundary condition, the moisture content of the soil was varied (10, 17 or 25%, by weight) to contrast the changes in frost penetration caused by the moisture-dependent differences in soil thermal conductivity and latent heat. The drier sand (3% moisture content by weight) with its smaller latent heat freezes more rapidly than does the soil under identical conditions, so initially (early winter) frost penetration is greater (by 5-6 cm) when the sand inclusion is present because the freezing front proceeds rapidly through the sand. Subsequently, the freezing front is deeper (by a maximum of 11 cm) in soil without a sand inclusion. The less conductive sand impedes heat flow toward the soil surface, resulting in higher soil temperatures beneath the inclusion, which in turn retards freezing of the soil. Frost penetration beneath a sand inclusion is deeper the drier the soil is; with no sand inclusion present, frost depth is greater the more

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moist the silty soil is. Under the conditions of this study, maximum frost penetration is 61 cm ("coldest" surface boundary condition, 25% moisture content soil, no sand inclusion). The change in maximum frost depth because of the presence of a wide sand inclusion is large relative to overall frost penetration. Similarly, the difference in soil temperature at a given depth, although small, can correspond to a large difference in unfrozen moisture content of the silty soil. Under either relatively mild or "normal New England" winter conditions, the presence of a sand inclusion is probably beneficial to the performance of a buried electromagnetic sensor system, which is more reliable in dry or frozen soil because of the soil's lower electrical conductivity. A sand inclusion may not lead to improved sensor performance under more severe winter conditions, which cause much lower soil temperatures and much deeper frost penetration.