Simulation Study of Shocked Particle Velocity Distributions in a HANE Disturbed Plasma

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August 1995

Technical Report

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This work was sponsored by the Defense Nuclear Agency under RDT&E RMC Code B4662D AF AN LAW07 3200A 25904D.

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A two dimensional, hybrid particle-in-cell simulation code is used to study the velocity distributions of a plasma disturbed by a collisionless shock wave generated by a high altitude nuclear explosion. Knowledge of the velocity distributions is crucial in understanding plasma instabilities that thermalize the disturbed plasma and produce the ion deposition spectrum. Variation in the deposition spectrum can cause orders of magnitude differences in IR radiation and ionization and strongly affect systems. The simulations indicate that the relevant anisotropic velocity distributions are shell-like rather than bimax wellian. Since almost all theoretical analysis of thermalizing plasma instabilities have been based on bimaxwellian distributions, this research motivates a reexamination of thermalizing instabilities using velocity distributions that are more physically realistic.
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CONVERSION TABLE

Conversion factors for U.S. customary to metric (SI) units of measurement.

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SECTION 1
INTRODUCTION

The deposition of energetic air and debris ions in the altitude range of 100-150 km is a major cause of IR, C³, and radar system degradation. Variations in the energy spectrum of the deposited particles can cause orders of magnitude differences in IR radiation and ionization. If the spectrum is "soft", ions are deposited at high altitude and produce long lived ionization because of the low recombination rate. However, the IR radiation will be lower because there are fewer molecules. If the spectrum is "hard", deposition will occur at lower altitudes (105-120 km). The ionization will be short lived but the IR radiation will be more intense because there are more excited molecules.

Current systems codes predictions depend upon realistic and reliable predictions of deposition spectra. The research under this contract is focused on providing the first principles calculations necessary to provide a better understanding of the acceleration and interaction processes between the expanding debris and the ionosphere of the earth. This report describes evolutionary steps toward an improved ion deposition model for DNA’s system codes.

A two dimensional, hybrid particle-in-cell simulation code is used to study the velocity distributions of a plasma disturbed by a collisionless shock wave generated by a high altitude nuclear explosion. Knowledge of the velocity distributions is crucial in understanding plasma instabilities that thermalize the disturbed plasma and produce the ion deposition spectrum that causes strong system effects. It is found that plasma particles are strongly accelerated when first encountering the shock wave. Rather than being specularly reflected, particles are scattered in various directions with respect to the shock because of irregularities in the shock front. On the basis of studies of velocity distributions at points successively deeper into the shock, it is found that the beam-like velocity distribution of particles in front of the shock is changed, over a relatively short distance, into a form gyrotropic with respect to the magnetic field. Over a longer distance scale the velocity distribution is partially isotropized, developing a Maxwellian shape in the component parallel to the magnetic field. In contrast, the perpendicular component maintains a shell-like rather than Maxwellian shape. This observed velocity distribution differs substantially from the bimaxwellian form typically used in analyses of plasmas disturbed by a collisionless shock. For example, bimaxwellian velocity distributions are assumed in the models used by codes such as CMHD.

The initial flow of kinetic energy is radial as debris expands away from the detonation point. The debris material expands faster than the local Alfven velocity, \( V_A = B/(4\pi n m) \) and generates a collisionless shock wave which energizes the background ionospheric plasma. Since the expanding shock wave pushes the Earth's magnetic field in front of it, much initial energization of the ionospheric plasma is perpendicular to the local magnetic field. It is the process by which energy is isotropized with respect to the magnetic field that especially interests us since the parallel energy can readily flow down along field lines and be deposited in the atmosphere. Ultimately, we seek a "turning model" -- a parameterized model of how energy that is initially predominantly perpendicular
to field lines is "turned" into parallel energy.

It is well known from kinetic theory that plasmas with free energy will develop unstable motions and relax to a lower energy state. A classic example is a plasma with more kinetic energy perpendicular to the magnetic field than kinetic energy parallel to the field. Such a plasma is said to be anisotropic, with $T_\perp > T_\parallel$, and is subject to instabilities such as the mirror mode [Chandrasekhar, 1958; Barnes, 1966; Hasegawa, 1975] or the Alfvén ion cyclotron [Kennel and Petschek, 1966; Davidson and Ogden, 1975]. These instabilities develop nonlinearly so as to reduce the temperature anisotropy of the plasma. The mirror mode and Alfvén ion cyclotron are both observed in magnetosheath [Tsurutani et al., 1982; Moustaizis et al., 1986; Hubert et al., 1989; Skopke et al., 1990, Brinca et al., 1990; Anderson and Fuselier, 1992], behind the collisionless bow shock that develops when the super-Alfvénic solar wind flow encounters the earth. They have been widely studied, with both theory and simulation, as probable mechanisms for thermalizing ion distributions in the magnetosheath [Gary et al., 1976, 1992; Tajima et al., 1977; Ambrosiano and Brecht, 1987; Winske and Quest, 1988; Brinca et al., 1990; Yoon, 1992; McKean et al., 1992]

The collisionless shock associated with a HANE is a dynamical event and differs from the quasistationary bow shock. A HANE produced shock is initially driven quite strongly but the impetus diminishes as the expanding debris material gives up its energy to the background. The bow shock is continuously driven by the solar wind. This difference has consequences for the development of plasma instabilities that isotropize the shock disturbed plasma. In the case of the bow shock, it is adequate to model the shocked plasma with a bimaxwellian velocity distribution function for which $T_\perp > T_\parallel$. In contrast, the dynamical, HANE induced, collisionless shock may have velocity space distortions of the distribution function in addition to the temperature anisotropy.

In this report we describe the use of a two-dimensional hybrid particle-in-cell simulation to ascertain the characteristic shape of the shocked plasma velocity distribution functions. It is found that plasma particles are strongly accelerated when first encountering the shock wave. Rather than being specularly reflected, particles are scattered in various directions with respect to the shock because of irregularities in the shock front. On the basis of studies of velocity distributions at points successively deeper into the shock, it is found that the beam-like velocity distribution of particles in front of the shock is changed, over a relatively short distance, into a form gyrotrropic with respect to the magnetic field. Over a longer distance scale the velocity distribution is partially isotropized, developing a Maxwellian shape in the component parallel to the magnetic field. In contrast, the perpendicular component maintains a shell-like rather than Maxwellian shape. This observed velocity distribution differs substantially from the bimaxwellian form typically used in analyses of plasmas disturbed by a collisionless shock.

In Section II we describe the 2-D hybrid PIC code and demonstrate how it is used to generate a collisionless shock typical of that produced by a HANE. The description of the macroscopic features of the shock, the nature of particle orbits in the presence of the shock, and the spatial evolution of the velocity distribution function within the shocked plasma are described in Section III. The observed velocity distribution function is characterized by an analytical distribution in Section IV and
the method by which the analytical distribution is fitted to observations is described. Some properties of the fitting analytical distribution are also presented. Conclusions are drawn in Section V.

The implications of the observed velocity distribution functions for the mirror mode and Alfvén ion cyclotron instabilities in a HANE disturbed plasma are examined in a separate report [Gladd and Brecht, 1994].
SECTION 2
DESCRIPTION OF SIMULATION

We are interested in the shape of the velocity distribution functions of ions energized by the passage of a typical, HANE induced, collisionless shock wave. To model such a shocked plasma we use a two-dimensional, hybrid particle-in-cell simulation code that treats plasma ions kinetically while treating electrons as a fluid. The details of the code are described in [Thomas and Brecht, 1986; Brecht and Thomas, 1988].

The collisionless shock is created in the simulation by streaming a light mass plasma onto a much heavier obstacle plasma. A shock wave is generated at the obstacle and flows back upstream into the light plasma. The shock formation process is dynamic and the simulation terminates when the shock wave has propagated across the computational grid. This situation is illustrated in Figure 2-1a. This method has some advantages over propagating the light plasma into a back wall [Winske and Quest, 1988] since the boundary conditions at the back wall are not involved. The simulation geometry is shown in Figure 2-1b. The simulation plane is the two-dimensional (x-z) plane. The ambient magnetic field is in the +x-direction, and perpendicular to the ambient flow (+z-direction) and the direction of propagation of the shock wave (-z-direction).

The particular simulation we discuss in this report is designed to produce a collisionless shock representative of a HANE. In Figure 2-2 we show representative profiles of the simulated collisionless shock at four different times (0, 500, 1000, 2000 $\Delta t$) during the simulation. The thick solid line represents the background material ($n_e\ A = 54$, Fe$^+$). The dashed line represents the lighter material that is flowing to the right ($n_e\ A = 16$, O$^+$). The thin solid line represents the magnetic field. Here, $B_x$ and $n_e$ have been averaged in the x direction and plotted as a function of z. These quantities have also been normalized to their values upstream of the shock. In this figure, the light material flows to the right and a shock develops when it encounters the stationary heavy material. This shock then propagates to left. After some transient development time, the characteristic overshoots of density and magnetic field develop, as does a small foot region on the density that corresponds to ions being reflected from the shock.
Figure 2-1.  a) Method for generating collisionless shock with simulation. A light specie is flowed onto a heavy specie, the shock develops and propagates upstream.
b) Geometry of the simulation. Simulation plane is x-z, magnetic field in x direction, and initial flow is in the -z direction.
Figure 2-2.  a) Profiles of main magnetic field component, $B_x$, light plasma density, $n_l$, and heavy plasma density, $n_h$, as a function of $z$. Profiled quantities are averaged in the x-direction. Snapshots taken after 0, 500, 1000 and 2000 time steps.
The parameters characterizing this simulation are

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<td>(N_z) (cells in z-direction)</td>
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<td>(dx) (cell size in x-direction)</td>
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<tr>
<td>(dz) (cell size in z-direction)</td>
<td>1.5 km</td>
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<td>(B_{x0}) (ambient B field)</td>
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<td>(n_{H0}) (density of heavy specie A = 54, Fe(^+))</td>
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<td>(n_{L0}) (density of light specie A = 16, O(^+))</td>
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<td>(v_{xL0}) (drift speed of light specie)</td>
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</tr>
<tr>
<td>(Z_H)</td>
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<tr>
<td>(Z_L)</td>
<td>1</td>
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<tr>
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<td>(\omega_{ri})</td>
<td>60 sec(^{-1})</td>
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<td>(\rho_i)</td>
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For these parameters the collisionless shock has Mach number 9.0, calculated as the ratio of \(v_{xL0}/v_{AL0}\) with \(v_{AL0}\) the Alfven speed in the light specie.
SECTION 3
ANALYSIS OF SIMULATION

Individual particle behavior: We first consider the behavior of simulation particles as they encounter the shock wave (see Figure 3-1). Here we show the positions of individual particles in a 4 cell \(\times\) 20 cell region of the simulation. The simulation time is 2000 time steps and the z-strip shown starts with cell \(z = 84\) (the front of the shock) and runs through cell \(z = 124\) (well into the shock). The x-strip contains the central cells 15, 16, 17, 18 and are taken to be representative of the 32 simulation cells transverse to the direction of shock propagation. The regular spacing of particles at the bottom of Figure 3-1 corresponds to the uniform light particles (O\(^+\)) propagating upward at a velocity of 2000 km/sec. The top of the Figure is well into the shocked region and the particles have been scattered by the turbulent electric and magnetic fields. The temporal and spatial variation of these turbulent fields is slow with respect to the velocity of the incoming particles so the sequence in space of incoming particles appears like a tracer in time of an individual particle. In this respect, we see that some of the incoming particles are strongly reflected back in the direction from which they came. Such reflection is expected for a high Mach number collisionless shock wave. This method of viewing particles is of limited utility in determining the overall effects of the shock on the plasma. It is more instructive to consider the shape of the velocity distribution function of the particles.

Evolution of velocity distribution functions: We are particularly interested in the change of the particle distribution function at successively greater penetration distances into to the shock. It is instructive to consider the distribution function with both spherical and cylindrical representations.

Some observed spherical velocity distribution functions are illustrated in Figure 3-2. The velocity of an individual particle is characterized by the magnitude of the velocity \(|v|\) and the spherical angles \(\theta\) (the polar angle) and \(\phi\) (the azimuthal angle). The orientation of the coordinate system is such that \(\theta = 0\) points in the +x direction (the direction of the ambient magnetic field) and \(\phi = 0\) points in the +z direction (the direction of ambient particle inflow). We have chosen to display the angular distribution in a planar form by unwrapping \(\theta\) and \(\phi\) from a sphere. Thus \(\theta\) has the extent \((0 < \theta < \pi)\) and \(\phi\) has the extent \((0 < \phi < 2\pi)\). To clarify what we have done, the actual spherical representation is shown in Figure 3-3 where \(f(|v|, \theta, \phi)\). Figure 3-3a) illustrates what a beam distribution would look like in a spherical representation; figure 3-3b) illustrates what a gyrotrropic distribution would look like. Gyrotrropic means that the distribution does not depend on the azimuthal angle \(\phi\). In constructing the distribution function for a given value of \(z\) we include all particles in 2 cell spatial \(z\) region starting with that value of \(z\) and the full 32 cell \(x\) region. Thus a given distribution is calculated with all of the particle in 2 \(\times\) 32 cell region. Figure 3-2a corresponds to a position \(z = 84\) near the shock front and 614 particles are used to construct the distribution functions. In this front region most particles have not yet been disturbed by the shock and have a velocity of 2000 km/sec. In the corresponding plot of the angular distribution, these largely undisturbed particles show up as a spike at \(\phi = 0\). The periodic symmetry of the planar display \(\phi = 0 = 2\pi\) makes it appear as if there were two spikes. The situation actually corresponds to that shown in Figure 3-3a). Also in Figure 3-2a) we see that a small number of particles have been accelerated to velocities of order 3000
km/sec. Inspection of the angular distribution shows that these particles are grouped near $\phi = \pi$, $\theta = \pi/2$ and thus correspond to reflected particles. Figure 3-2b) corresponds to a z value two cells deeper into the shock region where 935 particles are contributing to the illustrated distribution. Since we are only interested in the change of the shape of the distribution functions, all of them have been normalized to the same ordinate value. At z = cell 88 we see that the distributions of velocity magnitudes have been further scattered with a net slowing below the initial velocity of 2000 km/sec. In Figure 3-2c), at cell z = 92 where 1897 particles are included, this slowing and scattering of the distribution of velocity magnitudes is even more prominent. In addition, we see that a strong angular scattering in $\phi$ has occurred. The shock turbulence is driving the angular distribution of the particles toward gyrotropy with respect to the direction of the magnetic field.

The spherical representation of the particle distribution function is not convenient for stability analyses involving an ambient magnetic field. We need to know the velocity distribution functions with respect to the magnetic field direction. Such distributions are depicted in Figure 3-4, where $f(v_r)$ and $f(v_t)$ are shown with subscripts referring to the direction of the magnetic field. Again the distributions are depicted at different positions within the shock. They are calculated in the same manner as the spherical distribution function and are normalized to the same ordinate value. At z = 84, the front of the shock, almost all particles have velocity $v_r = 2000$ km/sec, $v_t = 0$. On considering the distribution function at values of z = 94, 104, and 112 cells, we see that the $v_r$ distribution has relaxed and that the $v_t$ distribution has developed a Maxwellian form. From the standpoint of developing a kinetic theory of the plasma instabilities responsible for these observed velocity space distributions, it is important to note that is that the $v_r$ distribution retains a "shell-like" form well into the shock. Thus, at least for the case of dynamically produced collisionless shocks such as the one simulated here, it is not appropriate to model shock instabilities with a simple bimaxwellian velocity distribution such as is used in the bulk of analyses of thermalizing instabilities in collisionless shocks.

**Summary of observations:** We have simulated a collisionless shock wave typical of HANE conditions and observed its interaction with particles. We saw that particles are strongly accelerated when they encounter the shock, with some fraction of them reflected back from the shock front. As one proceeds deeper and deeper into the shocked plasma, the particular distribution functions were observed to undergo a two stage relaxation process. Over a relatively short distance, the particles are strongly scattered in the plane perpendicular to the magnetic field and the distribution function becomes gyrotropic. Over a longer distance scale, the velocity components perpendicular and parallel to the magnetic field also relax. The parallel distribution, originally a spike at $v_t = 0$, becomes Maxwellian as perpendicular energy is scattered into parallel energy. The perpendicular distribution, originally a beam with $v_r = v_{r0}$ relaxes strongly but retains a distinct shell-like character.
Figure 3-1. Individual particle positions in 2 cell x 20 cell region.
Figure 3-2. Spherical representation of velocity distribution function at three points successive deeper into the shocked plasma. A rapid evolution of particle velocity distribution from beam to gyrotropic form is observed.
Figure 3-3. Spherical perspective plots showing what is meant by evolution from beam-like to gyrotropic distribution.
Figure 3-4. Longer time scale evolution of beam-like velocity distribution to form which is Maxwellian in $v_i$ and shell-like in $v_\perp$. Shown is a sequence of distributions that are successively deeper into the shocked plasma.
SECTION 4
VELOCITY DISTRIBUTION FUNCTION FOR
PARTICLES ENCOUNTERING COLLISIONLESS SHOCK

To model the shell-like $v_\perp$ distribution observed in the simulation of a collisionless shock we choose
the functional form

\[ f(v_\perp, v_\parallel) \sim \exp[-V_\perp^2] \exp[-(V_\perp - V_0)^2] \] (4.1)

where $V_\perp = v_\perp/v_\perp, V_\parallel = v_\parallel/v_\parallel, V_0 = v_\parallel/v_\perp$. Here the parameter $v_\parallel$ characterizes the shell-like form of
the distribution, while $v_\perp$ and $v_\parallel$ model the thermal spread in the directions perpendicular and parallel
to the magnetic field. When this distribution is normalized, $2 \pi \int v_\perp dv_\perp f(v_\perp, v_\parallel) = 1$, the
perpendicular part can be separated out to illustrate the normalization associated with the shell-like distribution

\[ f(v_\perp) = \frac{2\exp[-(V_\perp - V_0)^2]}{V_0^2 \exp[-V_0^2] + \sqrt{\pi} V_0 (1 + \text{Erf}(V_0))} \] (4.2)

This is characterized by the two parameters $V_0$ and $v_\perp$. To demonstrate that this is a reasonable
choice for representing the shell-like $v_\perp$ equilibrium we show, in Figure 4.1, the result of fitting the
distribution (2) to the observed $f(v_\perp)$ distribution at $z = 96$ and 104 cells. The fitting was achieved
with a nonlinear least-squares method using $v_\parallel$ and $v_\perp$ as parameters. Specifically, both distributions
were normalized and then a function $\text{Err}(v_\parallel, v_\perp) = \sum [f(v_\parallel, v_\perp) - f_{\text{obs}}(v_\parallel)]^2$ was constructed to
measure the error between the analytical distribution and the observed distribution. The sum is over
available $v_\perp$ data points. $\text{Err}(v_\parallel, v_\perp)$ was then numerically minimized with respect to its parameters
with the minimizing values of $v_\parallel$ and $v_\perp$ providing the best fits to the data. Notice that the
distribution in Figure 4.1b), which is measured some 8 cells further into the shocked plasma than the
distribution in Figure 4.1a), has relaxed its shell-like character ($v_\parallel$ is less) and is more thermal ($v_\perp$
is greater).

**Moments:** For purposes of reference we calculate some moments of this distribution. The
perpendicular speed moment is given by

\[ \langle V_\perp f \rangle = \frac{2V_0 + \sqrt{\pi} \exp(V_0^2)(1 + 2V_0^2)(1 + \text{Erf}(V_0))}{2[1 + \sqrt{\pi} \exp(V_0^2)V_0(1 + \text{Erf}(V_0))]} \] (4.3)

where $\langle \chi f \rangle = 2 \pi \int v_\perp dv_\perp f(v_\perp, v_\parallel)$. The perpendicular temperature moment is given by
\[ \langle V_1^2 \rangle = \frac{2(1 + V_0^2) + \sqrt{\pi} \exp(V_0^2)V_0(3 + 2V_0^2)(1 + \sqrt{\pi} \exp(V_0^2)V_0(1 + \text{erf}(V_0)))}{2[1 + \sqrt{\pi} \exp(V_0^2)V_0(1 + \text{erf}(V_0))]} \] (4.4)

These moments collapse back to the expected Maxwellian forms when \( V_0 \to 0 \).
Figure 4-1. Example of nonlinear least-squares fits of analytic $v_\perp$ velocity distributions with observed distributions at two positions successively deeper into the shock.
SECTION 5
SUMMARY AND CONCLUSIONS

A two-dimensional hybrid particle-in-cell simulation has been used to study the velocity distribution functions of a plasma shocked by collisionless shock wave typical of a HANE expansion. Such distribution functions are fundamentally important in ascertaining the dispersive and growth properties of plasma instabilities that regulate the flow of energy away from a HANE. Previously, these instabilities have been primarily studied for the bi-Maxwellian velocity distributions that are thought to be typical of the steady state collisionless bow shock formed where the solar wind encounters the Earth's magnetic field. The collisionless shock formed by a HANE, on the other hand, is dynamic and would be expected to lead to velocity distributions more severely distorted than the bimaxwellians on which most stability analysis is based.

In the simulation, it was observed that background particles are strongly accelerated on encountering the shock front. Because of the irregular nature of the shock front, the acceleration does not take the form of a specular reflection often associated with collisionless shocks based on 1-D analysis and simulation (Winske and Quest, 1988). Rather the acceleration occurs in various directions with respect to the shock front. Some of the belief that particles should specularly reflect from a collisionless shock stems from their study with 1-D simulations in which the spatial structure in directions perpendicular to shock propagation are not considered. By examining the velocity distribution with respect to the distance within the shock, we can study the evolution of the initially beam-like velocity distribution (with respect to the shock front). It was found that the most rapid evolution of the distribution was a rapid relaxation to a gyrotropic form, i.e., a form that was independent with respect to angle about the magnetic field. Over a somewhat longer distance scale, the distribution began to develop a Maxwellian form with respect to velocities parallel to the magnetic field.

The most significant finding of this study was that the velocity distribution perpendicular to the magnetic field did not have a Maxwellian form but maintains a distinct shell-like form with respect to $v_x$. We found that the observed distribution could be fitted reasonably well (nonlinear $\chi^2$) with the functional form $f_0(v_x) \sim \exp[-(v_x - v_0)^2]$.

In a companion report [Gladd and Brecht, 1994], we examine the implications of such a shell-like distribution function for the mirror mode and Alfvén ion cyclotron instabilities that are considered to be the most important mechanisms for isotropizing plasma energy. For bimaxwellian velocity distributions, the Alfvén ion cyclotron is known to be more important than the mirror mode. The principle finding in Gladd and Brecht, 1994, is that, despite the shell-like velocity distribution, the Alfvén ion cyclotron instability still dominates the mirror mode instability in thermalizing the anisotropic plasma produced by a collisionless shock.
SECTION 6
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