FINAL TECHNICAL REPORT

ON

COMPUTATIONAL METHODS FOR CONTROL AND
OPTIMAL DESIGN OF AEROSPACE SYSTEMS

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by

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This final technical report contains a summary of the research funded by AFOSR under Grant F49620-92-J-0078, titled "Computational Methods for Control and Optimal Design of Aerospace Systems", for the period 1 January 1992 to 31 March 1994. During this period, the investigators concentrated on five problems; (i) optimal design of forebody simulators, (ii) control of highly maneuverable aircraft, (iii) control of fluid flows, (iv) modeling and identification of smart materials, and (v) robust control of nonlinear conservation laws. A new sensitivity equation method was developed for shape optimization and flow tailoring. This method was transitioned into software at the Arnold Engineering Development Center and at Virginia Tech. A new mathematical model for aircraft agility was used to investigate feedback control laws for time optimal maneuvers. New vortex shedding patterns were discovered for unsteady flows about rotating cylinders. The work on smart materials produced new models and computational methods for the dynamics of shape memory alloys. In addition, robust control theory was applied to nonlinear conservation laws yielding a new approach to sensor location. The report contains a complete list of papers produced during this period.
1. INTRODUCTION AND OBJECTIVES

This research project was a continuation of an ongoing research program devoted to the development of practical computational methods for identification, control and optimal design of aerospace systems governed by partial differential equations. The effort concentrated on several problems involving the development and analysis of computational algorithms for optimal design and control of differential equations that model fluid/structure interactions, smart materials and highly maneuverable aircraft. The research had the following specific objectives:

1. To develop and analyze mathematical models for identification, control and optimal design of fluid, material and structural systems. The primary goal of this effort was the development of more realistic models of smart materials used in shape memory actuators and the analysis of models used in aircraft agility studies.

2. To develop approximation schemes needed in the construction of finite dimensional design models for use in conjunction with finite dimensional design models.

3. To conduct numerical experiments testing the algorithms developed in this effort.

4. To work with Air Force Facilities and industry to transition this research and to use Air Force problems as test models for computational experiments.

Applications played a central role in the research program. These applications include the control of fluid flows, the modeling and control of materials and structures that exhibit nonlinear behavior, optimal aerodynamic design and supermaneuverable aircraft. These applications were used to motivate the basic research, as test cases for computational experiments and as mechanisms to transition the research into Air Force facilities and industry.
2. ACCOMPLISHMENTS

Considerable progress was made towards achieving the project objectives. In addition to the development of computational methods, basic theoretical issues were addressed and fundamental understanding of these systems were obtained through a combined computational and theoretical analysis. This work established the feasibility of the SEM method as a practical design tool, produced software for various industrial and Air Force applications and demonstrated several new techniques for control design and optimization.

Summary of Accomplishments

During this period we made significant progress on the development of computational methods for control and optimization of systems governed by partial differential equations. In particular, we have:

- Produced more than 29 research papers and reports
- Supported 6 graduate students and 3 postdoctoral researcher
- Produced 2 Ph.D. students that held postdoctoral positions at Princeton University and at the Institute for Mathematics and its Applications, University of Minnesota.
- Given more than 32 scientific lectures on research supported under this grant
- Expanded our optimal design effort with scientists and engineers at the Air Force Arnold Engineering Development Center (AEDC) and initiated joint efforts with Tektronix, CALSPAN and BEAM Engineering.
- Developed a new computational approach to optimal design
- Identified some fundamental new issues in the area of optimal design
- Developed new mathematical models (body-rate-model) that are less complex than full rigid-body models and yet suitable for use in the study of flight maneuvers that exhibit aircraft 'agility'.
- Applied robust control to nonlinear conservation laws. A new approach to sensor location was developed and tested on boundary control of Burgers' equation.
Research Highlights

The following projects were considered during this period:

The Optimal Forebody Design Problem - The Air Force Arnold Engineering Development Center (AEDC) is responsible for the development of facilities and tools for testing integrated propulsion systems (i.e. engines, inlets and airframes). Since many full-scale propulsion systems are too large for the largest ground facility, the AEDC investigated a free-jet test technique for use in the Aeropropulsion System Test Facility (ASTF). This technique permits full-scale engines to be tested with real inlets and forebody simulators (FBS). A forebody simulator is a physical device attached to the engine. The purpose of the FBS is to produce a flow field at the inlet that approximates the actual aircraft forebody effects in flight. In a free-jet test, the engine, inlet and forebody simulator are placed in a test section and a free-jet nozzle is employed to control the flow over the test article in order to simulate angle-of-attack and sideslip effects. The goal of this test configuration is to produce (i.e. control) the flow field at the engine inlet that is approximately the same flow field produced by the total aircraft in free flight. A complete description of this problem and the AEDC program may be found in [2] and [3] and the references therein.

One needs to design an FBS that produces the inlet flow of the real aircraft to be tested. The shape of the FBS is to be determined so as to minimize the "distance" between the flow generated by the FBS and the desired flow conditions. There are several possible approaches to this problem, but initially AEDC considered two basic methods for attacking this type of "shape optimization" problem. The first is a trial-and-error approach and is done using empirical design techniques. The second approach is based on "black-box" optimization methods. In particular, it is a method that combines numerical optimization with computational fluid dynamics to produce an optimal design capability. The joint ASD/AEDC Aeropropulsion System Test Facility Free-Jet Development Technical Steering Committee proposed that such an optimization-based design technique be developed to solve the FBS design problem.

Initially the FBS optimization based design program combined the output of a CFD code (PARC3D or PARC2D) developed at AEDC with existing Gauss-Newton or quasi-Newton algorithms to produce sub-optimal designs. Gradients were computed by finite-differences. This procedure requires that the CFD code be "fully converged" in order to obtain "accurate" gradients. Consequently, the computational work required for each gradient evaluation is extremely large and the number of control and design variables must be kept small. This was an inhibiting factor in the development of a more practical optimization based design capability. Better methods were needed to compute sensitivities for the optimization loops. Existing techniques (based on finite differences) produced inaccurate sensitivities and were computationally expensive. In the papers [7] and [13] we developed a Sensitivity Equation Method (SEM) approach to this problem. This approach allows us to compute the sensitivities without using finite differences. An optimal design algorithm based on the SEM combined with a trust region optimization
algorithm was used to test this approach on a 2D version of the optimal forebody design problem. We observed faster and more robust convergence for this computational scheme and we are continuing to refine this idea. In addition, this approach seems to improve as the number of design parameters increase due to vectorization and parallelism.

This method is currently being transitioned into AEDC software. We have adapted the PARC2D code and it now contains a sensitivity module based on the SEM. Also, AEDC engineers have made considerable progress in adapting the PARC code so that the sensitivities can be computed by this new technique. Moreover, we initiated a new project with the CALSPAN Company (at AEDC) that has as its goal the use the SEM in their CFD codes. These projects are just now getting started and we hope to continue our efforts on these projects under the support of the new Air Force Center for Optimal Design and Control (CODAC). Similar efforts are getting underway with Tektronix, Aerosoft and Beam Engineering.

This research produced a new understanding of the role that dissipation plays in optimization based design. Contrary to intuition, the use of "shock capturing" schemes in optimal design problems with shocks can produce inaccurate and, in some cases, incorrect optimal designs. The use of such schemes can produce artificial local minima that causes the optimization algorithm to become trapped and unable to find the global minimum. This phenomenon is clearly shown in the paper [7].

As a result of this effort we now know that in black-box approaches to optimal design, the combination of simulation and optimization packages must be done carefully. Certain "shock capturing" schemes that are excellent "simulators" can produce inaccurate and, in some cases, incorrect designs when applied to optimal design problems with shocks. Numerical and artificial viscosity will often eliminate these artificial local minima and hence the optimization algorithm will perform better. However, there is a price to be paid in terms of accuracy and speed. Therefore, in order to avoid such problems we developed a new hybrid scheme that uses dissipation as a control to steer an optimization algorithm to a neighborhood of a global minimum and then switches to a high order shock capturing scheme to refine the optimal design. Other approaches to this problem should be pursued.

In particular we have shown [26] that the usual one-dimensional flow model with area change leads to a singular optimal control problem wherein the area change (the control variable) is alternately at its bounds or singular. For an interesting class of boundary conditions the control is saturated at the inflow boundary, the outflow boundary and around the shock, while it is singular in the interior regions of the nozzle.

**Control of Highly Maneuverable Aircraft** - One aspect that needed to be addressed when considering control of highly maneuverable aircraft was the rigid body control problem. In [3-6] optimal rigid body motions were investigated for a model including propulsive control moments. These results have application to control of aircraft in low dynamic pressure regime at high angles of attack where the usual aerodynamic control
surfaces are ineffective for generating control moments. These features have been incorporated in several technology demonstration flight programs (NASA's F-18 based High angle of Attack Research Vehicle and DARPA's X-31) and in the Air Force's Advanced Tactical Fighter (ATF) (Lockheed F-22). Since these capabilities are new, it is not reasonable to suppose that pilots will intuitively know how to best use them. Thus, studies of mathematically optimal flight trajectories are useful both in assessing the utility of these new propulsive features and in providing information for pilots.

Beyond these fuselage-pointing studies we have also investigated maneuvers for optimal control of the velocity vector. In [21] we proposed a mathematical model that is mid-way in complexity between traditional point-mass models and full rigid-body models. These body-rate-model has been used to study flight maneuvers that exhibit aircraft 'agility'. Some additional aspects of trajectory optimization are reported in [20].

Much of the work on distributed-parameter control has focused on extensions of the finite-dimensional theory of linear-quadratic control. While the quadratic performance index is useful in stabilization problems, such quadratic measures of performance are not commonly true measures of system performance. In many military cases one is interested in rapid response, that is, in time-optimal maneuvers.

In one formulation of these problems one imposes hard bounds on the control variables so that if the initial state error is increased, the control limits are fixed - that is they 'saturate' uncompromisingly. In other formulations one augments the performance index (time) with a quadratic measure of control usage so that increases in the initial state error produce a compromise in increased time and increased control. In [1-2] we continue earlier work and study a family of feedback laws in which the designer can 'trade-off' time and control saturation.

Control of Fluid Flows - Problems of control of external fluid flows are highly non-linear, and even the construction of open loop simulations is extremely complex. Our project with Professor D. Pelletier (Ecole Polytechnique de Montreal) involved the use of Dr. Pelletier's adaptive grid code in conjunction with an optimization code to investigate flow separation control. This work focused on a simple airfoil with a leading-edge control. The control (a rotating cylinder) can have significant effect on flow properties at angles-of-attack beyond stall for the uncontrolled surface. This research was done as part of Ms. S. Dufresne's Masters Thesis ("Optimization of an Airfoil's Performance through Moving Boundary Control", M.S. Thesis, Aerospace and Ocean Engineering Dept., VPI & SU, Blacksburg, VA, May 1993).

As a first step in the development of computational schemes for designing optimal control laws for various fluid flow systems, we used a central difference/pseudospectral method to compute (for the first time) optimal "lift-to-drag controllers" for a rotating cylinder [23, 24, 25]. Moreover, this work lead to the discovery of new vortex shedding patterns for flow about rotating cylinders and is reported in [13]. Other objectives were studied including vortex suppression, wake modification and synchronization of cylinder
and wake. Direct numerical simulations demonstrated that vortex shedding can be controlled or completely suppressed at certain Reynolds numbers.

In addition to numerical studies, we have begun to formulate a theoretical framework based on distributed parameter control. We have used linearization to find feedback control laws and the current effort is focused on computational methods and providing a rigorous theory. Preliminary results are reported in [14].

**Modeling and Identification of Smart Materials** - This project was concerned with the development of computational methods and new mathematical models of smart materials being used as control actuators and sensors. The goal of this effort was to develop models that capture the physics of these materials and that can be used in control and optimal design of fluid/structure systems. As a first step in this direction, it is essential to determine the which model parameters are the most sensitive to perturbations. This effort produced three specific accomplishments.

We developed a new mathematical model of materials with memory that for the first time captures the complete hysteresis loop as observed in experiments and at the same time is simple enough to be used in control design. This model is based on a simple parameterization of the hysteresis loop. Also, a finite element model was developed and tested on a Landau-Ginzburg model of phase transition. These results are summarized in [16].

Although some results on the well-posedness of the Landau-Ginzburg models of phase transition had been given in the literature, these results did not apply to the physically important case of zero thermal memory. This case is important because assuming non-zero thermal memory leads to a model that does not satisfy the second law of thermodynamics. We established the basic existence for the problem with zero thermal memory and developed a numerical algorithm to approximate the equations. Initial results are given in Ruben Spies' Ph.D. Thesis and papers [15] and[17].

In order to gather data for use in identifying model parameters from dynamic experiments, it is important to know the sensitivity of the solutions to these parameters. In [17] we investigated this problem for the Landau-Ginzburg model of shape memory alloys. As a result of these numerical experiments it was discovered that the coefficient for the cross terms (coupling temperature and strain in the Landau-Ginzburg potential) often used in theoretical studies was probably too large and needed to be estimated from real data.

**Robust Control of Nonlinear Conservation Laws** - Practical feedback controllers for nonlinear partial differential equations are needed in order to fully address problems in flow control. It is certainly important that these controllers be robust and, at the same time, the controllers must be low dimensional in order to implement on digital processors. Since it is not always possible to sense all states (an infinite dimensional sensor) one must resort to dynamic compensation. The use of observers to eliminate the need to "sense" the
entire flow in feedback control of fluid dynamic systems is one area that has not received enough attention. As a first step in this direction, we investigated this problem for a 1D conservation law (Burgers' equation) with boundary control and sensing. Burgers' equation has many of the characteristics found in Navier-Stokes equations and has been used as a test model in CFD for sometime now. We tested various controller reduction techniques on this quasilinear system. The first method we studied was the optimal projection method (Bernstein) and then we developed a new method based on approximating optimal functional gains.

Both approaches first constructed a linear feedback control law and combined this law with a low order (nonlinear) state estimator to produce the controller. The optimal projection method worked very well on the full nonlinear problem with a remarkably low order (of dimension 4) controller (see [12] and Marrekhi's Ph.D. Thesis). However, this approach required that the designer first select the controller dimension and then solve a complex system of nonlinear equations. When the controller dimension was too low (less than 4), the controller failed to stabilize the nonlinear plant and there seemed to be no theoretical results to guide the choice of controller dimension.

The second approach was based on distributed parameter robust control theory (MINMAX design). As before, a linearized plant was used to design a robust (linear) control law for the full infinite dimensional system. Distributed parameter control theory provided representations of the robust controllers as integral operators with so called functional gains as kernels. Having these functional gains allowed us to make rigorous approximations to produce reduced order controllers. Moreover, explicit information about the functional gains were used to "optimally" locate new sensors hence enhanced control effectiveness. This approach allowed us to construct a 4th order controller with almost the same performance as the optimal projection method and, at the same time, could be used to guide not only the size of the controller but the location of sensors. In order to apply this approach one must have specific integral representations of solutions to Riccati equations. In general, this is a extremely difficult task (a form of the Schwartz Kernel Theorem). However, for specific PDE models it is possible to obtain such results and a preliminary investigation into this problem is reported in [11].
3. RESEARCH PUBLICATIONS

The following papers were produced under this grant:


4. PROFESSIONAL PERSONNEL

Principal Investigators

J. A. Burns and E. M. Cliff

Postdoctoral Researchers

M. Bikdash, H. Marrechki, R. Y. Ou and M. Tadi

Graduate Students

J. Borggaard, J. Burkhardt and S. Dufresne
H. Marrechki, D. Rubio, R. Spies, and B. Thompson

Advanced Degrees Awarded

R. D. Spies -- Ph.D. 1992

S. Dufresne -- M.S. 1993

H. Marrekchi -- Ph.D. 1993

D. Rubio -- M.S. 1994 (Non Thesis)
5. INTERACTIONS

Talks and Lectures

J.A. Burns

7. Rice University, Houston, TX, January, 1992
8. Texas Tech University, Lubbock, TX, September, 1992
9. ICASE, Hampton, VA, November, 1992
10. IEEE Conference on Decision and Control, Tucson, AZ, December 1992
11. Southeastern Regional Meeting of the AMS, Knoxville, TN, March, 1993
12. NSF Conference on Graduate Education in the Mathematical Sciences, Clemson, SC, April, 1993
13. Air Force Meeting on Control, Ann Arbor, MI, May, 1993
14. IEEE Conference New Directions in Control, Crete, Greece, June, 1993
15. AIAA Shear Flow Control Conference, Orlando, FL, July, 1993
17. Sixth International Conference on Distributed Parameter Control, Vorau, Austria, July 1993
18. International Conference on Mathematical Theory of Networks and Systems, Regensburg, Germany, July 1993
19. SIAM Conference on Linear Algebra, Systems and Control, Seattle, WA, August, 1993
21. Tektronix Printing and Ink Division, Wilsonville, OR, January, 1994
E.M. Cliff
1. Fifth Workshop on Control Mechanics, in Honor of the Seventieth Birthday of Angelo Miele, University of Southern California, January 1992
3. Aircraft Guidance and Control Branch, NASA Langley Research Center, October 1992
4. 31st IEEE Conf. on Decision and Control, Tucson, AZ, December 1992
5. Aircraft Guidance and Control Branch, NASA Langley Research Center, March 1994

Y.R. Ou
1. First World Congress on Nonlinear Analysis, Tampa, FL, August, 1992
2. IMA Workshop on Fluid Flow Control, University of Minnesota, Minneapolis, MN, November, 1992
3. Naval Command, Control and Ocean Surveillance Center, San Diego, CA, December, 1992
Laboratory and Industrial Interactions

In addition to the research described above, we had the following interactions with Air Force personnel and industries.

Air Force Arnold Engineering Development Center, Tulahoma, TN
  Captain Scott Tennent
  Captain Mark Briski

Sverdrup Company, Tulahoma, TN
  Mr. Donald Todd
  Mr. Karl Kneile

CALSPAN Company, Tulahoma, TN
  Dr. John Benek
  Dr. Peter Hoffman
  Dr. Steve Keeling

Phillips Laboratory
  Dr. Alan Weston
  Dr. Alok Das

Tektronix
  Dr. Ron Burr
  Ms. Sharon Berger

Beam Engineering
  Dr. Gal Berkooz